Testing Grand Unification:

Upper Bound on the Proton Lifetime

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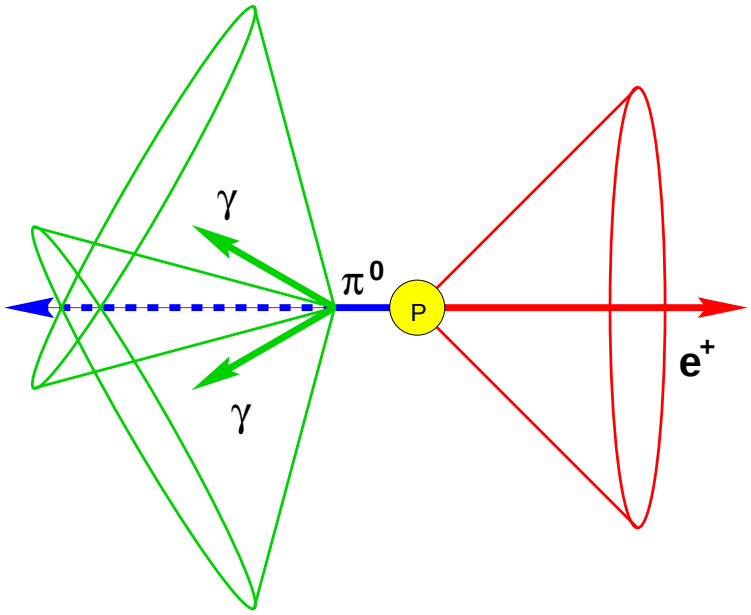

Underground Detectors Investigating Grand Unification (UDiG)
BNL, October 16, 2008

Aim

How could we test grand unification through proton decay ?

What is the upper bound on the proton lifetime in GUTs ?

References

- P. Fileviez Pérez, PLB 595 (2004) 476.
- B. Bajc, P. Fileviez Pérez, G. Senjanović, PRD 66 (2002) 075005; hep-ph/0210374.
- I. Dorsner, P. Fileviez Pérez, PLB 625 (2005) 88.
- I. Dorsner, P. Fileviez Pérez, NPB 723 (2005) 53.
- I. Dorsner, P. Fileviez Pérez, G. Rodrigo, PRD 75 (2007) 125007.
- P. Fileviez Pérez, PLB 654 (2007) 189.
- P. Fileviez Pérez, PRD 76 (2007) 071701.
- P. Fileviez Pérez, H. Iminniyaz, G. Rodrigo, PRD 78 (2008) 015013.

REVIEW:

- P. Nath, P. Fileviez Pérez, Physics Reports 441 (2007) 191.

Outline

- Introduction
- Baryon Number Violating Effective Operators
  - $d = 6$
  - $d = 5$
  - $d = 4$
- $d = 6$ Nucleon Decay in $SU(5)$ and $SO(10)$ Theories
  - Testing Realistic GUTs Through $p \to K^+\bar{\nu}$ and $p \to \pi^+\bar{\nu}$
- Upper Bound on the Total Proton Decay Lifetime
- Summary

Proton decay

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In the Standard Model the Baryon number is conserved, $U(1)_B$ is a global symmetry at the classical level. Therefore, the lightest baryon, the proton, should be stable!

Now if the Baryon number is broken, we have a new prediction:

the decay of the proton (Pati, Salam, 1973)

$$p \longrightarrow \text{meson} + \text{(anti)lepton}$$

it is a generic prediction of any grand unified theory ($SU(5)$, $SO(10)$, E_6,...)

Georgi, Glashow 1974; Georgi, 1975
Fritzsch, Minkowski, 1975

We have to look for several decay channels:

$$p \longrightarrow (K^+, \pi^+, \rho^+)\bar{\nu}_i, \qquad n \longrightarrow (\pi^0, \rho^0, \eta, \omega, K^0)\bar{\nu}_i,$$

where $i = 1, 2, 3$, and

$$p \longrightarrow (K^0, \pi^0, \eta, \rho^0, \omega, \gamma)e_j^+, \qquad n \longrightarrow (K^-, \pi^-, \rho^-)e_j^+, \quad j = 1, 2.$$

Channel	τ_p (10^{30} years)
$p \longrightarrow invisible$	0.21
$p \longrightarrow e^+\pi^0$	1600
$p \longrightarrow \mu^+\pi^0$	473
$p \longrightarrow \nu\pi^+$	25
$p \longrightarrow e^+\eta^0$	313
$p \longrightarrow \mu^+\eta^0$	126
$p \longrightarrow e^+\rho^0$	75
$p \longrightarrow \mu^+\rho^0$	110
$p \longrightarrow \nu\rho^+$	162
$p \longrightarrow e^+\omega^0$	107
$p \longrightarrow \mu^+\omega^0$	117
$p \longrightarrow e^+K^0$	150
$p \longrightarrow e^+K^0_S$	120
$p \longrightarrow e^+K^0_L$	51
$p \longrightarrow \mu^+K^0$	120
$p \longrightarrow \mu^+K^0_S$	150
$p \longrightarrow \mu^+K^0_L$	83
$p \longrightarrow \nu K^+$	670
$p \longrightarrow e^+\gamma$	670
$p \longrightarrow \mu^+\gamma$	478

S. Eidelman *et al.* (PDG) Phys. Lett. B **592** (2004)

Georgi, Glashow, Phys.Rev.Lett.32:438-441,1974

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Georgi-Glashow Model

Gauge Symmetry: $SU(5)$

Gauge Bosons:

$$A_\mu = \frac{1}{2}\begin{pmatrix} G_\mu, B_\mu & V_\mu \\ V_\mu^* & W_\mu, B_\mu \end{pmatrix} \quad V_\mu = \sqrt{2}\begin{pmatrix} X_{1\mu} & Y_{1\mu} \\ X_{2\mu} & Y_{2\mu} \\ X_{3\mu} & Y_{3\mu} \end{pmatrix}$$

Fermions:

$$\mathbf{10} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u_3^C & -u_2^C & u_1 & d_1 \\ -u_3^C & 0 & u_1^C & u_2 & u_2 \\ u_2^C & -u_1^C & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^C \\ -d_1 & -d_2 & -d_3 & -e^C & 0 \end{pmatrix}_L , \ \overline{\mathbf{5}} = \begin{pmatrix} d_1^C \\ d_2^C \\ d_3^C \\ e \\ -\nu \end{pmatrix}_L$$

Higgs Sector:

$$\mathbf{5_H} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ H^+ \\ H^0 \end{pmatrix} , \ \mathbf{24_H} = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_3 \end{pmatrix} + \frac{\mathbf{1}}{\mathbf{2}\sqrt{\mathbf{15}}}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \mathbf{\Sigma_{24}}$$

Ruled out by unification and/or fermion masses !

New B-L Conserving Interactions:

$$\bar{5}^\dagger \gamma^0 i\gamma^\mu D_\mu \bar{5} \quad \rightarrow \quad \mathrm{g}_5 \, \overline{(d^C)}_L \, \gamma^\mu \, (X_\mu e_L \; - \; Y_\mu \nu_L)/\sqrt{2}$$

$Q(X) = 4/3$ and $Q(Y) = 1/3$

$$Tr\overline{10} i\gamma^\mu D_\mu 10 \quad \rightarrow \quad \mathrm{g}_5 \, \overline{(e^C)}_L \, \gamma^\mu \, (X_\mu d_L \; - \; Y_\mu u_L)/\sqrt{2} \, +$$

$$g_5 \, \left(\overline{u}_L \, \gamma^\mu \, X_\mu (u^C)_L \; + \; \overline{d}_L \, \gamma^\mu \, Y_\mu (u^C)_L \right) / \sqrt{2}$$

$$10 \; Y_U \; 10 \; 5_H = Q Y_U u^C H \; + \; Q Y_U Q T \; + \; u^C Y_U e^C T$$

$$10 \; Y_D \; \bar{5} \; 5_H^* = Q Y_D d^C H^* + L Y_D^T e^C H^* + Q Y_D L T^* + u^C Y_D d^C T^*$$

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MINIMAL SUPERSYMMETRIC SU(5)

S. Dimopoulos and H. Georgi NPB (1981); N. Sakai Z. Phys. C (1981)

Chiral Superfields: $\hat{\bar{5}}_i$, $\hat{10}_i$, $\hat{5}_H$, $\hat{\bar{5}}_H$, $\hat{24}_H$

Vector Superfields: $\hat{24}_G$

$$\hat{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \hat{U}_3^C & -\hat{U}_2^C & \hat{U}_1 & \hat{D}_1 \\ -\hat{U}_3^C & 0 & \hat{U}_1^C & \hat{U}_2 & \hat{U}_2 \\ \hat{U}_2^C & -\hat{U}_1^C & 0 & \hat{U}_3 & \hat{D}_3 \\ -\hat{U}_1 & -\hat{U}_2 & -\hat{U}_3 & 0 & \hat{E}^C \\ -\hat{D}_1 & -\hat{D}_2 & -\hat{D}_3 & -\hat{E}^C & 0 \end{pmatrix}_L$$

$$\hat{\bar{5}} = \begin{pmatrix} \hat{D}_1^C \\ \hat{D}_2^C \\ \hat{D}_3^C \\ \hat{E} \\ -\hat{N} \end{pmatrix}_L \qquad \hat{5}_H = \begin{pmatrix} \hat{T}_1 \\ \hat{T}_2 \\ \hat{T}_3 \\ \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix} \qquad \hat{\bar{5}}_H = \begin{pmatrix} \hat{\bar{T}}_1 \\ \hat{\bar{T}}_2 \\ \hat{\bar{T}}_3 \\ \hat{H}_1^- \\ -\hat{H}_1^0 \end{pmatrix}$$

$$\hat{24}_H = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{24}$$

If we neglect higher-dimensional operators the model is ruled out by fermion masses since $Y_D = Y_E^T$!!!

New B Violating Interactions in SUSY GUTs:

$$\mathcal{W}_Y = \hat{10}\ Y_U\ \hat{10}\ \hat{\bar{5}}_H\ +\ \hat{10}\ Y_D\ \hat{\bar{5}}\ \hat{\bar{5}}_H\ +$$

$$+\ \mathcal{O}^4(\hat{24}_H/M_{Pl})\ +\ \mathcal{O}^5(\hat{24}_H^2/M_{Pl}^2)$$

$$\mathcal{W}_Y = \hat{Q}\ Y_U\ \hat{U}^C\ \hat{H}\ +\ \hat{\bar{H}}\ \hat{Q}\ Y_D\ \hat{D}^C\ +\ \hat{\bar{H}}\ \hat{L}\ Y_E\ \hat{E}^C$$

$$+\tfrac{1}{2}\hat{Q}\ \underline{A}\ \hat{Q}\ \hat{T}\ +\ \hat{U}^C\ \underline{B}\ \hat{E}^C\ \hat{T}\ +\ \hat{Q}\ \underline{C}\ \hat{L}\ \hat{\overline{T}}\ +\ \hat{U}^C\ \underline{D}\ \hat{D}^C\ \hat{\overline{T}}$$

$$\mathcal{W}_{NR} = \epsilon_i\ \hat{\bar{5}}_i\ \hat{5}_H\ +\ \lambda_{ijk}\ \hat{10}_i\ \hat{\bar{5}}_j\ \hat{\bar{5}}_k\ +\ \delta_i\ \hat{\bar{5}}_i\ \hat{24}_H\ \hat{5}_H$$

$$\lambda_{ijk}\ \hat{10}_i\ \hat{\bar{5}}_j\ \hat{\bar{5}}_k =$$

$$= 2\lambda_{ijk}\ \hat{Q}_i\ \hat{L}_j\ \hat{D}_k^C\ +\ \lambda_{ijk}\ \hat{E}_i^C\ \hat{L}_j\ \hat{L}_k\ +\ \lambda_{ijk}\ \hat{U}_i^C\ \hat{D}_j^C\ \hat{D}_k^C$$

where

$$\lambda_{ijk} = -\lambda_{ikj}$$

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Weinberg, 1979-1982; Wilczek, Zee,'79; Sakai, Yanagida,'82

Baryon Number Violating Effective Operators

$$\mathcal{L}_{eff} = c_d \frac{\mathcal{O}^d}{M^{d-4}}$$

Supersymmetric Models

$$\mathcal{O}_I^4 = \Lambda_1^{cba} \int d^2\theta \; \epsilon_{ijk} \; \hat{U}_{ci}^C \; \hat{D}_{bj}^C \; \hat{D}_{ak}^C$$

$$\mathcal{O}_{II}^4 = \Lambda_2^{cba} \int d^2\theta \; \epsilon_{\alpha\beta} \; \hat{Q}_{c\alpha} \; \hat{L}_{b\beta} \; \hat{D}_a^C$$

$$\mathcal{O}_I^5 = \frac{C_{LLLL}^{abcd}}{M_T} \int d^2\theta \; \epsilon_{\alpha\beta} \; \epsilon_{\gamma\delta} \; \epsilon_{ijk} \; \hat{Q}_{a\alpha i} \; \hat{Q}_{b\beta j} \; \hat{Q}_{c\gamma k} \; \hat{L}_{d\delta}$$

$$\mathcal{O}_{II}^5 = \frac{C_{RRRR}^{abcd}}{M_T} \int d^2\theta \; \epsilon_{ijk} \; \hat{U}_{ai}^C \; \hat{D}_{bj}^C \; \hat{U}_{ck}^C \; \hat{E}_d^C$$

$$\mathcal{O}_{III}^5 = \frac{\tilde{C}_{RRRR}^{abcd}}{M_T} \int d^2\theta \; \epsilon_{ijk} \; \hat{U}_{ai}^C \; \hat{D}_{bj}^C \; \hat{D}_{ck}^C \; \hat{N}_d^C$$

where $T = (3, 1, -2/3)$ and $\overline{T} = (3^*, 1, 2/3)$

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Non-Supersymmetric Contributions

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Gauge $d = 6$

$$
\begin{aligned}
\mathcal{O}^6_I &= k_1^2\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u^C_{ia}}\, \gamma^\mu\, Q_{j\alpha a}\, \overline{e^C_b}\, \gamma_\mu\, Q_{k\beta b} \\
\mathcal{O}^6_{II} &= k_1^2\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u^C_{ia}}\, \gamma^\mu\, Q_{j\alpha a}\, \overline{d^C_{kb}}\, \gamma_\mu\, L_{\beta b} \\
\mathcal{O}^6_{III} &= k_2^2\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{d^C_{ia}}\, \gamma^\mu\, Q_{j\beta a}\, \overline{u^C_{kb}}\, \gamma_\mu\, L_{\alpha b} \\
\mathcal{O}^6_{IV} &= k_2^2\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{d^C_{ia}}\, \gamma^\mu\, Q_{j\beta a}\, \overline{\nu^C_b}\, \gamma_\mu\, Q_{k\alpha b}
\end{aligned}
$$

where: $k_1 = g_{GUT}/\sqrt{2}M_V$, $k_2 = g_{GUT}/\sqrt{2}M_{V'}$

$V = (X, Y) = (3, 2, 5/3)$, $V' = (X^{'}, Y^{'}) = (3, 2, -1/3)$

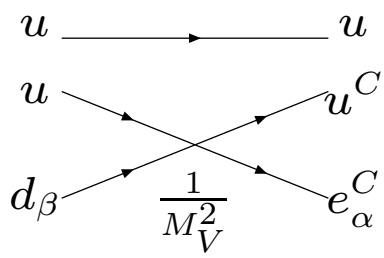
Higgs $d = 6$

$$
\begin{aligned}
O_H(d_\alpha, e_\beta) &= a(d_\alpha, e_\beta)\, u^T\, P_L\, C^{-1}\, d_\alpha\, u^T\, P_L\, C^{-1} e_\beta \\
O_H(d_\alpha, e^C_\beta) &= a(d_\alpha, e^C_\beta)\, u^T\, P_L\, C^{-1}\, d_\alpha\, e^{C\dagger}_\beta\, P_L\, C^{-1} u^{C*} \\
O_H(d^C_\alpha, e_\beta) &= a(d^C_\alpha, e_\beta)\, d^{C\dagger}_\alpha\, P_L\, C^{-1}\, u^{C*}\, u^T\, P_L\, C^{-1} e_\beta \\
O_H(d^C_\alpha, e^C_\beta) &= a(d^C_\alpha, e^C_\beta)\, d^{C\dagger}_\alpha\, P_L\, C^{-1}\, u^{C*}\, e^{C\dagger}_\beta\, P_L\, C^{-1} u^{C*} \\
O_H(d_\alpha, d_\beta, \nu_i) &= a(d_\alpha, d_\beta, \nu_i)\, u^T\, P_L\, C^{-1}\, d_\alpha\, d^T_\beta\, P_L\, C^{-1}\, \nu_i \\
O_H(d_\alpha, d^C_\beta, \nu_i) &= a(d_\alpha, d^C_\beta, \nu_i)\, d^{C\dagger}_\beta\, P_L\, C^{-1}\, u^{C*}\, d^T_\alpha\, P_L\, C^{-1}\, \nu_i
\end{aligned}
$$

Buras, Ellis, Gaillard, Nanopoulos '78; Hisano '00; P. Fileviez Pérez '04

d=6 Contributions: Example

Channel: $p \to \pi^0 \; e^+$ ($\tau_{exp} > 1.6 \times 10^{33}$ years)



in the physical basis we get:

$$\mathcal{O}(e^C_\alpha, d_\beta) \quad = \quad k_1^2 \; c(e^C_\alpha, d_\beta) \; \epsilon_{ijk} \; \overline{u^C_i} \; \gamma^\mu \; u_j \; \overline{e^C_\alpha} \; \gamma_\mu \; d_{k\beta}$$

Naive calculation:

$$\Gamma_6 \propto \alpha^2_{GUT} \frac{m_p^5}{M_V^4}$$

$$\alpha_{GUT} = (1/40 - 1/25) \;\; \to \;\; M_V > (2.57 - 3.23) \times 10^{15} \; GeV$$

$$M_{GUT} \sim (10^{14} - 10^{16}) \; GeV !!!$$

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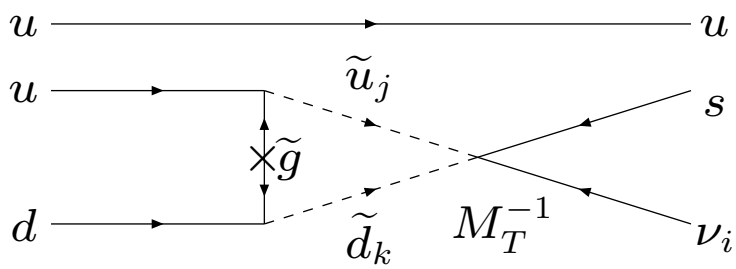
Dimopoulos, Raby, Wilczek '82; Arnowitt, Chamseddine, Nath '85; Hisano, Murayama, Yanagida '93; Lucas, Raby '97; Goto, Nihei '99; Nath, Syed '01; Babu, Pati, Wilczek'00; Murayama, Pierce'02; Bajc, P. Fileviez Pérez, Senjanovic '02.

d=5 Contributions: Example

Channel: $p \to K^+ \bar{\nu}_i$ ($\tau_{exp} > 2.3 \times 10^{33}$ years)



$$A_5 \propto \frac{1}{(4\pi)^2} \frac{G}{M_T} I(m_{\widetilde{g}}, m_{\widetilde{u}_j}, m_{\widetilde{d}_k}) (\widetilde{U}^\dagger\ U)_{j1} (\widetilde{D}^\dagger\ D)_{k1} .....$$

$$I(m, m_1, m_2) = \frac{m}{m_1^2 - m_2^2} \left(\frac{m_1^2}{m_1^2 - m^2} \ln \frac{m_1^2}{m^2} - \frac{m_2^2}{m_2^2 - m^2} \ln \frac{m_2^2}{m^2} \right)$$

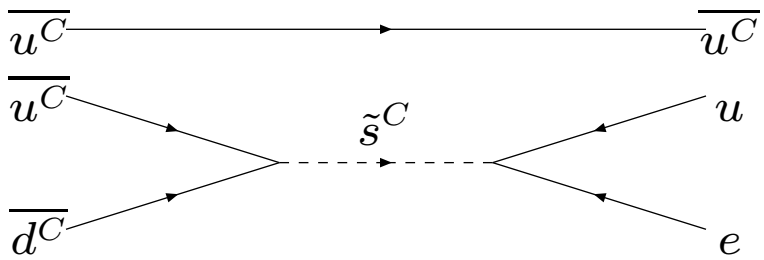
In minimal SUSY $SU(5)$, assuming $m_{\widetilde{q}_3, \widetilde{l}_3} \sim 200$ GeV, $m_{\widetilde{q}_{1,2}, \widetilde{l}_{1,2}} \sim 1 - 10$ TeV, and $Y_E = Y_D^T$ (WRONG !!):

$$M_T > 10^{17} \text{ GeV} \quad (\text{ NAIVE !! })$$

Note that in general we do not know $(\widetilde{U}^\dagger\ U)_{j1} (\widetilde{D}^\dagger\ D)_{k1}$!!!

d=4 Contributions: Example

Channel: $p \to \pi^0 \, e^+$ ($\tau_{exp} > 1.6 \times 10^{33}$ years)



Naive calculation:

$$\Gamma_4 \propto \frac{m_p^5}{M_{SUSY}^4}$$

$$M_{SUSY} \sim (10^3 - 10^{12}) \; GeV \;\; \to \;\; \tau_4 \sim 2.3 \times (10^{-20} - 10^{16}) \; years$$

Matter parity: $M = (-1)^{3(B-L)} = (-1)^{2S} R$

$M = -1$ for $\hat{Q}$, $\hat{L}$, $\hat{U}^C$, $\hat{D}^C$, $\hat{E}^C$; $M = 1$ for $\hat{H}$, $\overline{\hat{H}}$, and $\hat{G}_\mu$.

See for example: P. Fileviez Pérez, J. Phys. G **31** (2005) 1025

P. Fileviez Pérez, Physics Letters **B** 595 (2004) 476-483

$d = 6$ Nucleon Decay in $SU(5)$ and $SO(10)$ Theories

$$
\begin{aligned}
O(e^C_\alpha, d_\beta) &= c(e^C_\alpha, d_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}\ \gamma^\mu\ u_j\ \overline{e^C_\alpha}\ \gamma_\mu\ d_{k\beta}, \\
O(e_\alpha, d^C_\beta) &= c(e_\alpha, d^C_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}\ \gamma^\mu\ u_j\ \overline{d^C_{k\beta}}\ \gamma_\mu\ e_\alpha, \\
O(\nu_l, d_\alpha, d^C_\beta) &= c(\nu_l, d_\alpha, d^C_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}\ \gamma^\mu\ d_{j\alpha}\ \overline{d^C_{k\beta}}\ \gamma_\mu\ \nu_l
\end{aligned}
$$

$$
\begin{aligned}
c(e^C_\alpha, d_\beta) &= k_1^2 \left[V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta} (V_2 V_{UD}^\dagger)^{\alpha 1} \right] \\
c(e_\alpha, d^C_\beta) &= k_1^2\ V_1^{11} V_3^{\beta\alpha} + \ k_2^2\ (V_4 V_{UD}^\dagger)^{\beta 1} (V_1 V_{UD} V_4^\dagger V_3)^{1\alpha} \\
c(\nu_l, d_\alpha, d^C_\beta) &= k_1^2\ (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l} + k_2^2\ V_4^{\beta\alpha} (V_1 V_{UD} V_4^\dagger V_3 V_{EN})^{1l}
\end{aligned}
$$

$$\alpha = \beta \neq 2$$

$$V_1 = U_C^\dagger U, \quad V_2 = E_C^\dagger D, \quad V_3 = D_C^\dagger E, \quad V_4 = D_C^\dagger D,$$

$$V_{UD} = U^\dagger D, \quad V_{EN} = E^\dagger N, \quad V_{UD} = U^\dagger D = K_1\ V_{CKM} K_2,$$

$$V_{EN} = K_3 V_l^M$$

$$U_C^T\ Y_U\ U = Y_U^{diag}, \ \ D_C^T\ Y_D\ D = Y_D^{diag}, \ \ E_C^T\ Y_E\ E = Y_E^{diag}$$

$$\alpha, \beta = 1, 2,\ l = 1, 2, 3.$$

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P. Fileviez Pérez, Physics Letters **B** 595(2004) 476-483

$p \to K^+\bar{\nu}$ and $p \to \pi^+\bar{\nu}$ Channels and the Test of GUTs

$$\Gamma(p \to K^+\bar{\nu}) = \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2} A_L^2 \left|\alpha_H\right|^2 \times$$

$$\times \sum_{i=1}^{3} \left| \frac{2m_p}{3m_B} D\ c(\nu_i, d, s^C) + [1 + \frac{m_p}{3m_B}(D + 3F)] c(\nu_i, s, d^C) \right|^2$$

$$\Gamma(p \to \pi^+\bar{\nu}) = \frac{m_p}{8\pi f_\pi^2} A_L^2 \left|\alpha_H\right|^2 (1 + D + F)^2 \sum_{i=1}^{3} \left| c(\nu_i, d, d^C) \right|^2$$

A GUT model with symmetric Yukawa couplings

$$k_1 = \frac{g_{GUT}}{\sqrt{2} M_V} = \left[\frac{Q_1\ \Gamma^{exp}(p \to K^+\bar{\nu})}{\left|A_1\right|^2 \left|V_{CKM}^{11}\right|^2 + \left|A_2\right|^2 \left|V_{CKM}^{12}\right|^2} \right]^{1/4}$$

$$Q_1 = \frac{8\pi m_p^3 f_\pi^2}{(m_p^2 - m_K^2)^2 A_L^2 \left|\alpha_H\right|^2}, \quad A_1 = \frac{2m_p}{3m_B} D, \quad A_2 = 1 + \frac{m_p}{3m_B}(D + 3F)$$

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$$k_2 = \frac{g_{GUT}}{\sqrt{2} M_{V'}} = k_1 \left| V_{CKM}^{11} \right| \{-1 + \sqrt{Q_2}\}^{1/2}$$

$$Q_2 = 1 + \frac{8\pi f_\pi^2 \, \Gamma^{exp}(p \to \pi^+ \bar{\nu})}{k_1^4 \left| V_{CKM}^{11} \right|^4 m_p A_L^2 \left| \alpha_H \right|^2 (1 + D + F)^2} - \left| V_{CKM}^{11} \right|^{-2}$$

- $$\frac{\tau(p \to K^+ \bar{\nu})}{\tau(p \to \pi^+ \bar{\nu})} > 30,$$
- $$\frac{\tau(n \to K^0 \bar{\nu})}{\tau(p \to K^+ \bar{\nu})} = 0.37,$$
- $$\frac{\tau(n \to \pi^0 \bar{\nu})}{\tau(p \to \pi^+ \bar{\nu})} = 1.99,$$
- $$\frac{\tau(n \to \eta^0 \bar{\nu})}{\tau(p \to \pi^+ \bar{\nu})} = 54.50$$

I. Dorsner, P. Fileviez Pérez, Physics Letters B **625** (2005) 88-95;
P. Fileviez Pérez, AIP Conf. Proc. 903: 385, 2006

Upper Bound on the Proton Lifetime in GUTs

$$c(e^C_\alpha, d_\beta) = k_1^2 \left[V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta} (V_2 V_{UD}^\dagger)^{\alpha 1} \right]$$

$$c(e_\alpha, d^C_\beta) = k_1^2 \; V_1^{11} V_3^{\beta\alpha} + \; k_2^2 \; (V_4 V_{UD}^\dagger)^{\beta 1} (V_1 V_{UD} V_4^\dagger V_3)^{1\alpha}$$

$$c(\nu_l, d_\alpha, d^C_\beta) = k_1^2 \; (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l} + k_2^2 \; V_4^{\beta\alpha} (V_1 V_{UD} V_4^\dagger V_3 V_{EN})^{1l}$$

$$\alpha = \beta \neq 2$$

- 1-) **There are no decays into a meson and antineutrino**
- 2-) **There are no decays into a meson and charged anti-lepton**

$$(SU(5) \text{ where } k_2 = 0)$$

There are no decays into a meson and antineutrino

$$\begin{aligned}
(V_1 V_{UD})^{1\alpha} &= (U_C^\dagger D)^{1\alpha} = 0 \Longrightarrow U_C = D A^\dagger \quad \text{(C.I)} \\
V_2^{\beta\alpha} &= (E_C^\dagger D)^{\beta\alpha} = 0 \Longrightarrow E_C = D B_1 \quad \text{(C.II)} \\
V_3^{\beta\alpha} &= (D_C^\dagger E)^{\beta\alpha} = 0 \Longrightarrow D_C = E B_2 \quad \text{(C.III)}
\end{aligned}$$

$$\Gamma_p \quad = \quad 8\pi^2 C(p, K^0)\ |V^{13}_{CKM}|^2\ \alpha^2_{GUT}\ M_V^{-4}$$

$$C(p, K^0) = \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2}\ A_L^2\ |\alpha_H|^2 \times \left[1 + \frac{m_p}{m_B}(D - F)\right]^2$$

In the case of Majorana neutrinos

$$\tau_p^M \leq 6.0 \times 10^{39}\ \frac{(M_X/10^{16}\,\mathrm{GeV})^4}{\alpha^2_{GUT}}\ (0.003\,\mathrm{GeV}^3/\alpha_H)^2\,\mathrm{years}$$

Using $\tau_p \geq 50 \times 10^{32}$ years and $\alpha_H = 0.003\,\mathrm{GeV}^3$ the bound on M_X is :

$$M_X \ > \ 3.04 \times 10^{14}\sqrt{\alpha_{GUT}}\,\mathrm{GeV}$$

$$\alpha_{GUT} = 1/39 - 1/25 \rightarrow M_X \ > \ (4.9 - 6.1) \times 10^{13}\,\mathrm{GeV}$$

I. Dorsner, P. Fileviez Pérez, Physics Letters B **625** (2005) 88-95

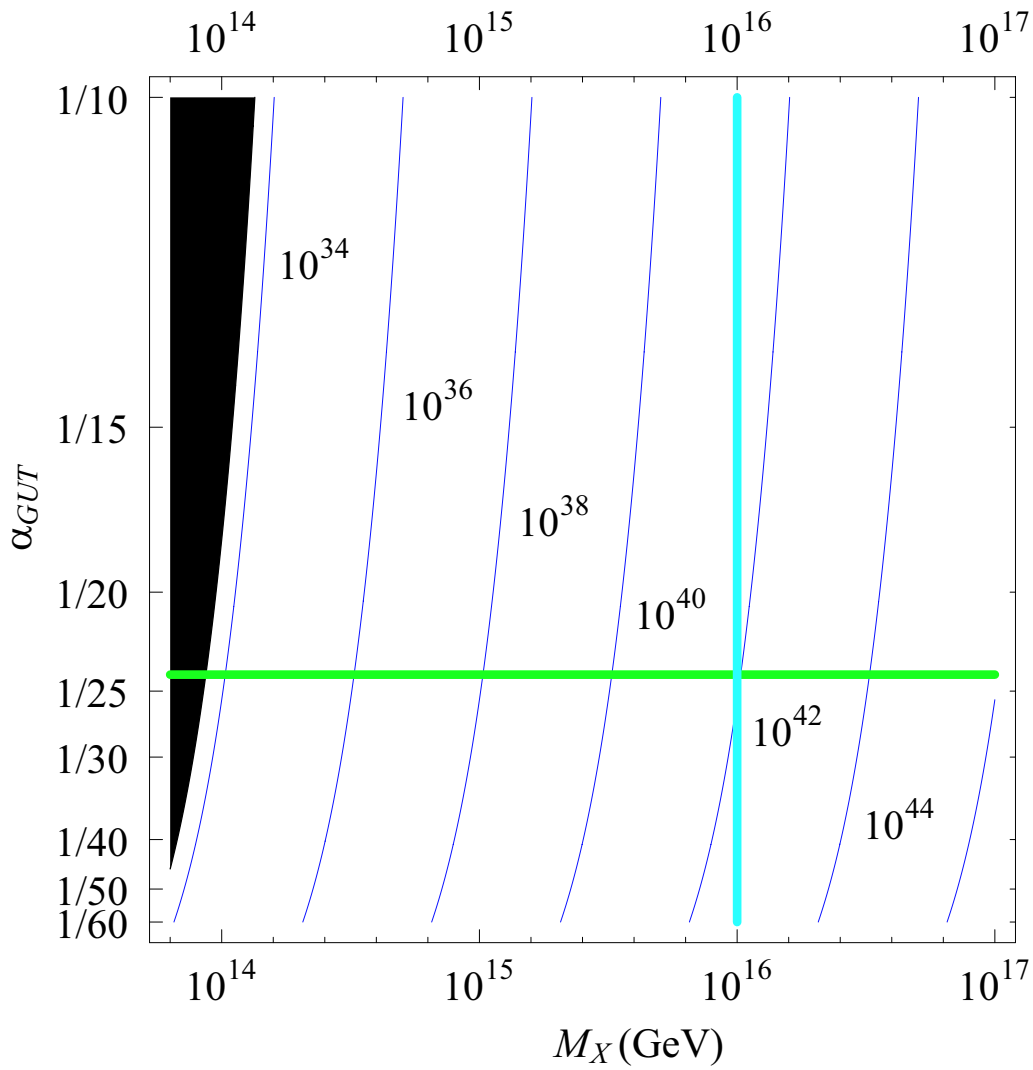


Figure 1: Isoplot for the upper bounds on the total proton lifetime in years in the Majorana neutrino case in the M_X–α_{GUT} plane. The value of the unifying coupling constant is varied from 0.02 to 1. The conventional values for M_X and α_{GUT} in SUSY GUTs are marked in thick lines. The experimentally excluded region is given in black.

I. Dorsner, P. Fileviez Pérez, Physics Letters B **625** (2005) 88-95

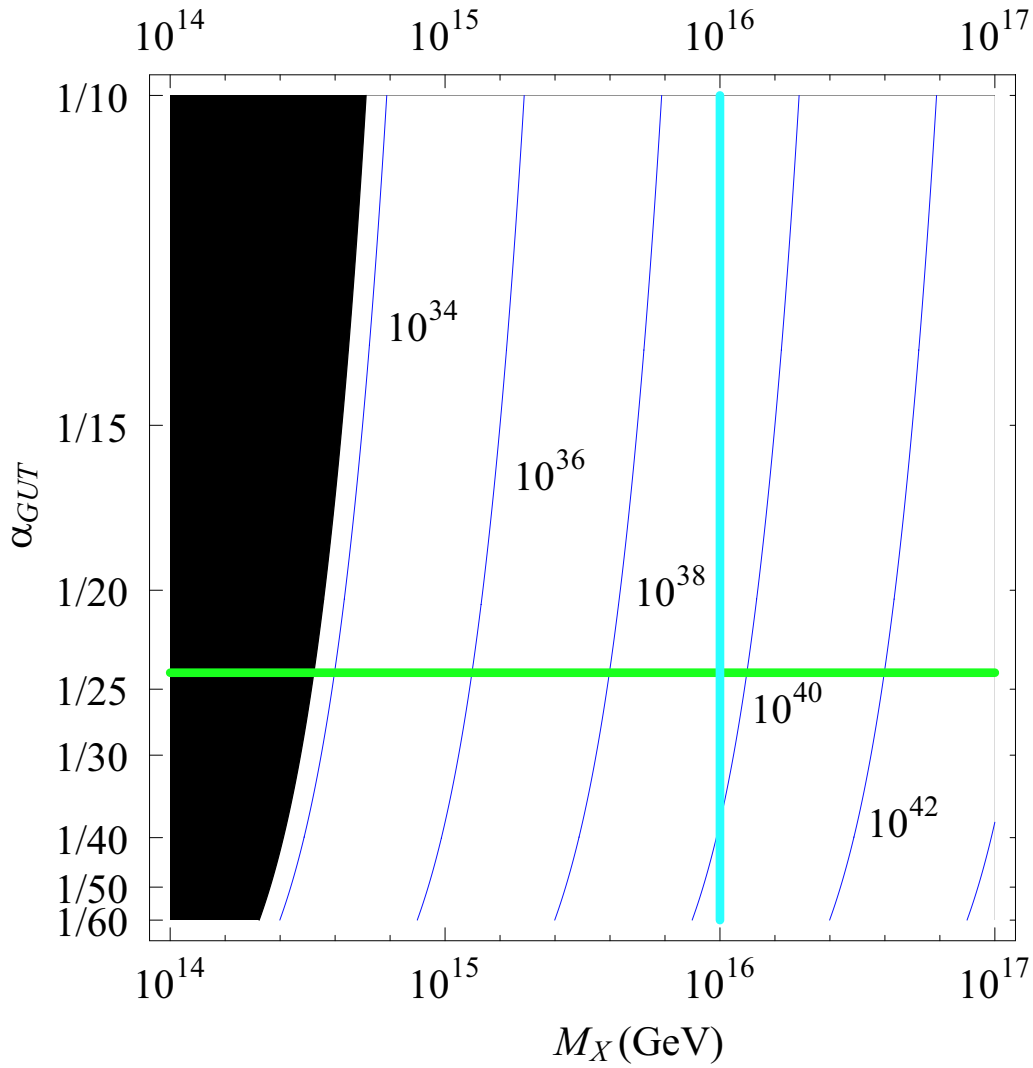


Figure 2: Isoplot for the upper bounds on the total proton lifetime in years in the Dirac neutrino case in the M_X–α_{GUT} plane. The value of the unifying coupling constant is varied from $1/60$ to $1/10$. The conventional values for M_X and α_{GUT} in SUSY GUTs are marked in thick lines. The experimentally excluded region is given in black.

Type II–SU(5)

I. Dorsner, P. Fileviez Pérez Nucl.Phys.B723:53-76,2005

Matter: $\bar{5} = (d^C, e, \nu)$, $10 = (u^C, Q, e^C)$

Higgs Sector: 5_H, 24_H, 15_H

$$15_H = \underbrace{(1,3,1)}_{\Phi_a} \bigoplus \underbrace{(3,2,1/6)}_{\Phi_b} \bigoplus \underbrace{(6,1,-2/3)}_{\Phi_c}$$

Neutrino Masses: Type II seesaw mechanism

$$V_\nu = Y_\nu \; \bar{5} \; \bar{5} \; 15_H \; + \; \mu \; 5_H^* \; 5_H^* \; 15_H + \text{h.c.}$$

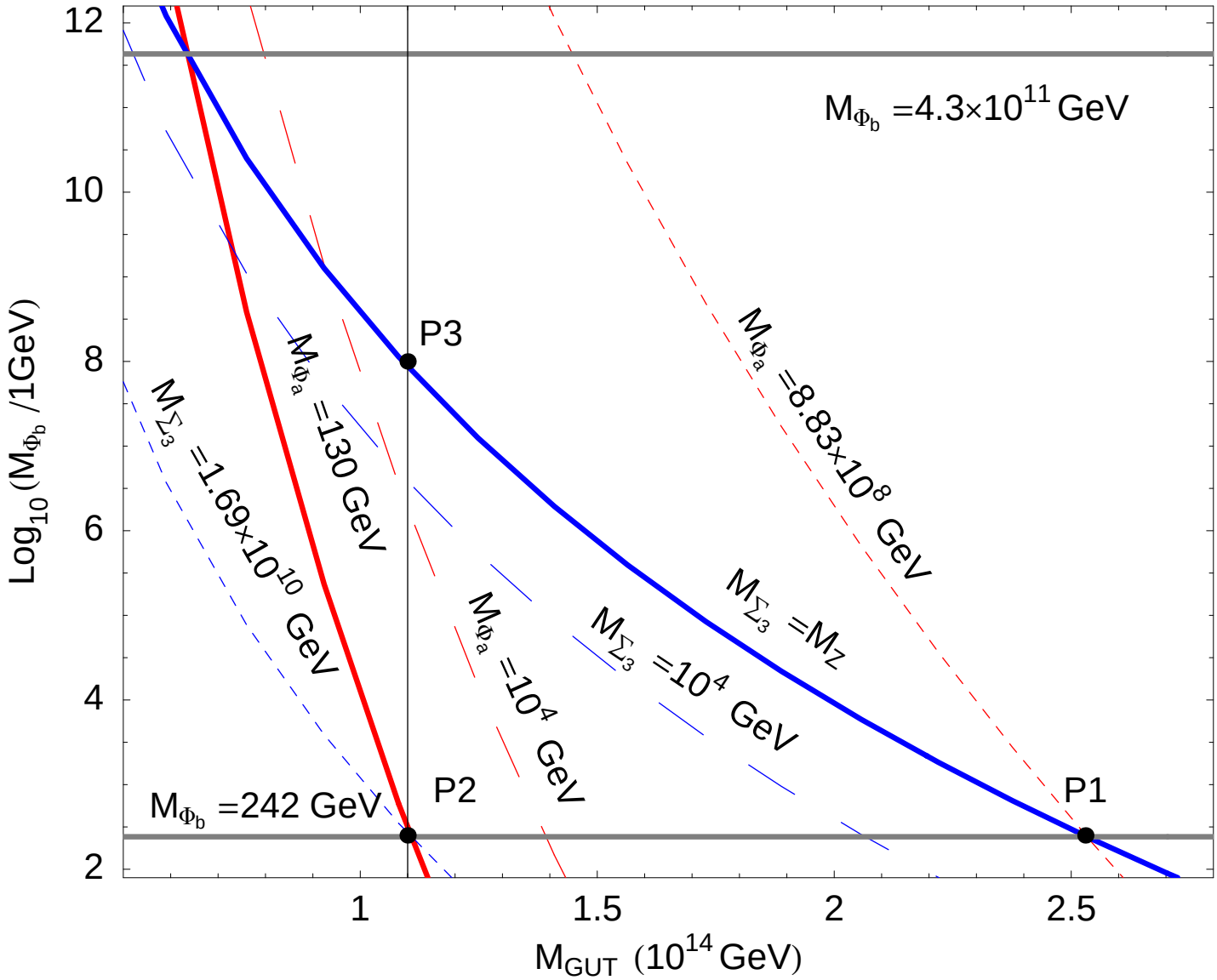
$$M_\nu = \sqrt{2} Y_\nu v_\Delta = Y_\nu \; \mu \; v_0^2 / M_\Delta^2.$$

Charged Fermion Masses: $Y_E \neq Y_D^T$ using higher-dimensional operators

Unification: O.K.

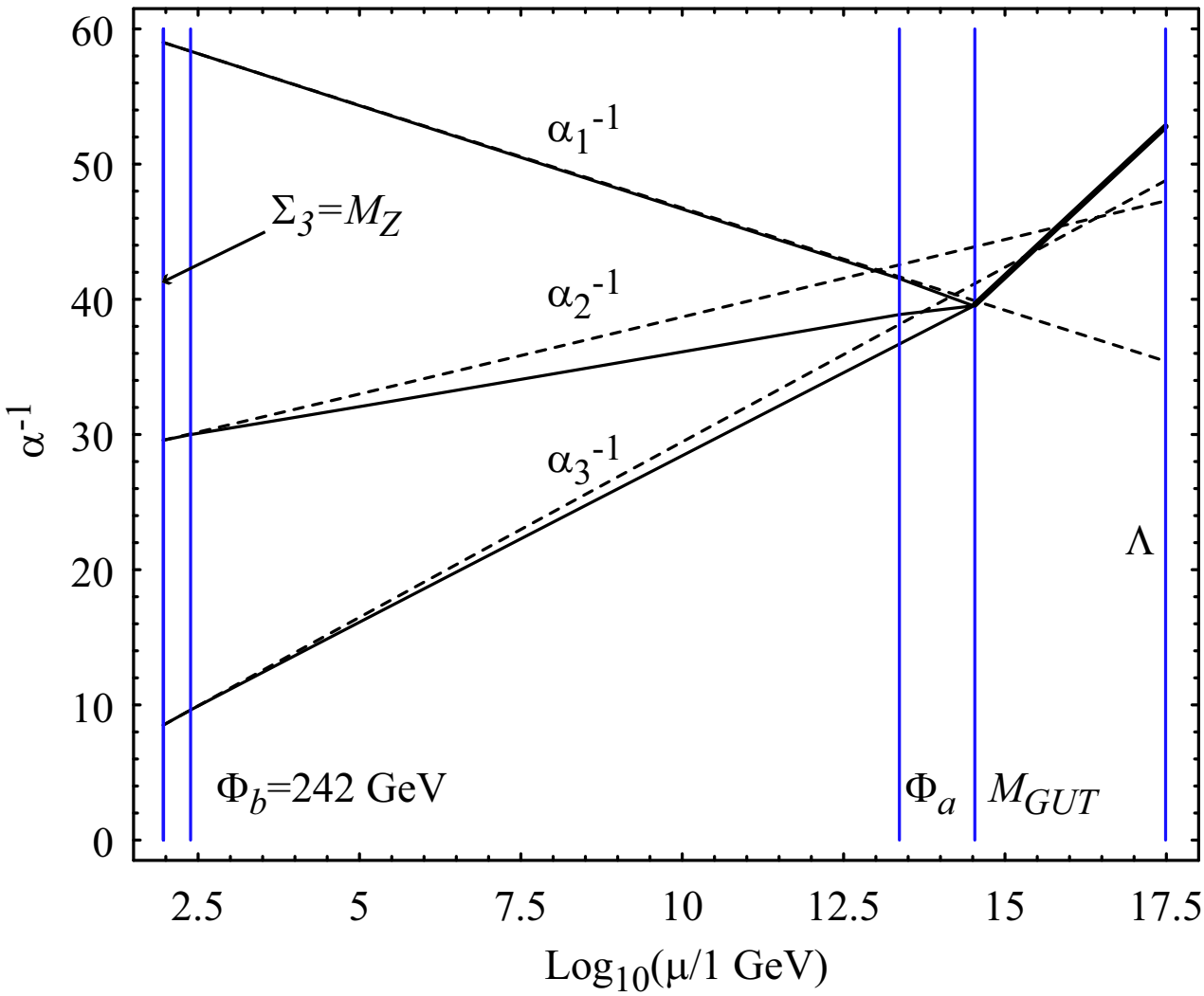
See also:
I. Dorsner, P. Fileviez Pérez, R. González Felipe, NPB 747:312,2006
I. Dorsner, P. Fileviez Pérez, G. Rodrigo, PRD: 125007,2007

I. Dorsner, P. Fileviez Pérez, G. Rodrigo, PRD: 125007,2007



The whole parameter space where one has unification at one-loop.

I. Dorsner, P. Fileviez Pérez, G. Rodrigo, PRD: 125007,2007



Unification at the two-loop level.

Using $\alpha_H = 0.011$ GeV^3 and $M_{GUT}^{max} = 4.5 \times 10^{14}$ GeV one gets

$\tau_p \; < \; 2 \times 10^{36}$ years

Adjoint SU(5)

P. Fileviez Pérez, PLB 654 (2007) 189.

Matter: $\bar{5} = (d^C, e, \nu)$, $10 = (u^C, Q, e^C)$, 24

Higgs Sector: 5_H, 24_H, 45_H

$$24 = \underbrace{(8,1)}_{\rho_8} \oplus \underbrace{(1,3)}_{\rho_3} \oplus \underbrace{(3,2)}_{\rho_{(3,2)}} \oplus \underbrace{(\bar{3},2)}_{\rho_{(\bar{3},2)}} \oplus \underbrace{(1,1)}_{\rho_0}$$

Neutrino Mass: Type I and Type III seesaw

$$V_\nu = \alpha_i \; \bar{5}_i \; 24 \; 5_H \; + \; p_i \; \bar{5}_i \; 24 \; 45_H$$

$$M_\nu^{ij} = a^i \; a^j / M_{\rho_3} \; + \; b^i \; b^j / M_{\rho_0}$$

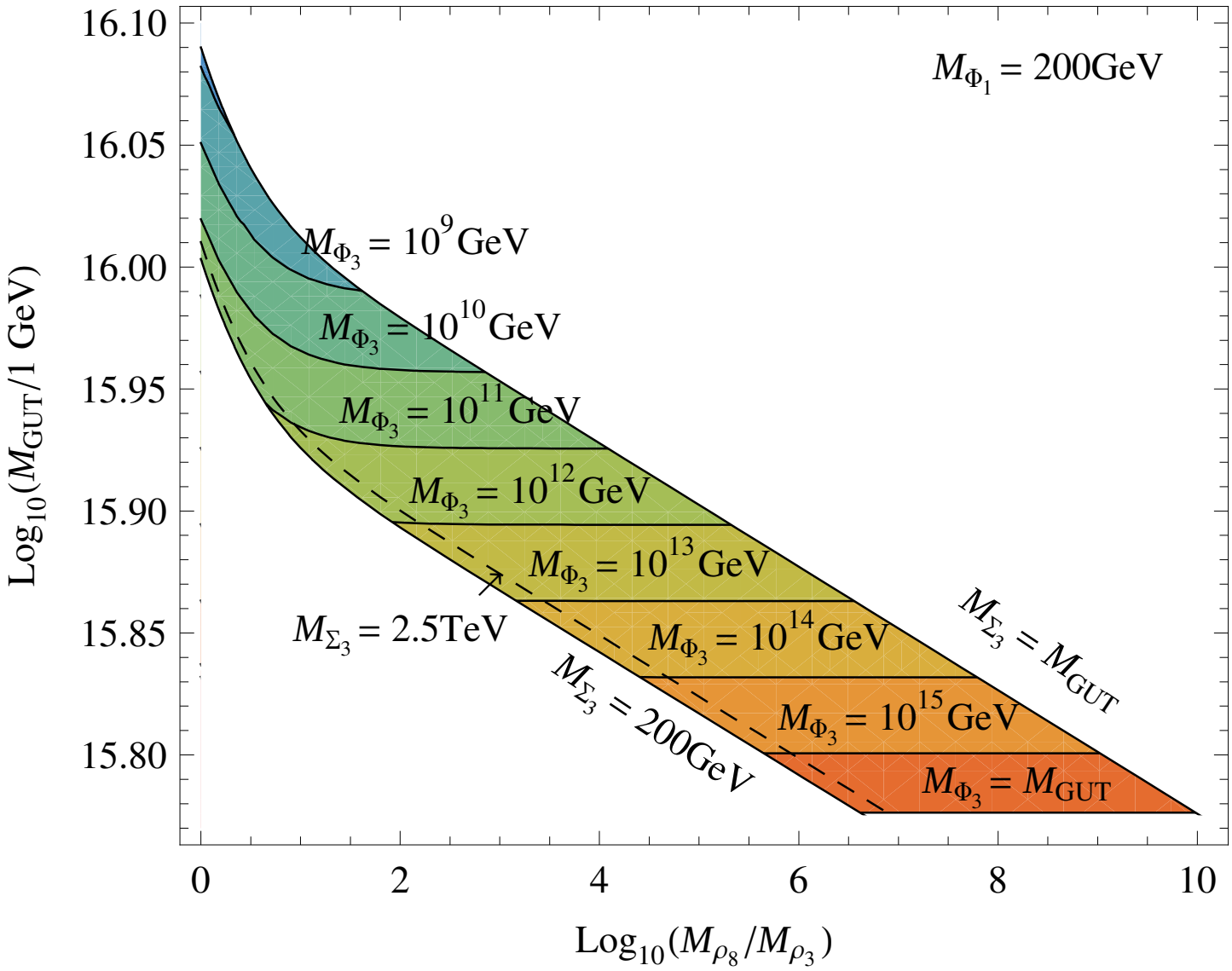
Charged Fermion Masses: $Y_E \neq Y_D^T$ using 45_H

$$V_{de} = 10 \; \bar{5} \; (Y_1 \; 5_H^* \; + \; Y_2 \; 45_H^*) \; + \; h.c.$$

Unification: O.K.

See also: P. Fileviez Pérez, PRD 76 (2007) 071701.

P. Fileviez Pérez, H. Iminniyaz, G. Rodrigo, PRD 78 (2008) 015013.



Unification at the one-loop level.

P. Fileviez Pérez, H. Iminniyaz, G. Rodrigo, PRD 78 (2008) 015013.

Using $\alpha_H = 0.011$ GeV^3 (Y. Aoki et al'08)

and $M_{GUT}^{max} = 7.8 \times 10^{15}$ GeV one gets

- $\tau(p \to K^+ \bar{\nu}) < 9 \times 10^{36}$ years
- $\tau(p \to \pi^+ \bar{\nu}) < 3 \times 10^{35}$ years
- $\tau(p \to e^+ \pi^0) < 1.2 \times 10^{35}$ years

SUMMARY-MODEL INDEPENDENT

- It is very difficult to test grand unification through proton decay if the supersymmetric contributions, $d = 5$, are the most important.

- The gauge $d = 6$ contributions are the least model dependent and if they are dominant one can hope to test grand unification.

- The channels $p \longrightarrow K^+\bar{\nu}$ and $p \longrightarrow \pi^+\bar{\nu}$ can help us to test most of the realistic grand unified theories with symmetric Yukawa couplings.

- There is a VERY conservative and model independent upper bound on the total proton decay lifetime:

$$\tau_p^M \leq 6.0 \times 10^{39} \, \frac{(M_X/10^{16}\,\mathrm{GeV})^4}{\alpha_{GUT}^2} \, (0.003\,\mathrm{GeV}^3/\alpha_H)^2 \,\mathrm{years}$$

SUMMARY-MODEL DEPENDENT

- Non-SUSY GUTs could be viable even if the unification scale is low. This is the case of Type II–SU(5) where the neutrino masses are generated through Type II seesaw and

  $\tau_p < 2 \times 10^{36}$ years $\quad (\alpha_H = 0.011\ \mathrm{GeV}^3)$.

- Adjoint SU(5) is an appealing GUT where neutrino masses are generated through Type I and Type III seesaw. In this case:

  - $\tau(p \to K^+\ \bar{\nu}) < 9 \times 10^{36}$ years

  - $\tau(p \to \pi^+\ \bar{\nu}) < 3 \times 10^{35}$ years

  - $\tau(p \to e^+\pi^0) < 1.2 \times 10^{35}$ years