

TRANSPORT OF $\overrightarrow{^3\text{He}}^{2+}$ IN BOOSTER AND AGS

CONTEXT :

- C-AD is exploring the possibility of polarized $^3\text{He}^{2+}$ storage in RHIC, at time scale of a few years from now.
- $^3\overrightarrow{\text{He}}^{2+}$ provides the polarized neutrons in $^3\overrightarrow{\text{He}}^{2+}$ - $^3\overrightarrow{\text{He}}^{2+}$ or p- $^3\overrightarrow{\text{He}}^{2+}$ collisions,
500 GeV $^3\text{He}^{2+}$, 170 GeV p.

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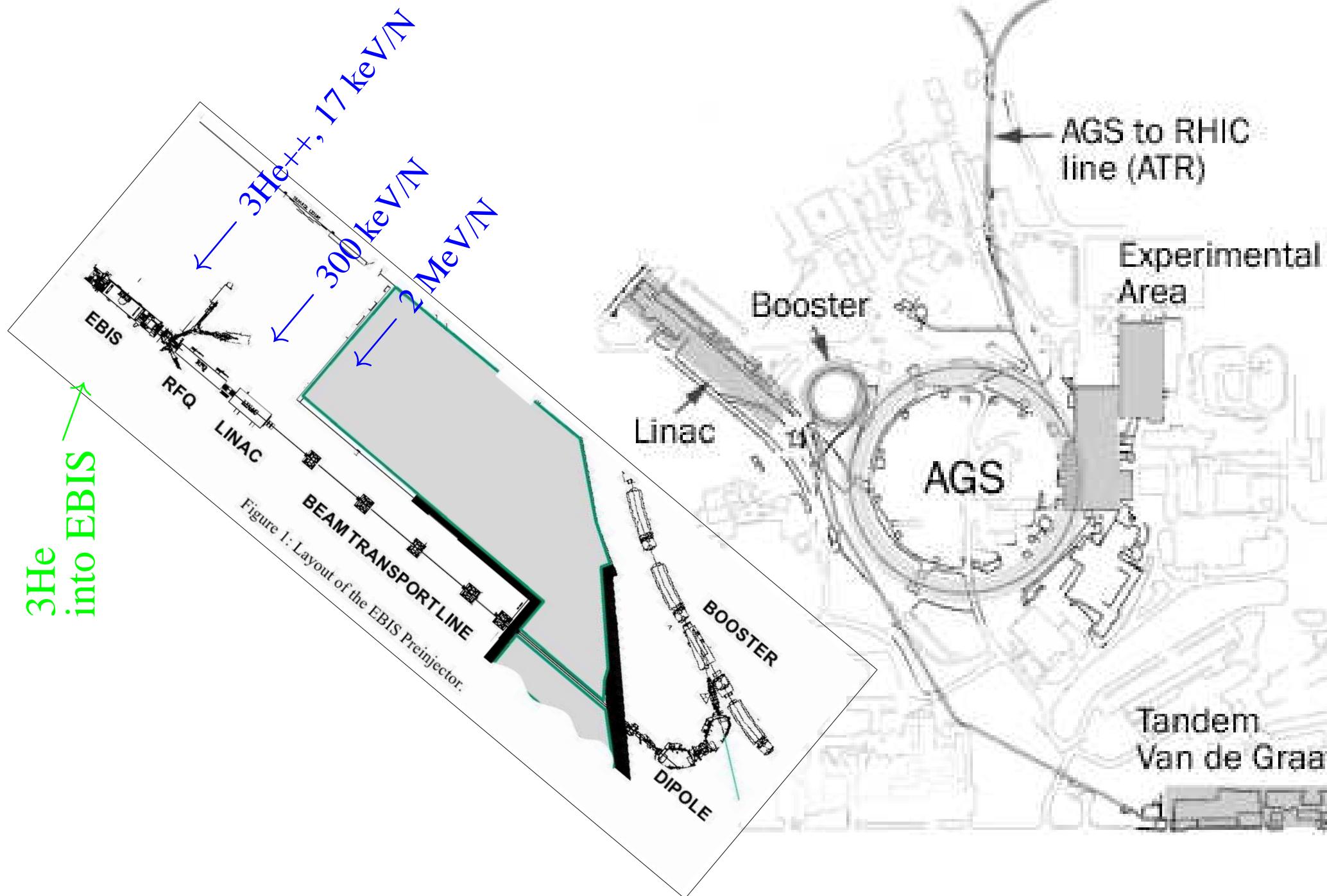
1 Layout

Reminder, RHIC at max. $B\rho$, 833.9 T.m :

proton : $p=250 \text{ GeV}/c$, $G\gamma=477.7$

$^3\text{He}^{2+}$: $p=500 \text{ GeV}/c$, $G\gamma=-745$, $\sim 10^{11} \text{ ppb}$

APPEX W



2 Particle data

	units	proton (for comparison)	${}^3\text{He}^{2+}$
mass	MeV/c^2	938.272046	2808.39148
mass/amu	10^{-27}kg	1.672621777	5.00641234
charge Q	$ e $	1.00727646	3.0149322
number of nucleons, N		+1	+2
number of protons, Z		1	3
spin quantum number		1	2
magnetic moment, μ	μ_N	2.7928474	-2.1276253
gyromagnetic ratio $g = \frac{\mu}{\mu_N} \times 2 \frac{m_p \mu_N}{Z e \hbar I}$	μ/μ_N	2	2.99315281
anomalous mag. moment $G = \frac{g-2}{2}$		1.7928474	-4.18415382
Imp. resonance interval $ mc^2/G $	MeV	523.3	671.2

nuclear magneton unit, $\mu_N = \frac{e\hbar}{2m_p}$	eV/T	3.1524512×10^{-8}
atomic mass unit, amu	MeV/c^2	931.494061
elementary charge, e	C	$1.602176565 \times 10^{-19}$
speed of light in vac.	m/s	2.99792458×10^8
\hbar	J.s	$1.054571726 \times 10^{-34}$
	eV.s	$6.58211928 \times 10^{-16}$

3 Booster and AGS energy ranges

Proton, for comparison

		Comments
BOOSTER INJECTION :		
kin-E (GeV)	0.2	Hyp. : Energy out of linac (Ref. CJG)
$B\rho$ (T.m)	2.1496	
γ	1.2132	
$\beta\gamma$	0.6868	
$G\gamma$	2.175	
BOOSTER EXTRACTION / AGS INJECTION :		
kin-E (GeV)	1.4167	
$B\rho$ (T.m)	7.2051	Max. $B\rho$ in Booster is 17 T.m
γ	2.5100	w/ $Qx/y = 4.73/4.82$ optics
$\beta\gamma$	2.3021	
$G\gamma$	4.5	
AGS EXTRACTION :		
kin-E (GeV)	22.8738	
$B\rho$ (T.m)	79.3668	
γ	25.3786	
$\beta\gamma$	45.5	Half-int. for AtR match ; not more due to AtR matching limitations.
$G\gamma$		

- Note : Transition γ :
Booster : 4.8 ; AGS : 8.4 ; RHIC : 23.6.

${}^3\overline{\text{He}}^{2+}$, possible data :

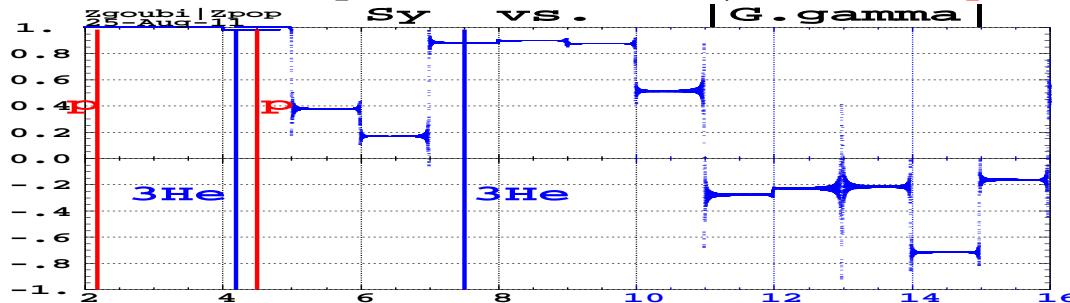
		Comments
BOOSTER INJECTION :		
kinetic E (MeV/N)	2.0146	Hyp. : $\beta = 0.0655$ from EBIS IH DTL
$B\rho$ (T.m)	0.3075	Note : $B\rho_{inj,^3\text{He}^{2+}} \ll B\rho_{inj,p}$
γ	1.0022	
$\beta\gamma$	0.0656	
$G\gamma$	-4.1932	
BOOSTER EXTRACTION / AGS INJECTION :		
kinetic E (GeV)	2.2256	0.74186 GeV/N
$B\rho$ (T.m)	6.9678	Consistent w/ p : $B\rho_{inj,^3\text{He}^{2+}} \approx B\rho_{inj,p}$.
γ	1.7925	
$\beta\gamma$	1.4876	
$G\gamma$	-7.5	
	-10.5	Interest : injection into AGS beyond 0+.
AGS EXTRACTION :		
kinetic E (GeV)	30.4	
$B\rho$ (T.m)	55.2	$B\rho_{xtr,^3\text{He}^{2+}} \ll B\rho_{xtr,p}$, whereas $B\rho = 79.36 \rightarrow G\gamma = -71$, this leaves room for extraction beyond 60-.
$G\gamma$	-49.5	
$ G\gamma $	>51 ?	Higher injection E into RHIC

4 Depolarizing resonances in booster

Imperfection resonances, $|G\gamma - n| = 0$, induced by vertical closed-orbit

- Acceleration range, \vec{p} : $G\gamma : 2.175 \rightarrow 4.5$: **2 resonances**, at $G\gamma = \begin{cases} 3 \\ 4 \end{cases}$: **harm. orbit correction reinforced/spin flip**
- Possible “conservative” ${}^3\overrightarrow{\text{He}}^{2+}$ range : $G\gamma : -4.19 \rightarrow -7.5$: **3 resonances**, at $-G\gamma = 5, 6, 7$.
- Example of S_y motion, ${}^3\overrightarrow{\text{He}}^{2+}$, $|G\gamma| : 4.19 \rightarrow 16$.

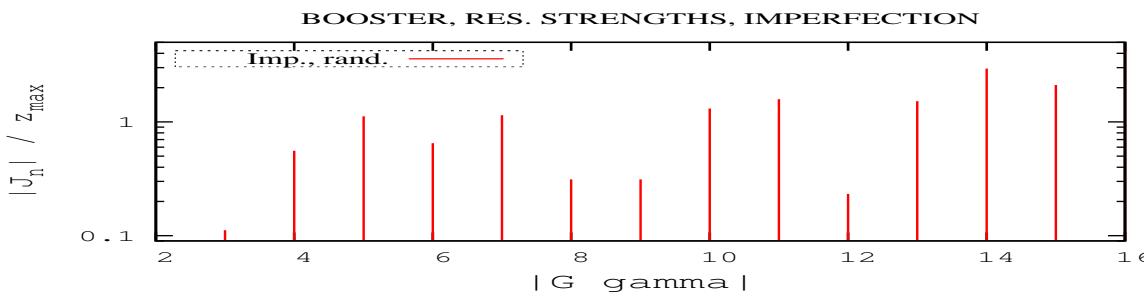
Range shown covers up to max. booster $B\rho \approx 17$ T.m (proton $G\gamma = 9.9$, ${}^3\overrightarrow{\text{He}}^{2+} G\gamma = -15.75$) :



$$\frac{P_f}{P_i} = 2 \exp \left(-\frac{\pi |J_n|^2}{2 \alpha} \right) - 1$$

Single ${}^3\overrightarrow{\text{He}}^{2+}$ on V c.o. ($\hat{y}_{co} = 1$ mm), ϵ_x , ϵ_y zero.

- Strengths of imperfection resonances in Booster, $Q_x / Q_y = 4.73 / 4.82$ optics.



Thin lens model :

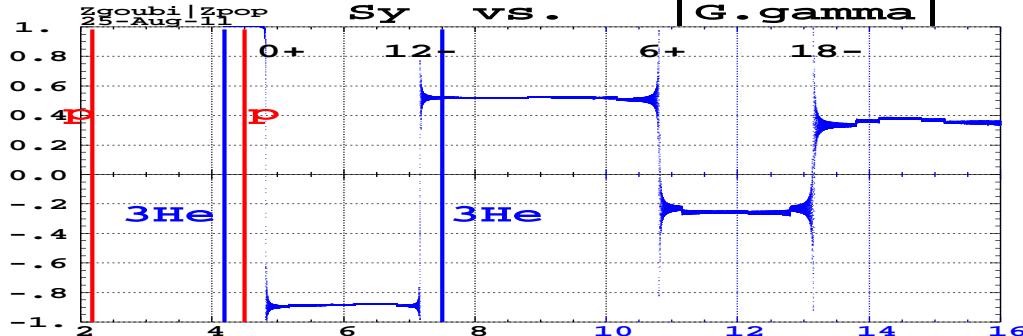
$$J_n = \frac{1+G\gamma}{2\pi} \sum Q_{poles} (\cos(G\gamma\alpha_i) + i \sin(G\gamma\alpha_i)) (KL)_i z_{co,i}$$

- Means for correction of imperfection resonances, practical feasibility to be explored : harmonic orbit correction, helical partial snake.

Intrinsic resonances, $[G\gamma \pm Q_y - n = 0]$, induced by betatron motion

- Acceleration range, proton : $G\gamma$: $2.175 \rightarrow 4.5$: no systematic ($n=6 \times$ integer) resonance.
- Possible range, ${}^3\overline{\text{He}}^{2+}$: $G\gamma$: $-4.193 \rightarrow -7.5$: 2 systematic resonances, at $G\gamma = 0 + Q_y$, $12 - Q_y$.

- S_y motion for ${}^3\overline{\text{He}}^{2+}$ in $|G\gamma|$: $4.19 \rightarrow 7.5$. Vertical lines show $G\gamma$ ranges for p and for ${}^3\overline{\text{He}}^{2+}$.

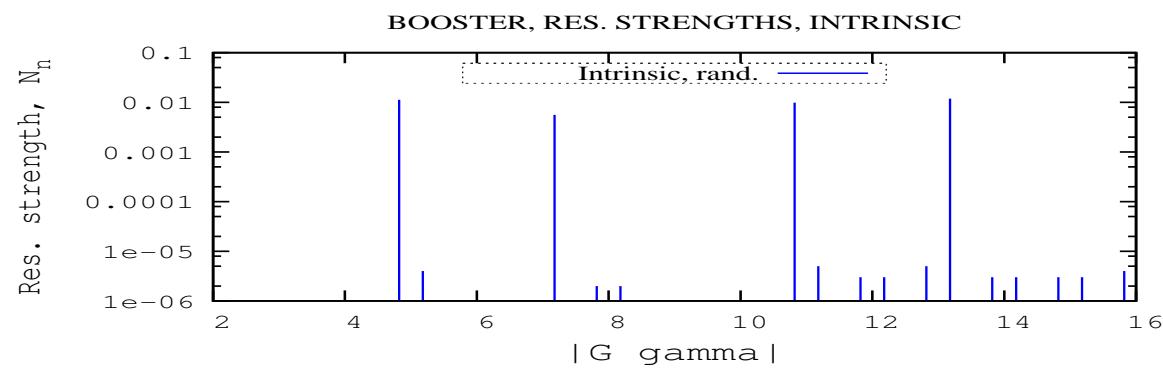


$$\frac{P_f}{P_i} = 2 \exp \left(-\frac{\pi |J_n|^2}{2 \alpha} \right) - 1, \quad \dot{\gamma} = 16.$$

Single ${}^3\overline{\text{He}}^{2+}$ on zero vertical orbit with $\epsilon_y \approx 0.6 \pi \text{mm.mrad}$ invariant.

- Strengths of intrinsic resonances in Booster, $Q_y = 4.82$.

Major lines are at $G\gamma = 0 + Q_y$, $12-$, $6+$, $18-$.



$$N_n^\pm = \frac{1+G\gamma}{4\pi} \sum_{Qpoles} (\cos(G\gamma\alpha_i \pm \psi_i) + i \sin(G\gamma\alpha_i \pm \psi_i)) (KL)_i \sqrt{\beta_{z,i} \epsilon_z / \pi}$$

- Foreseen means for overcoming intrinsic resonances, to be assessed : AC dipole (was done in AGS).

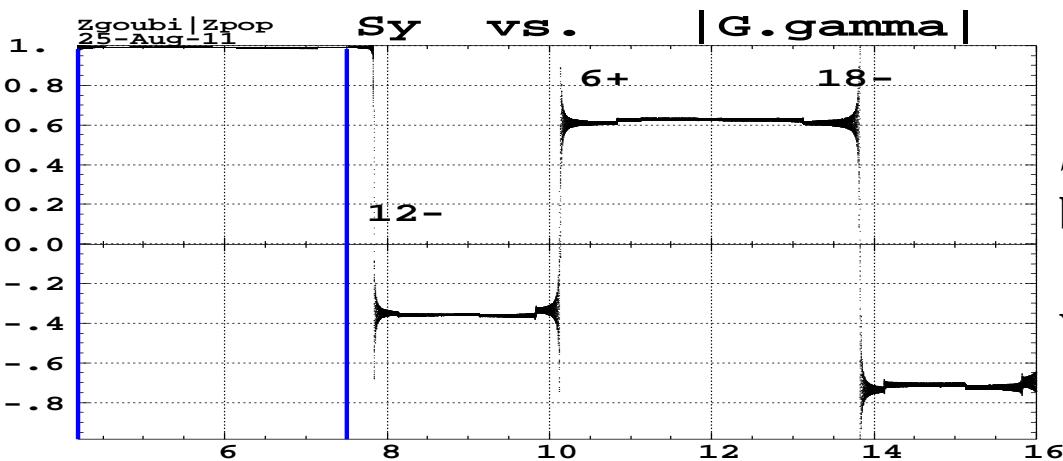
- Possible different settings for ${}^3\overrightarrow{\text{He}}^{2+}$ – to be assessed:

(i) Could foresee pushing both $\begin{cases} 0 + Q_y \\ 12 - Q_y \end{cases}$ resonances beyond [4.19,7.5] range.

Hence, no systematic resonance in acceleration range, no compensation needed.

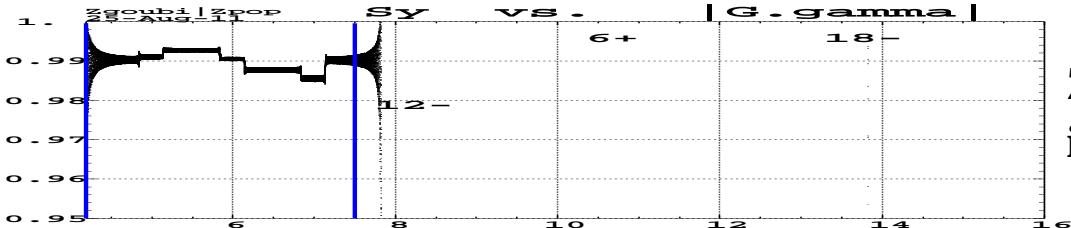
How that works :

- Strength of 0^+ for $\epsilon_y = 2.5\pi\text{mm.mrad}$, norm., is $N_n \approx 0.012$.
- Take distance to the resonance $\Delta = |G\gamma + Q_y - n| > 7 \times N_n$, then
- given $n = 0$ here, then tune at injection should satisfy $Q_y < -G\gamma|_{inj} - 7 \times N_n \approx 4.11$.



S_y motion. ${}^3\overrightarrow{\text{He}}^{2+}$ is on zero vertical orbit with non-zero vertical invariant.

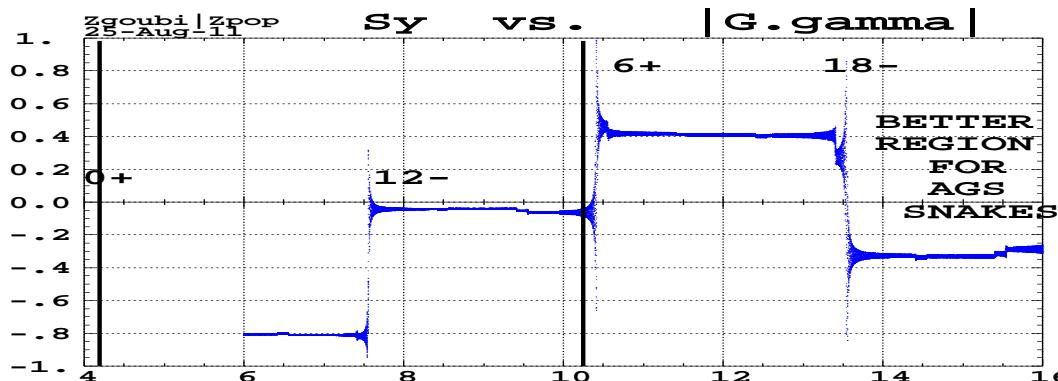
Vertical lines show $G\gamma$ range for ${}^3\overrightarrow{\text{He}}^{2+}$.



Zoom on random intrinsic resonances in acceleration range.

(ii) Could foresee extracting at higher rigidity.

- For instance, below **6+Qy** and **beyond AGS[0+]**, e.g., $G\gamma = -10.5$ ($B\rho = 10.8$ T.m, needs $Q_y > 4.5$).
- Or even further, $B\rho \rightarrow 15$ T.m ($G\gamma \rightarrow -14$).
- Note : still below transition γ ($|G\gamma|_{tr} > 16$), and below max. $B\rho = 17$ T.m.

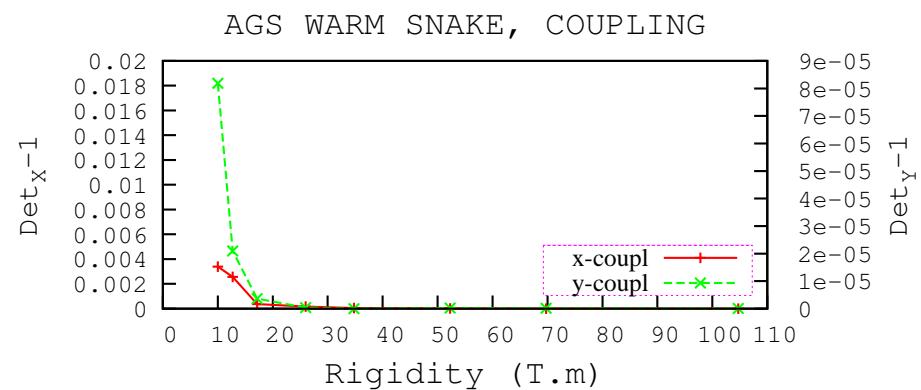
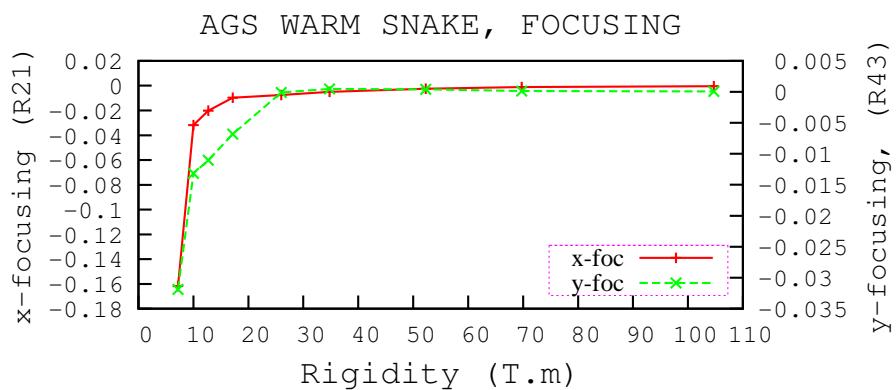


Strengths ($Q_x/Q_y=4.73/4.82$) :

$G\gamma$	$0+$	$12-$	$6+$	$18-$
N_n at 2.5π	0.011	0.0006	0.010	0.012

• Interest : improved optics in AGS with snakes if rigidity is higher :

- $G\gamma = -7.5$ is $B\rho = 6.97$ T.m, even lower than proton $G\gamma = 4.5$, $B\rho = 7.21$;
- lower rigidity is less favorable as to snake optics : focusing perturbed, coupling.



5 Depolarizing resonances in AGS

Imperfection resonances,

$$G\gamma - n = 0$$

- Proton, acceleration range :

$G\gamma : 4.5 \rightarrow 45.5$,

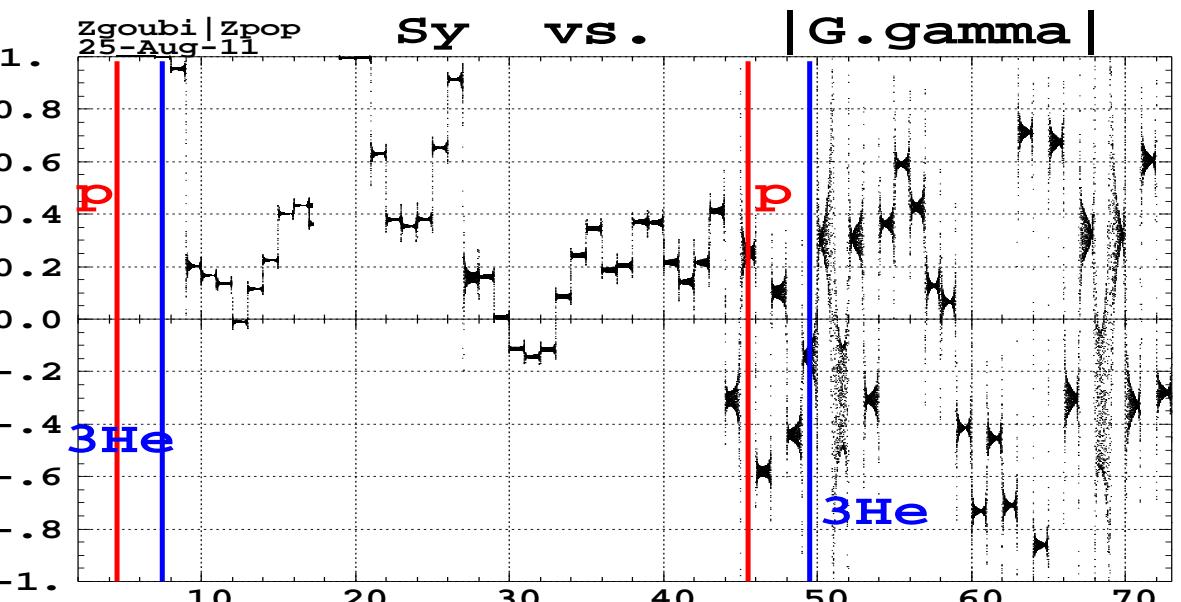
41 integer resonances crossed.

- ${}^3\overline{\text{He}}^{2+}$, possible range :

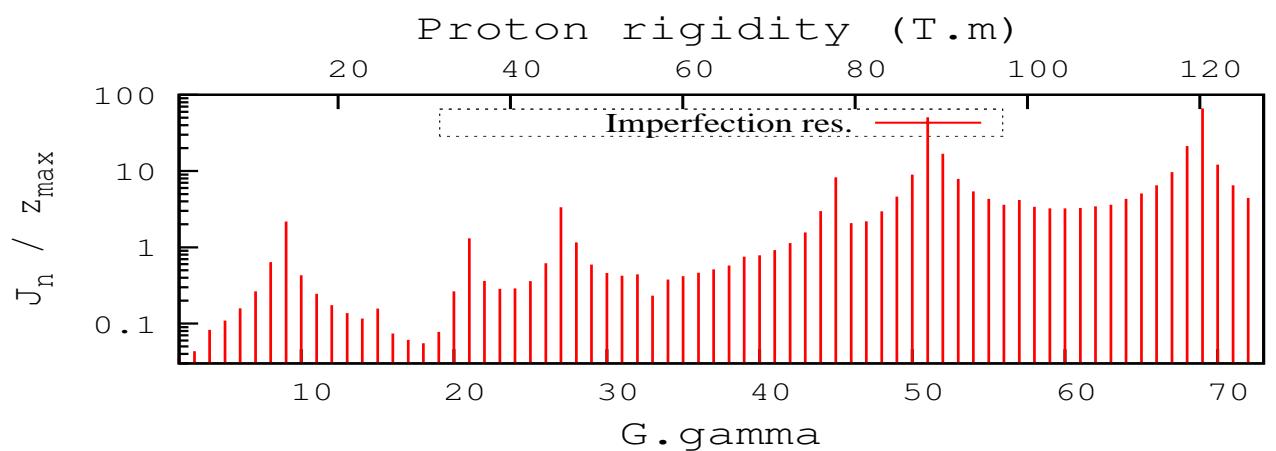
$G\gamma : -7.5 \rightarrow -49.5$,

42 resonances crossed.

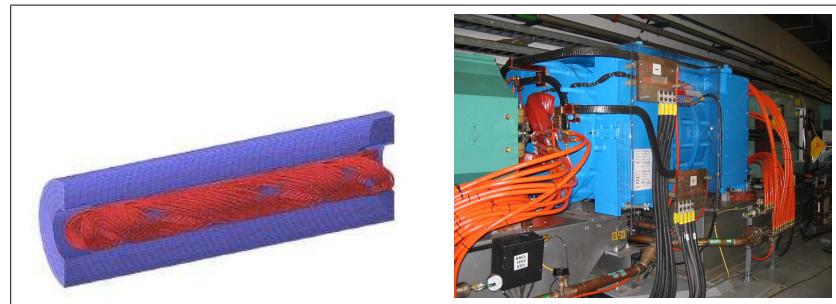
Spin motion, typical :



- Strengths of imperfection resonances in AGS :



- Imperfection resonances in AGS are overcome by spin-flipping, using two helical partial snakes



- A partial snake forces polarization to flip : at Xing of any integer resonance one gets

$$\frac{P_{final}}{P_{initial}} = \left(2 \exp\left(-\frac{\pi}{2\alpha} \left| \frac{\phi}{2\pi} + J_n \right|^2\right) - 1 \right) = -1 \quad \text{at better than } 10^{-3}$$

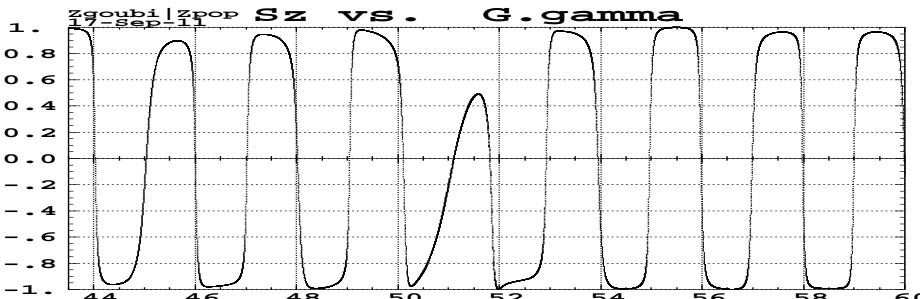
if snake strength $\phi/2\pi$ dominates over strength of imperfection resonances : $\frac{\phi}{2\pi} \gtrsim |J_n| + 2\sqrt{\alpha}$.

- In the AGS, snakes ensure spin rotations $\phi = \begin{cases} \text{proton : Cold/2.1T/21°, Warm/1.5T/11°} \\ {}^3\bar{\text{He}}^{2+} : \text{Cold/2.1T/-44°, Warm/1.5T/-26°} \end{cases}$

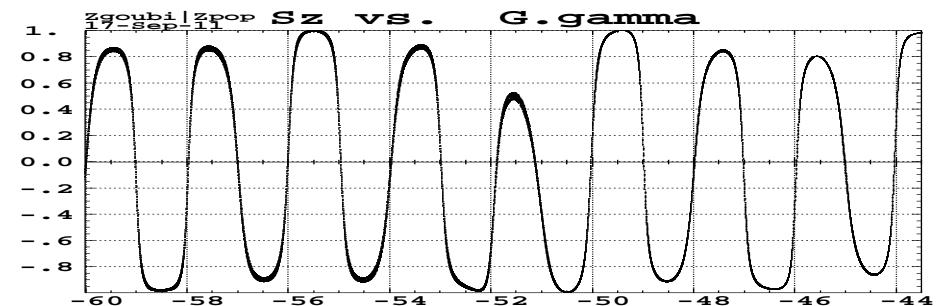
- Example spin flip, below : $G\gamma : 43.5 \rightarrow 60$, rigidities : $\begin{cases} \text{proton : } 75.9 \rightarrow 104.7 \text{ T.m} \\ {}^3\bar{\text{He}}^{2+} : 48.47 \rightarrow 67.0 \text{ T.m} \end{cases}$,

- Single particle ; vertical orbit : $\begin{cases} \text{proton : } \hat{y}_{co} \approx 8 \text{ mm} \\ {}^3\bar{\text{He}}^{2+} : \hat{y}_{co} \approx 5 \text{ mm} \end{cases} ; \epsilon_x, \epsilon_y \approx 0, \delta p/p|_{synch.} < 10^{-4}$.

Proton snakes 12% + 6%



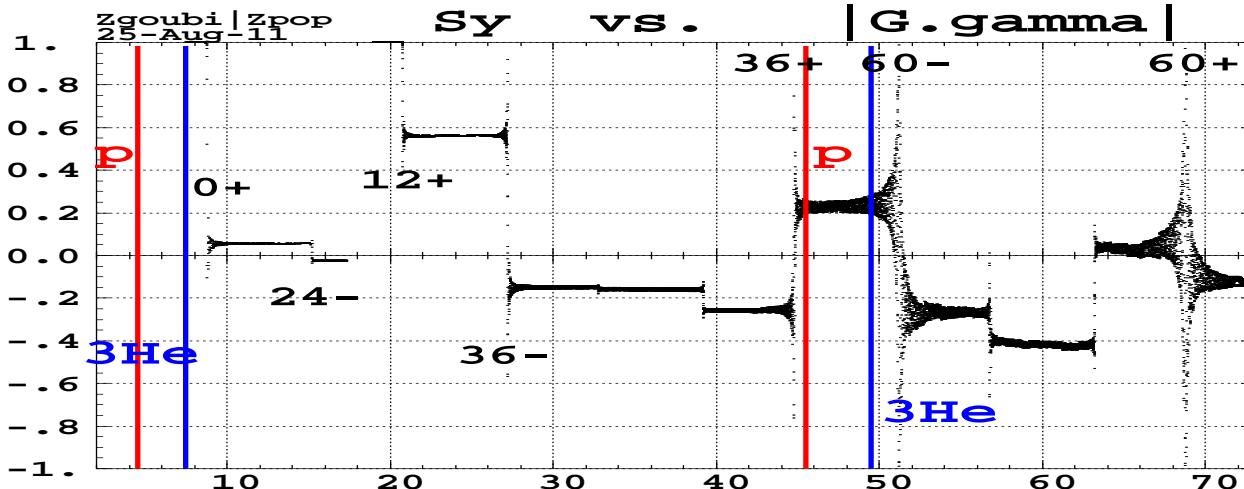
${}^3\bar{\text{He}}^{2+}$, snake settings as p's, i.e. 25% + 15%



Intrinsic resonances, $G\gamma \pm Q_y - n = 0$:

- S_y motion, typical :

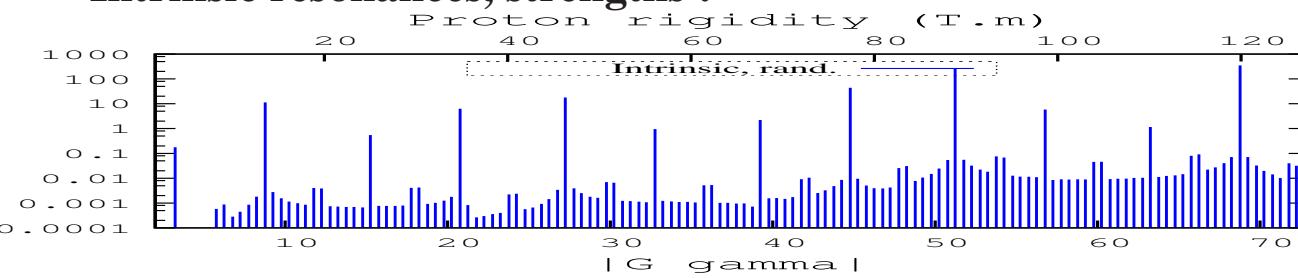
Major lines are systematic resonances ($n=12 \times \text{integer}$), at $G\gamma = 0+Q_y, 12+Q_y, 36-Q_y, 36+Q_y, 60-Q_y$



- Proton, acceleration range :
 $G\gamma : 4.5 \rightarrow 45.5$,
4 strong resonances crossed.

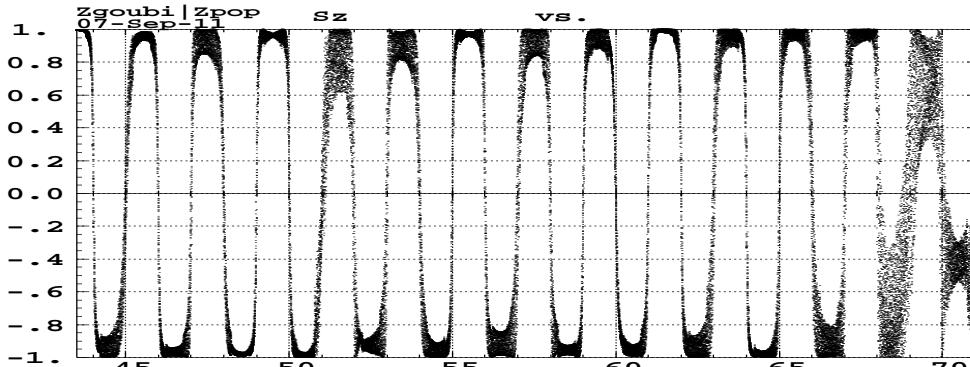
- $^{3}\text{He}^{2+}$, possible range :
 $G\gamma : -7.5 \rightarrow -49.5$,
4 strong resonances crossed.

- Intrinsic resonances, strengths :



$^{3}\text{He}^{2+}$ resonance strengths are stronger than p 's , in the ratio
 $\sqrt{|G_{^{3}\text{He}^{2+}}| / G_p} \approx 1.5$

- Intrinsic resonances in AGS are overcome by pushing Q_y in the snake-induced, forbidden Q_s band



However, non-zero ϵ_x excites

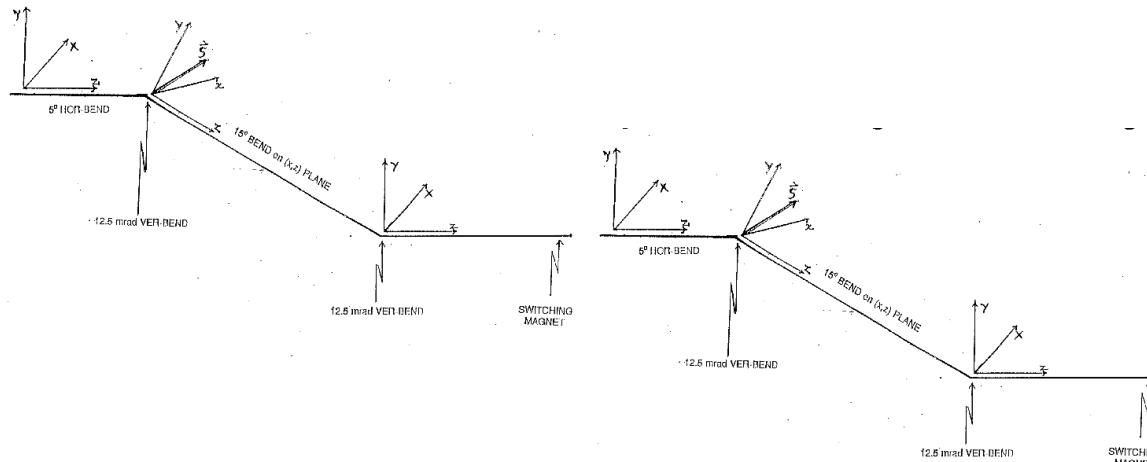
$$G\gamma = \text{integer} \pm Q_s$$

$$(\vec{p}, \epsilon_x \approx 15\pi 10^{-6} \text{ norm.}, \epsilon_y \approx 2\pi 10^{-6} \text{ norm., here})$$

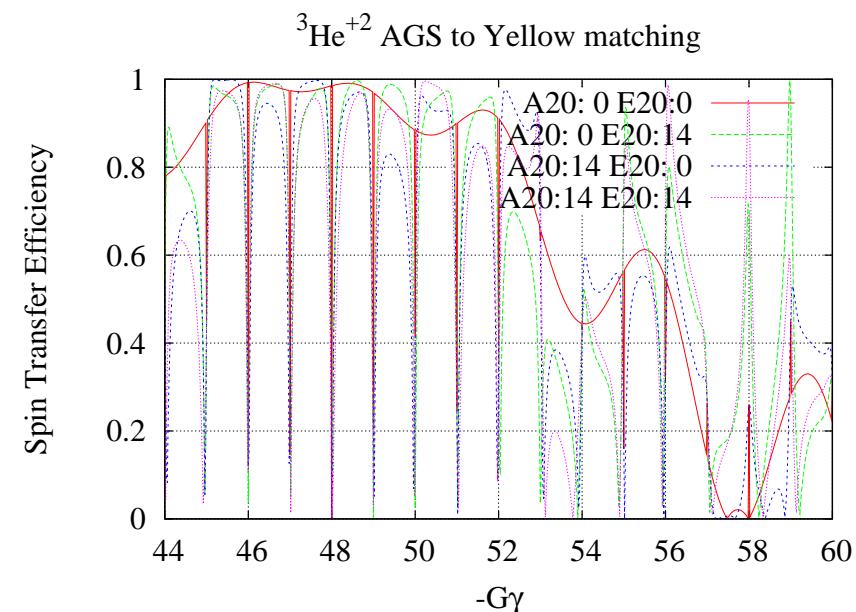
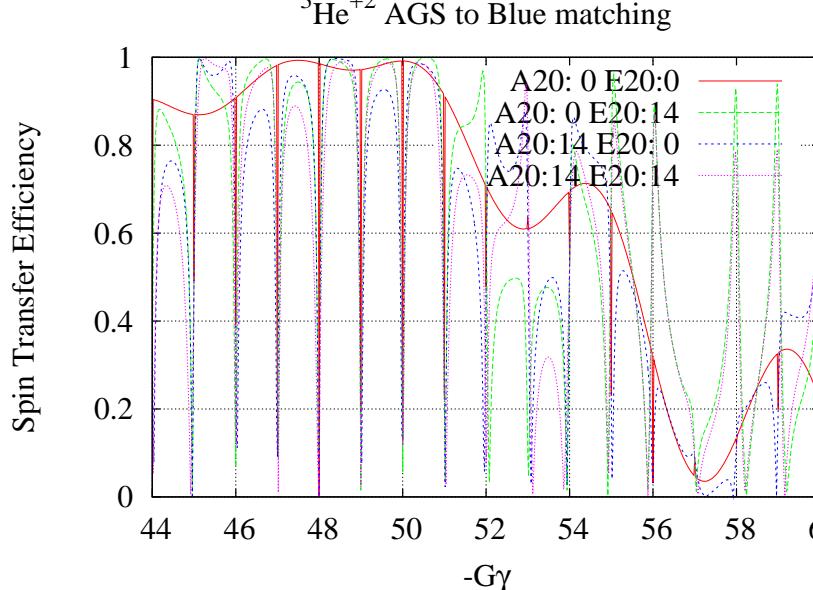
6 AtR matching

- Horizontal bends interleaved with vertical deviations in AtR cause the line to be non-transparent to spin motion.

Hence matching between AGS and RHIC stable spin directions is necessary



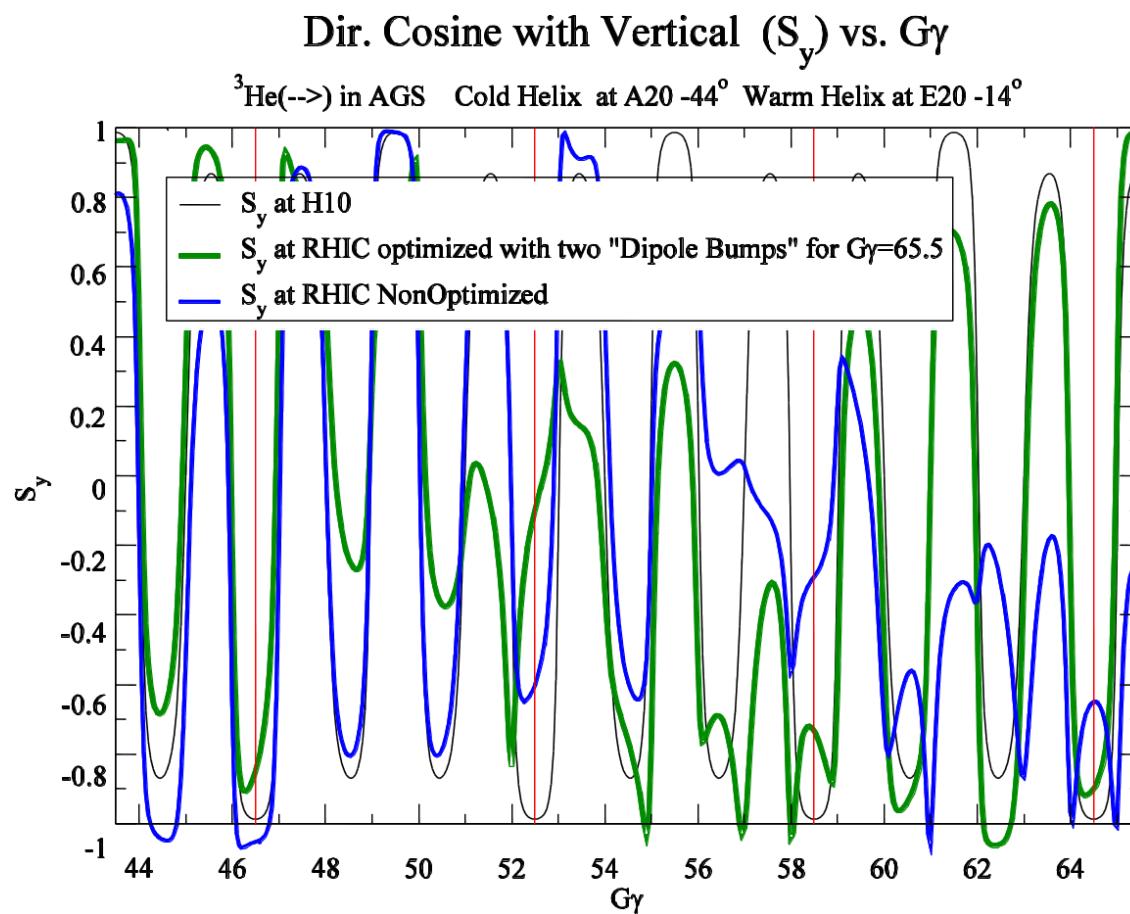
(i) Matching to vertical \vec{n}_0 in RHIC is achievable in the range $G\gamma \simeq 45.5 - 49.5$ – depending on snake settings [Ref. W. MacKay] :



AtR matching

(ii) Extracting ${}^3\text{He}^{2+}$ at higher $B\rho$ from AGS ? (this requires overcoming $G\gamma = 60$ -Qy and higher $G\gamma$ resonances in AGS - to be assessed)

- That raises the question of extending the AtR matching to higher $G\gamma$.
- Matching is possible using two horizontal dipole bumps located at vertical deviations in the line [ref. Nick Tsoupas] :



7 Comments

- In view of preparing future acceleration of ${}^3\overrightarrow{\text{He}}^{2+}$ in RHIC, we might plan the following experiments, in booster and AGS :
 - (i) Assess operation of Booster at Q_y close to ≈ 4 : $G\gamma_3\text{He}^{2+}$ span $4.19 \rightarrow 7.5$ is then free of intrinsic resonances.
 - (ii) Acceleration in booster beyond $B\rho = 7$ T.m : in view of
 - higher rigidity in AGS snakes,
 - bringing $G\gamma_3\text{He}^{2+}$ at AGS injection beyond $0 + Q_y$ AGS depolarizing resonance.
 - (iii) AGS : investigate use of warm snake at higher field.

Possible APEX :

- ${}^3\text{He}^{2+}$ in RHIC :

Demonstration of feasibility : accelerate (non polarized) ${}^3\text{He}^{2+}$ up to 500GeV in RHIC.

THANK YOU

