Prospects for the computation of rare kaon-decay amplitudes

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New Horizons for Lattice Computations with Chiral Fermions
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Motivation and Introduction

- Lattice simulations are contributing to a wide variety of fundamental issues in particle physics and increasingly to nuclear physics.
- For me personally, a major motivation is the important complementarity of high-energy experiments (most notably the LHC) and precision flavour physics in discovering and unraveling the next layer of fundamental physics.
    - If, as expected/hoped the LHC experiments discover new elementary particles, then precision flavour physics will be necessary to understand the underlying framework.
    - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
    - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.
- For many years we have been calculating matrix elements of the forms:

$$\begin{aligned} \langle 0\,|\,O(0)\,|\,h\rangle &\to f_P\,,\ \text{for example} \\ \langle h_2\,|\,O(0)\,|\,h_1\rangle &\to B_P,\ \text{semileptonic form factors}, \cdots . \end{aligned}$$

$\Rightarrow$ unitarity triangle tests of the SM, determination of CKM matrix elements etc.

- Recent RBC-UKQCD extensions to the above include the evaluation of $K \to \pi\pi$ decay amplitudes. arXiv:1106.2714, arXiv:1111.1699
- More recently we have began to consider long-distance contributions to physical quantities. These are not given in terms of matrix elements of local operators, but require the evaluation for example of:

$$\int d^4x \int d^4y \, \langle h_2 \,|\, T\{O_1(x)\, O_2(y)\} \,|\, h_1 \rangle \,.$$

- The most advanced of our projects is on the evaluation of long-distance contributions to the $K_L - K_S$ mass difference.

Jianglei Yu, arXiv:1111.6953; paper in preparation.

$$\int d^4x \int d^4y \, \langle \bar{K}^0 \,|\, T\{H_W(x)\, H_W(y)\} \,|\, K^0 \rangle \,.$$

- Here instead, I will present some preliminary thoughts about the rare kaon decays $K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$:

$$\int d^4x\, e^{-iq\cdot x} \int d^4y \, \langle \pi \,|\, T\{J^\mu(x)\, H_W(y)\} \,|\, K^0 \rangle \,.$$

- Up to now, the main theoretical tool for these processes has been Chiral Perturbation Theory with its many limitations and uncertainties.

Example: $K_L \to \pi^0 \ell^+ \ell^-$

F.Mescia, C,Smith, S.Trine hep-ph/0606081

- Rare kaon decays which are dominated by short-distance FCNC processes, $K \to \nu\bar{\nu}$ in particular, provide a potentially valuable window on new physics at high-energy scales.
- The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are also considered promising because the long-distance effects are reasonably under control using ChPT.
  - They are sensitive to different combinations of short-distance FCNC effects and hence in principle provide additional discrimination to the neutrino modes.
  - A challenge for the lattice community is therefore either to calculate the long-distance effects reliably or at least to determine the Low Energy Constants of ChPT.

$K_L \to \pi^0 \ell^+ \ell^-$ cont.

There are three main contributions to the amplitude:

1. Short distance contributions: F.Mescia, C,Smith, S.Trine hep-ph/0606081

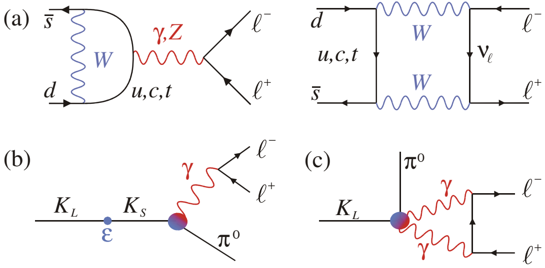
$$H_{\rm eff} = -\frac{G_F \alpha}{\sqrt{2}} V_{ts}^* V_{td} \{ y_{7V} (\bar{s}\gamma_\mu d)\,(\bar{\ell}\gamma^\mu \ell) + y_{7A} (\bar{s}\gamma_\mu d)\,(\bar{\ell}\gamma^\mu \gamma_5 \ell)\} + \text{h.c.}$$

   - Direct CP-violating contribution.
   - In BSM theories other effective interactions are possible.

2. Long-distance indirect CP-violating contribution

$$A_{ICPV}(K_L \to \pi^0 \ell^+ \ell^-) = \varepsilon A(K_1 \to \pi^0 \ell^+ \ell^-)\,.$$

3. The two-photon CP-conserving contribution $K_L \to \pi^0 (\gamma^* \gamma^* \to \ell^+ \ell^-)$.



- The current phenomenological status for the SM predictions is nicely summarised by: V.Cirigliano et al., arXiv1107.6001

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right\}$$

  - $\lambda_t = V_{td} V_{ts}^*$ and Im $\lambda_t \simeq 1.35 \times 10^{-4}$.
  - $|a_S|$, the amplitude for $K_S \to \pi^0 \ell^+ \ell^-$ at $q^2 = 0$ as defined below, is expected to be $O(1)$ but the sign of a_S is unknown. $|a_S| = 1.06^{+0.26}_{-0.21}$.
  - For $\ell = e$ the two-photon contribution is negligible.
  - Taking the positive sign (?) the prediction is

$$\begin{aligned} \mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} &= (3.1 \pm 0.9) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} &= (1.4 \pm 0.5) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPC}} &= (5.2 \pm 1.6) \times 10^{-12}\,. \end{aligned}$$

  - The current experimental limits (KTeV) are:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \quad \text{and} \quad \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}\,.$$

G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026

- We now turn to the CPC decays $K_S \to \pi^0\ell^+\ell^-$ and $K^+ \to \pi^+\ell^+\ell^-$ and consider

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\,\mathrm{T}\{J^\mu_{\mathrm{em}}(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

  where Q_i is an operator from the effective Hamiltonian.
- Gauge invariance implies that

$$T_i^\mu = \frac{\omega_i(q^2)}{(4\pi)^2}\left\{q^2(p+k)^\mu - (m_K^2 - m_\pi^2)\,q^\mu\right\}.$$

- Within ChPT the Low energy constants a_+ and a_S are defined by

$$a = \frac{1}{\sqrt{2}}V_{us}^*V_{ud}\left\{C_1\omega_1(0) + C_2\omega_2(0) + \frac{2N}{\sin^2\theta_W}f_+(0)C_{7V}\right\}$$

  where $Q_{1,2}$ are the two current-current GIM subtracted operators and the C_i are the Wilson coefficients. (C_{7V} is proportional to y_{7V} above).

G.Ambosio, G.Ecker, G.Isidori and J.Portoles, hep-ph/9808289

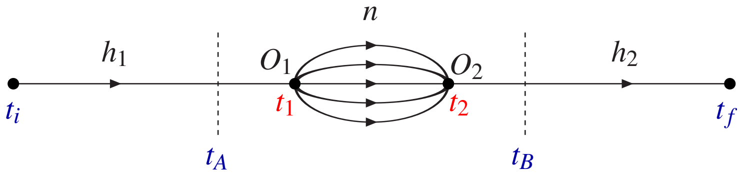
- Phenomenological values: $a_+ = -0.578 \pm 0.016$ and $|a_S| = 1.06^{+0.26}_{-0.21}$.

Can we do better in lattice simulations?

- Ideas developed (or being developed) from our ΔM_K project.
- How do you prepare the states $h_{1,2}$ in

$$\int d^4x \int d^4y \,\langle h_2 \,|\, T\{O_1(x)\, O_2(y)\} \,|\, h_1\rangle \,,$$

  when the time of the operators is integrated.
- The practical solution is to integrate over a large subinterval in time $t_A \leq t_{x,y} \leq t_B$, but to create h_1 and to annihilate h_2 well outside of this region:



- This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time.
- This approach has been successfully implemented in the ΔM_K project.

N.Christ arXiv:1012.6034; Jianglei Yu arXiv:1111.6953; paper in preparation

- The generic non-local matrix elements which we wish to evaluate is

$$\begin{aligned} X &\equiv \int_{-\infty}^{\infty} dt_x\, d^3x\, \langle \pi(p)\,|\,\mathrm{T}\,[\,J(0)\,H(x)\,]\,|K\rangle \\ &= i\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{m_K - E_n + i\varepsilon} - i\sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi + i\varepsilon}\,, \end{aligned}$$

  - $\{|n\rangle\}$ and $\{|n_s\rangle\}$ represent complete sets of non-strange and strange sets.
- In Euclidean space we envisage calculating correlation functions of the form

$$C \equiv \int_{-T_a}^{T_b} dt_x\, \langle \phi_\pi(\vec{p}, t_\pi)\,\mathrm{T}\,[\,J(0)\,H(t_x)\,]\,\phi_K^\dagger(t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-m_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,,$$

where

$$\begin{aligned} X_{E_-} &= -\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{m_K - E_n}\left(1 - e^{(m_K - E_n)T_a}\right) \quad \text{and} \\ X_{E_+} &= \sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s} - E_\pi)T_b}\right). \end{aligned}$$

Removing the single-pion intermediate state

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- Chiral ward identities imply that we can add a term proportional to the scalar density $\bar{s}d$ to the Hamiltonian without changing physical results. We can therefore subtract the single pion intermediate state by imposing $\langle\pi|H + c_S\,\bar{s}d|K\rangle = 0$.
- It is instructive to see how this works in the present case at lowest order in chiral perturbation theory. The scalar density in the effective theory can be written as

$$S^{sd} = \mathrm{Tr}\left[\lambda^{sd}\left(\Sigma+\Sigma^\dagger\right)\right] \quad \text{where} \quad \lambda^{sd} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

- The em current is of the form

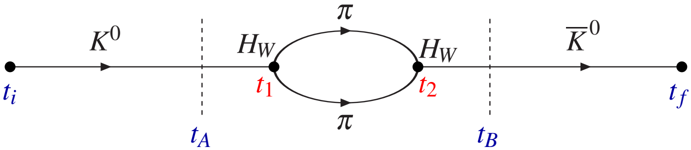
$$J^\mu = i\frac{f^2}{4}\,\mathrm{Tr}\left[Q\left(\Sigma\partial^\mu\Sigma^\dagger + \Sigma\partial^\mu\Sigma\right)\right]$$

- The c_S term leads to additional diagrams:

which are proportional to

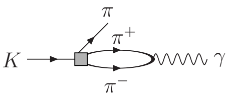
$$\frac{(p_\pi+p_K)^\mu}{p_K^2-m_\pi^2} + \frac{(p_\pi+p_K)^\mu}{p_\pi^2-m_K^2}\,.$$

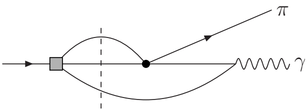
On shell, when $p_K^2 = m_K^2$ and $p_\pi^2 = m_\pi^2$, the sum of the two terms indeed gives zero.



- In the ΔM_K computation, there is, of course, a two-pion intermediate state and we have had to control the corresponding finite-volume effects.
- This has been done on the assumption that the dominant intermediate states below m_K are the two-pion states.

- We have seen that we can remove the single pion intermediate state.
- Which intermediate states contribute?
  - Are there any states below M_K?
  - We can control q^2 and stay below the two-pion threshold.





  - Are there two-pion intermediate states as a result of the Wess-Zumino term?
  - Do we need to consider three-pion intermediate states?
  - Answers to the above questions will affect what the finite-volume corrections are?
  - The ChPT-based phemomenology community neglect such possibilities.

All to be investigated further!

- It looks as though the FV corrections are much simpler than for ΔM_K and may be exponentially small?

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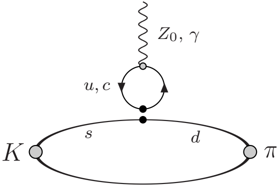
$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\,\langle \pi(p)\,|\,\mathrm{T}\{J^\mu(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

- Each of the two local Q_i operators can be normalized in the standard way and J can be normalized.
- Calculation of long-distance effects $\Rightarrow$ must treat additional divergences as $x \to 0$.

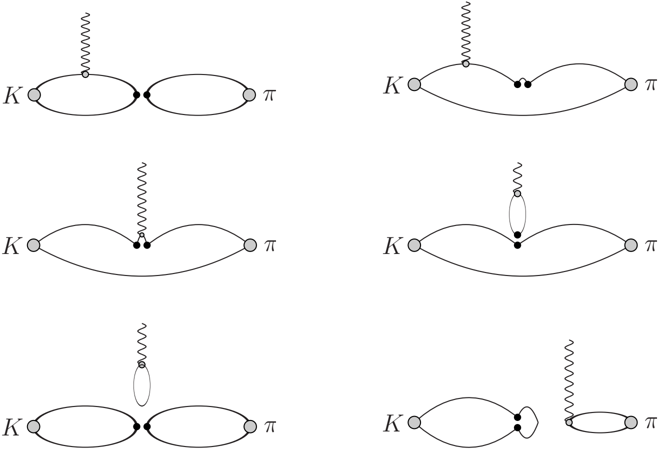


- Quadratic divergence is absent by gauge invariance $\Rightarrow$ Logarithmic divergence.
  - Checked explicitly for Wilson and Clover at one-loop order.

    G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - Absence of power divergences does not require GIM.
  - Logarithmic divergence cancelled by GIM.
  - For DWF the same applies for the axial current.
- Control of short-distance effects also appears to be much simpler than for ΔM_K.

To be investigated further!

- Sample diagrams:



+ lots more

- The last two diagrams are examples of *disconnected* diagrams.

Summary and Conclusions

- Our community must continue to strive to
  1. improve the precision of computations of quantities we know well how to compute;
  2. extend the range of quantities which can be computed.

  This is necessary of precision flavour physics is to play a complementary rôle to large $p_\perp$ experiments in exploring the limits of the standard model and unravelling the basic framework of new physics.
- I have reviewed one such possible future extension, that of the calculation of long-distance physics in rare kaon decays.
  - There are many similarities to our calculation of the long-distance contributions to ΔM_K which is further advanced.
  - Although much still remains to be done, the theoretical background appears to be simpler than for ΔM_K (both finite-volume effects and short distance subtractions).
  - The calculations will rely on progress in the computation of disconnected diagrams.
- There are many important rare B-decays which should be studied but
- I still don't know how to tackle nonleptonic B decays, even in principle.