Nucleon EDM from Lattice QCD

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for RBC/UKQCD collaboration

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CP symmetry breaking in the SM

- EW
  - It has been known the CP violation occurs by the phase of CKM matrix
  - K, D, B meson decay via direct and indirect CP violation
  - Contribution to EDM is very tiny, $\Rightarrow d_N^{\mathrm{KM}} \simeq 10^{-30\sim-33}\,\mathrm{e\cdot cm}$

    6-orders magnitude below the exp. upper limit: $|d_N^{\mathrm{exp}}| < 2.9\times 10^{-26}\,\mathrm{e\cdot cm}$
- QCD
  - θ term in the QCD Lagrangian:

$$\mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}\int d^4x G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M$$

    renormalizable and CP-violation comes due to topological charge density.
  - EDM experiment provides very strong constraint on $\bar{\theta} < 10^{-9}$

    $\Rightarrow$ θ and $\arg\det$ M need to be unnaturally canceled ! (strong CP problem)

CP symmetry breaking beyond the SM

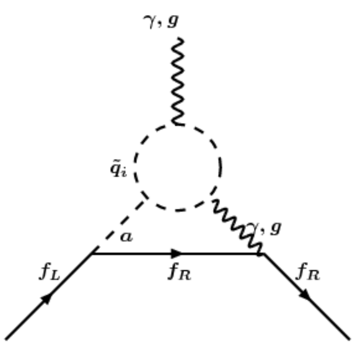
- Possible higher dimension operators
  - Effective Hamiltonian with higher dimension than 4

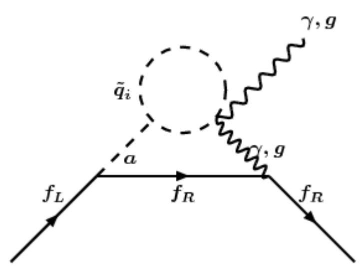
$$H_{\mathrm{C}\not{P}} = \sum_k C_k(\mu)\,\mathcal{O}_k$$

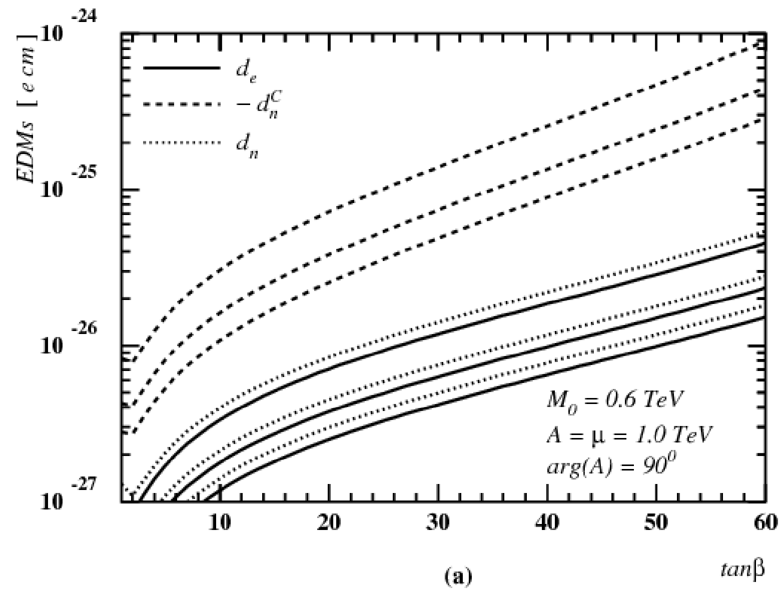
$$\begin{aligned}
\mathcal{O}_{\mathrm{qEDM}} &= \bar{q}(\sigma\cdot F)\gamma_5 q && \text{: Quark-photon} \\
\mathcal{O}_{\mathrm{cEDM}} &= \bar{q}(\sigma\cdot G)\gamma_5 q && \text{: Quark-gluon} \\
\mathcal{O}_{\mathrm{Weinberg}} &= GG\widetilde{G} && \text{: Pure gluonic} \\
&\vdots
\end{aligned}$$

SUSY model

$$d_N \sim d_d^{\mathrm{cEDM}} \sim -\frac{\alpha m_d \tan\beta}{64\pi^3 m_A^2}\sum_{q=t,b}\xi_q Q_q^2 F$$



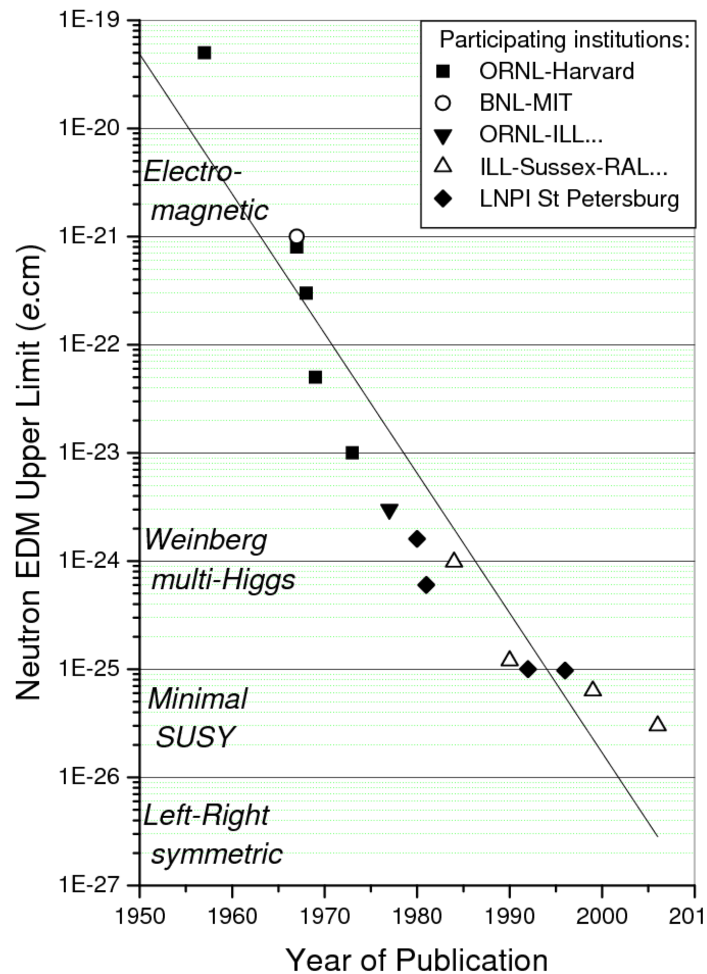




Chang, et al. (99), Ibrahim and Nath (08)

Constraint on nEDM

- The present and future experiment is close to "exclude" of MSSM
  pEDM experiment @ BNL,
  nEDM experiment @ J-PARC, …
  ⇒ reaching a sensitivity of 10^{-29} e･cm !
- Current theoretical bound is based on quark model.
- Non-perturbative computation is necessary to draw more reliable conclusion.



Harris, 0709.3100

What lattice QCD can do for nEDM

- In principle
  - Direct estimate of neutron and proton EDM from θ term, higher dim. $\not{CP}$ operators
  - Matrix elements of higher dimension operators

    $$\langle N|\bar{q}\gamma_5\sigma\cdot Fq|N\rangle,\ \langle N|\bar{q}\gamma_5\sigma\cdot Gq|N\rangle, \langle N|GG\tilde{G}|N\rangle, \cdots$$
- In practice there are some difficulties
  - Statistical error

    Source of CP violation comes from gauge background (topological charge, sea quark) which is intrinsically noisy.

    Disconnected diagram is necessary because of flavor singlet contraction.
  - Systematic error

    Volume effect may be significant.

    Chiral behavior is important, $d_N \sim O(m)$?

Possible lattice methods

- Spectrum method

  Aoki-Gocksch(89), ES et al. CP-PACS(06, 07)

  - Spin splitting of nucleon energy in external electric field and θ term, which is given by 2-pt function: $m_{\uparrow} - m_{\downarrow} = 2d_N\theta E$
  - Computational cost is cheap, and directly obtain EDM.

- Form factor

  ES et al. CP-PACS(05), RBC(06)

$$\langle n(P_1)|J_\mu^{\mathrm{EM}}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\mathrm{P,T\text{-}odd}} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\mathrm{P,T\text{-}even}} + \cdots \Big] u_N^\theta$$

  - F_3 in $Q^2 \to 0$ provides d_N
  - Subtraction to contribution of CP-odd phase in n propagator.

- Imaginary θ
  - Generate new configurations with imaginary θ term, which may enhance signal.

  Izubuchi (07), Horsley et al. (08)

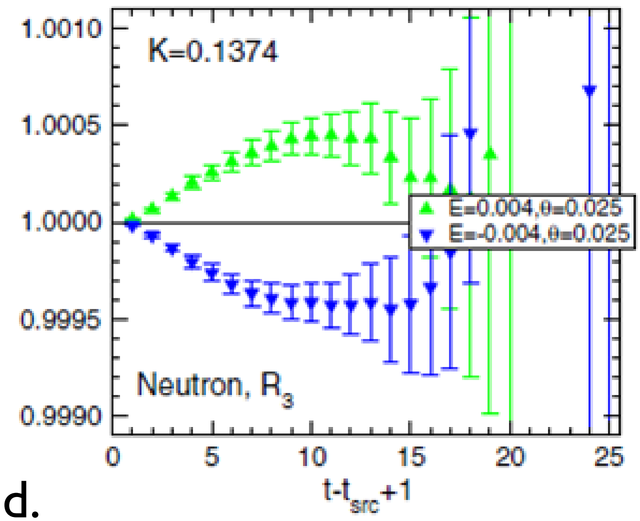
Spectrum method

ES et al. (06, 07)

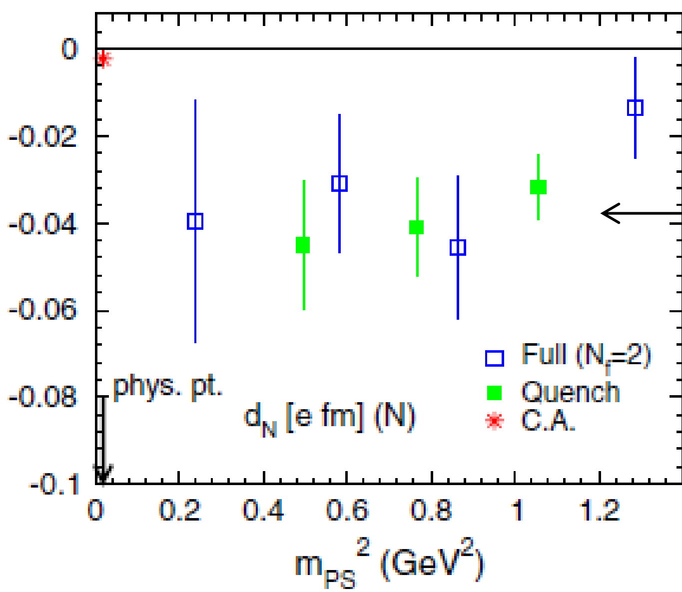
- Ratio of spin up and down

$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\mathrm{up}}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\mathrm{down}}_{\theta,E}} \simeq 1 + d_N E\theta t$$

Linear response, gradient is a signal of EDM.



- Remarks
  - Reweighting works well for small real θ
  - Temporal periodicity is broken by electric field.



Full QCD with clover fermion:

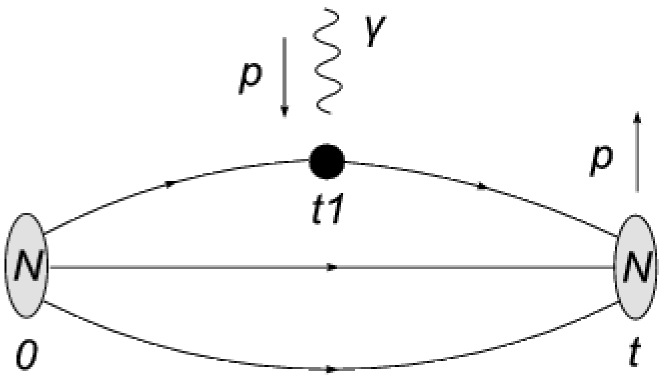
- There seems to be no significant difference between quench and full QCD.
- Statistical error is still large.

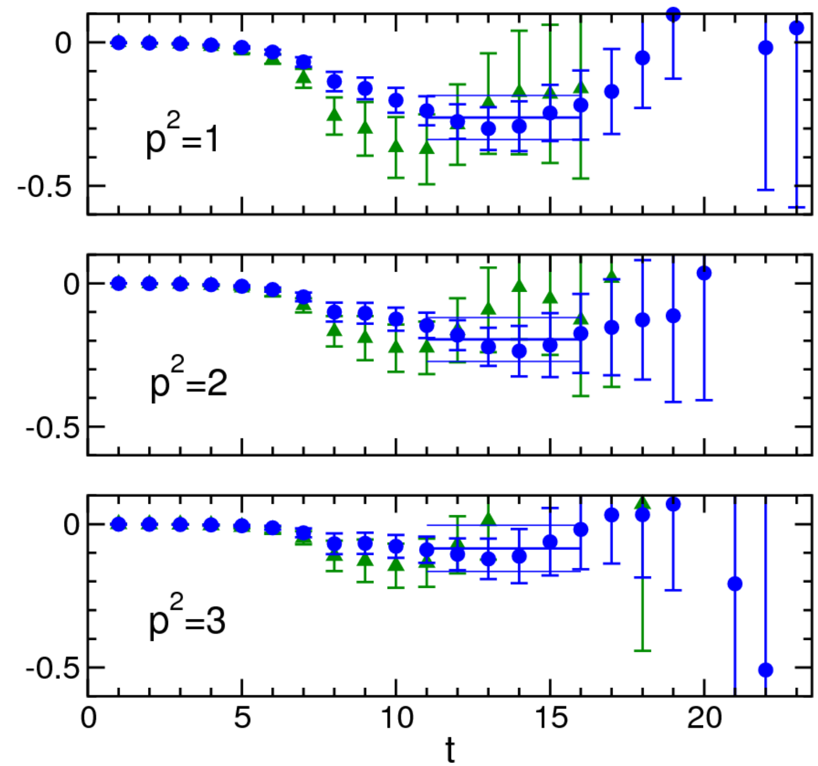
- Finite size effect from breaking of temporal periodicity is also significant

Form factor method

ES(08)

- F_3 signal





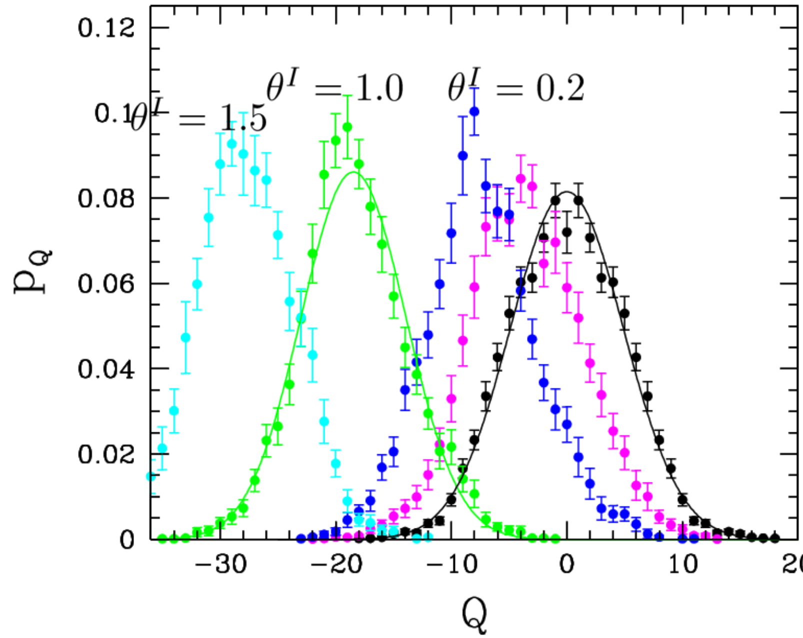
  - Nf=2 clover fermion
  - Sequential source for V current

Imaginary θ

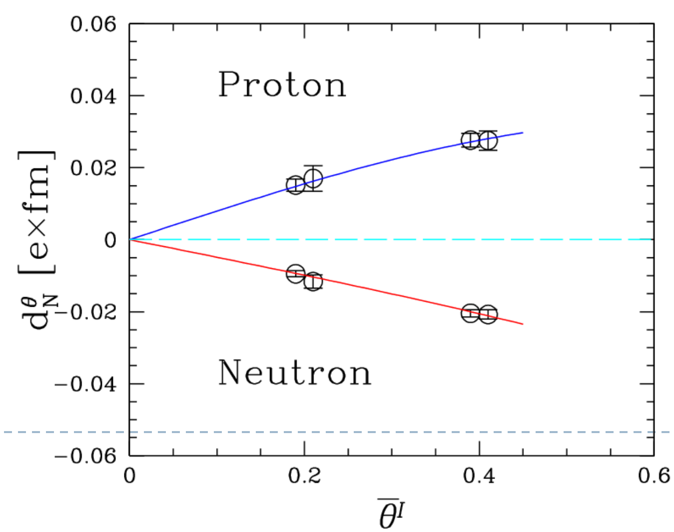
Izubuchi(07), Horsley et al. (08)

- Analytically continued to pure imaginary, $\theta \to i\theta^I$

$$\langle O e^{i\theta Q} \rangle \to \langle O e^{-\theta^I Q} \rangle$$



  - There is no sign problem, expect better signal.
  - Generate the QCD ensemble with θ^I: distribution of topological charge is shifted by θ^I



Full QCD with clover fermion

- EDM is given by the slope.
- Clear signal, but systematic error due to chiral symmetry breaking of clover fermion has not been taken into account.

Comparison of results

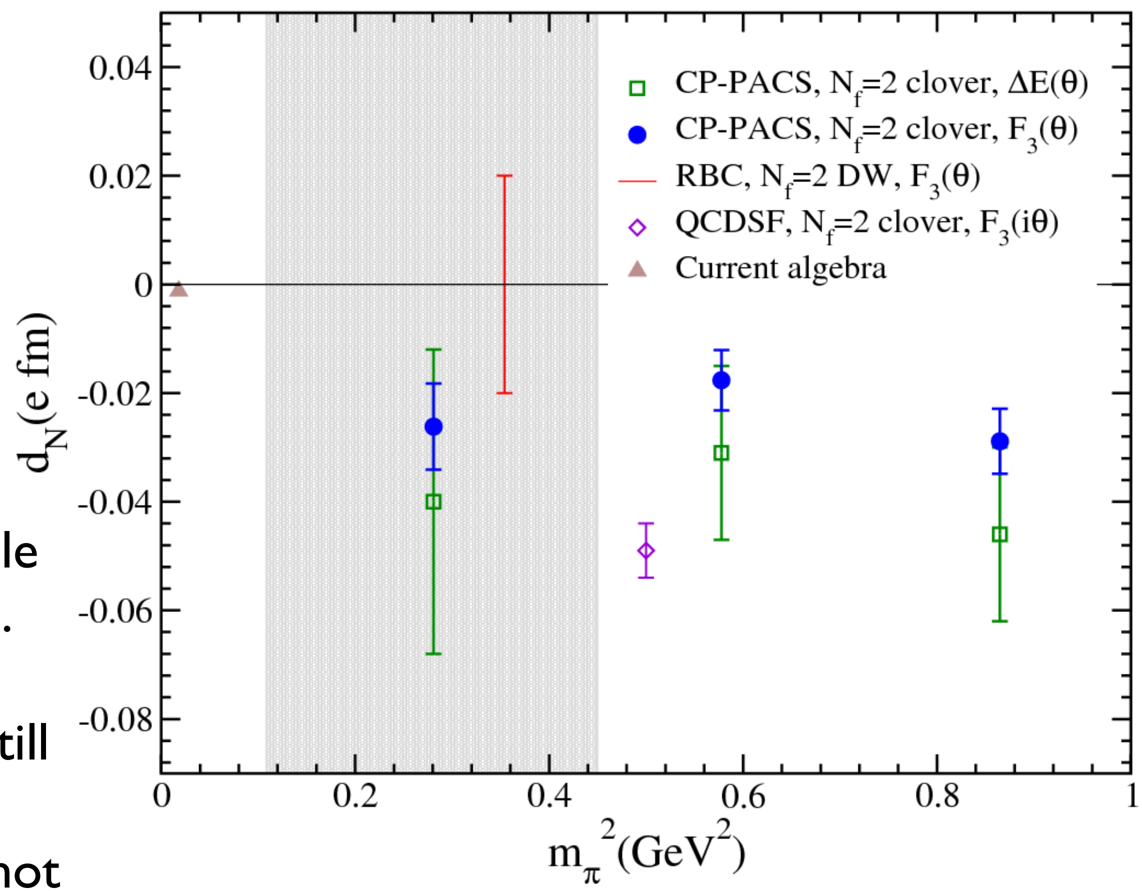
Full QCD

- Lattice results are consistent within 1σ.

- An order of magnitude larger than the results of current algebra.

- N_f = 2+1 DWF configs. (RBC/UKQCD) are available for near physical pion mass.

- Large statistical error is still problem.
(O(100) measurements is not enough)



Error reduction techniques

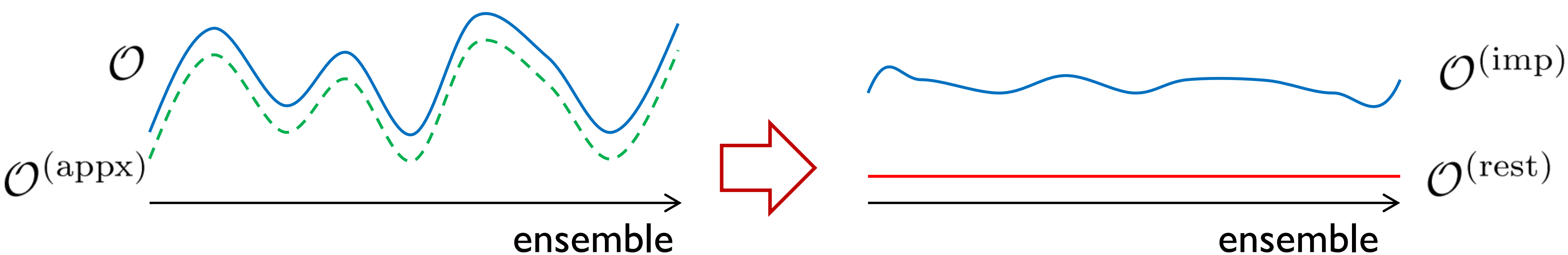
RBC in prep.

- Covariant approximation averaging (CAA)
  - For original observables O, (unbiased) improved estimator

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

which satisfies $\langle O\rangle = \langle O^{\mathrm{imp}}\rangle$ if approximation is covariant under lattice symmetry g, and error becomes $\boxed{\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}}$

  - Ideal approximation



- Ignoring the error from $O^{(\mathrm{rest})}$
- There may be many candidates of $O^{(\mathrm{appx})}$ e.g. LMA, heavy mass, …
- The cost of approximated observable need to be smaller than the original.

Examples of CAA

- Lowmode averaging (LMA)

  Guisti et al.(04), Neff et al.(01), DeGrand et al. (04)

  - Using lowlying eigenmode of Dirac operator to approximate propagator:

$$\mathcal{O}^{(\mathrm{appx})} = \sum_{\lambda}^{N_\lambda} \mathcal{O}_\lambda^{\mathrm{low}}$$

  where N_λ is number of lowmode computed by Lanczos.

  Except for computational cost of eigenmode, Cost(LMA) $\backsimeq$ 0, but approximation is only lowmode part (long distance contribution).

- All-mode averaging (AMA)
  - Using sloppy CG (loose stopping condition),

$$\mathcal{O}^{(\mathrm{appx})} = O^{\mathrm{sloppy}}$$

  If stopping cond. is 0.003, Cost(AMA) $\backsimeq$ Cost(CG)/50(without deflation).

  Approximation becomes better than LMA for other than lowmode dominanted observables (nucleon, finite momentum hadron, …).

Examples of Covariant Approximations

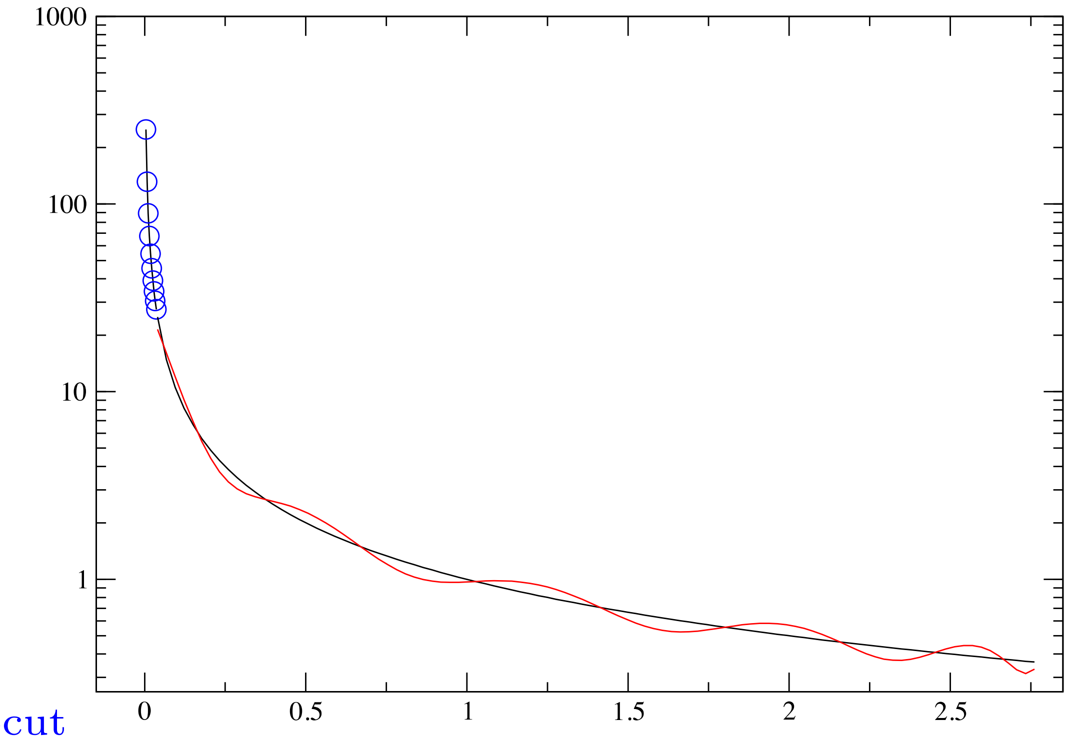
- **All Mode Averaging**
  AMA
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$



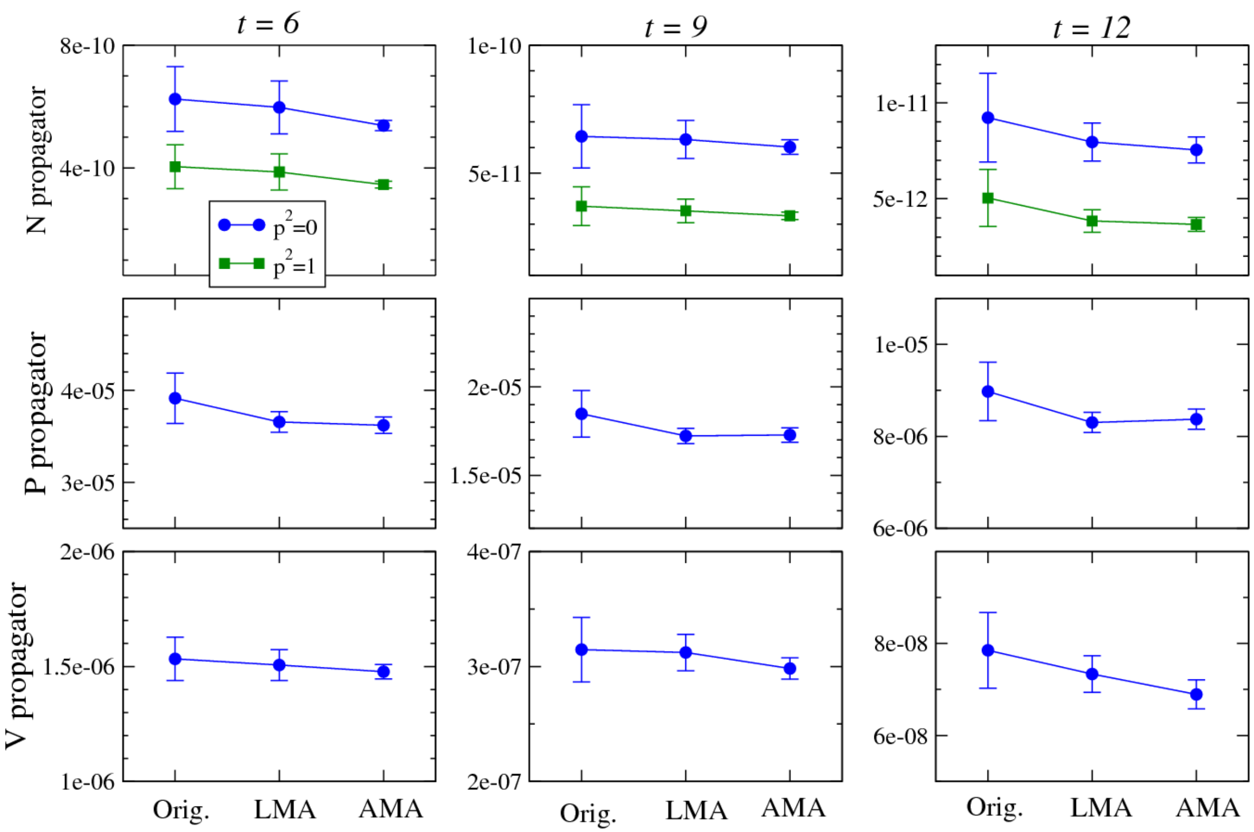
accuracy control :

- low mode part : # of eig-mode
- mid-high mode : degree of poly.

Comparison between LMA/AMA

RBC in prep.

- Preliminary result
  - 8 configs, Gaussian smearing, $N_G = 2^3 \times 4 = 32$ sources, $24^3 64 \times 16$ DWF

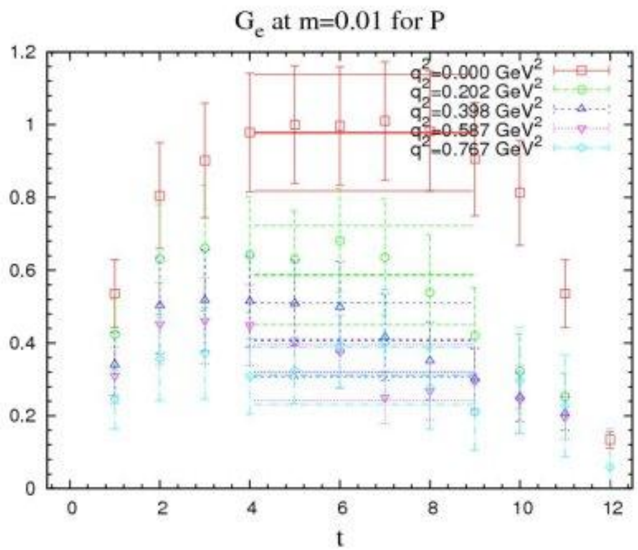


- t = 6:
  Error in AMA is actually reduced by factor 5 compared with orig. and LMA.

- t = 12
  Error in AMA/LMA is reduced by factor 3--4 compared with original.

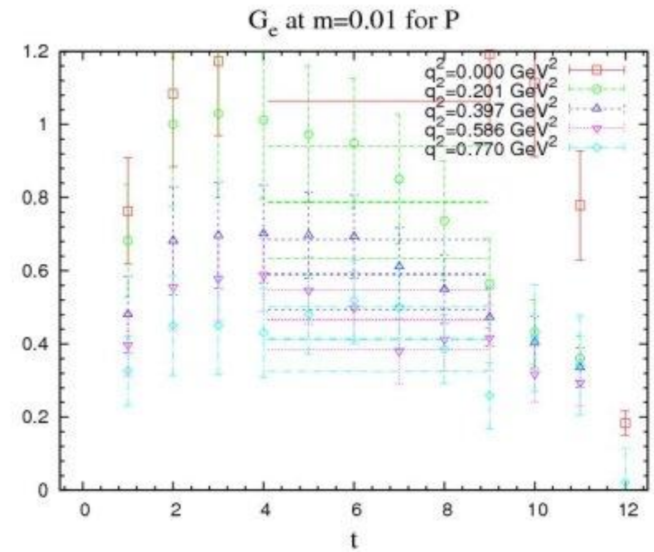
Comparison between LMA/AMA

- Very preliminary

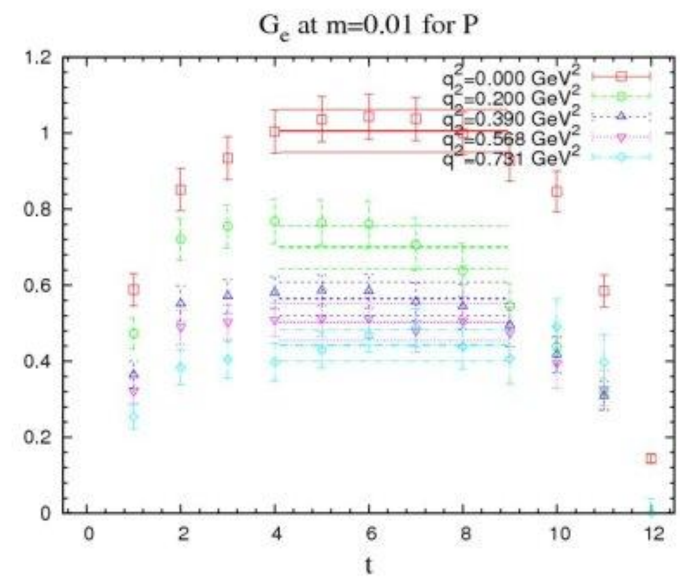


Proton Ge (Original)

Proton Ge (LMA)



Proton Ge (AMA)



Conclusion and future work

- Nucleon EDM in lattice QCD
  - Large statistical error is problem.
  - LMA/AMA may work well.
  - Aim for less than 10% statistical error.
  - Systematic study of finite size effect, chiral behavior, …
  - Other source ~~CP~~ effect

Thank you.

Backup

Electric dipole moment (EDM)

- P(arity), T(ime reversal)[=CP] symmetry breaking

EDM:

$$\vec{D} = d\vec{S}, \quad \Delta H = \vec{D} \cdot \vec{E}$$

under discrete symmetries, spin and E have different behavior

$$\begin{array}{lll} \mathrm{P}: & \vec{E} \to -\vec{E} & \vec{S} \to \vec{S} \\ \mathrm{T}: & \vec{E} \to \vec{E} & \vec{S} \to -\vec{S} \end{array}$$

If $d \neq 0$,

$$\mathrm{P}, \mathrm{T} : \Delta H \to -\Delta H$$

Non-vanishing EDM is a signal of the P, CP violation.

- In EW P, CP violation following Kobayashi-Maskawa mechanism.
- In QCD, it is natural to exist but there has been no signal the breaking would be also.

Strong CP problem ?

- Possible solution
  - Massless quark

    One of the quark flavor is massless (m_u = 0 or m_d = 0),

    i.e. $\arg\det M \propto m_u m_d m_s/(m_u+m_d+m_s) = 0$

    This has been refused by spectrum study in lattice QCD+QED.

  - Axion model
    - Pecci-Quinn (additional chiral) symmetry is spontaneously broken.

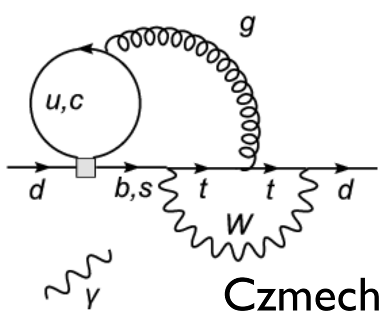
      Axion of (in-)visible model has been almost excluded by cosmology.

  - Spontaneous breaking

CP symmetry breaking in the SM

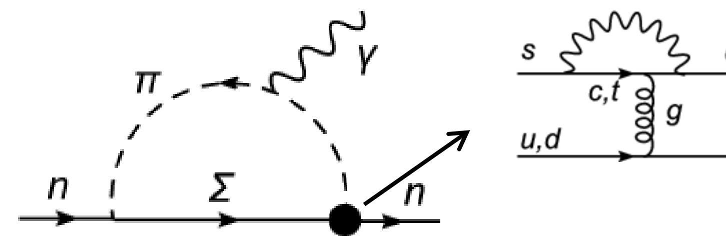
- Contribution to EDM from weak interaction is very small
  - Vanishing 1-loop (no Im part), 2-loop diagram
  - Three-loop order(short) and pion loop correction (long):

Short distance



Czmechi, Krause (1997)

Long distance



Khriplovich, Zhitnitsky (1982)

$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G^2) \sim -10^{-34}\,\mathrm{e\cdot cm}$ $\qquad$ $d_N^{\mathrm{KM\,long}} \sim 10^{-30\sim -33}\,\mathrm{e\cdot cm}$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30\sim -33}\,\mathrm{e\cdot cm}$$

which is the 6-order magnitude below the exp.
upper limit: $|d_N^{\mathrm{exp}}| < 2.9 \times 10^{-26}\,\mathrm{e\cdot cm}$

Spectrum method

Aoki-Gocksch(89), ES(06,07), Horsley(07), QCD-SF(08)

- Energy difference between nucleon spin
  - Energy eigenvalue if θ exist in the background

$$\mathcal{E}_{+S_3}(\vec{E}) - \mathcal{E}_{-S_3}(\vec{E}) = 2d_N\bar{\theta}S_3E_3 + \mathcal{O}(E^2, \bar{\theta}^2, E\bar{\theta})$$

in the case of $\vec{E}(t) = (0, 0, E_3)$

  - 2-pt function provides EDM as exponents

$$\langle N(t)\bar{N}(0)\rangle^{\mathrm{upper}}_{\theta,E} \simeq Z(1 + A_N(E^2)\theta E_3)\exp(-m_N t - d_N\theta E_3 t/2)$$

$$\langle N(t)\bar{N}(0)\rangle^{\mathrm{lower}}_{\theta,E} \simeq Z(1 - A_N(E^2)\theta E_3)\exp(-m_N t + d_N\theta E_3 t/2)$$

d_N is given by fitting with the above asymptotic function.

  - Two reweighting method

$$\begin{aligned}\langle N\bar{N}\rangle_{\theta,E} &= \langle N\bar{N}(Ue^{q_e E})e^{i\theta Q}\rangle && \text{Eucleadian E, real } \theta \\ &= \langle N\bar{N}(Ue^{iq_e E})e^{\theta m_q \bar{q}\gamma_5 q}\rangle && \text{Minkowski E, imaginay } \theta\end{aligned}$$

Form factor

ES(05, 08)

- Matrix element

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\rm P,T\text{-}odd} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\rm P,T\text{-}even} + \cdots \Big] u_N^\theta$$

  - F_3 in $Q^2 \to 0$ is equivalent to d_N
  - Expansion of 3pt func. at $O(\theta)$ into different CP-odd sources:

$$\langle N(t)J_\mu^{\rm EM}(t_1)\bar{N}(0)Q\rangle$$
$$= \Big[u_N \otimes \bar{u}_N \frac{F_3}{2m_N}Q_\nu\sigma_{\mu\nu}\gamma_5 u_N \otimes \bar{u}_N$$
$$+u^{\theta^1} \otimes \bar{u}_N^{\theta^1}\Big(F_1\gamma_\mu + \frac{F_2}{2m_N}Q_\nu\sigma_{\mu\nu}\Big)u_N \otimes \bar{u}_N$$
$$+u_N \otimes \bar{u}_N\Big(F_1\gamma_\mu + \frac{F_2}{2m_N}Q_\nu\sigma_{\mu\nu}\Big)u^{\theta^1} \otimes \bar{u}_N^{\theta^1}\Big]e^{-E_N(P_1)t_1 - E_N(P_2)(t-t_1)}$$

Subtraction of CP-odd phase in n propagator (2nd and 3rd terms) is essential.

Error reduction technique

- Statistical error

$$\mathrm{err} \simeq C/\sqrt{N_{\mathrm{mes}}}$$

  - In order to reduce error,

    do more N_{mes} independent measurements.

    change to C of observables with small fluctuation.

  - Due to limited gauge ensembles, usually covariant observables under lattice symmetry O^g are regarded as independent measurements:

$$\langle O \rangle = \langle O^g \rangle$$

    e.g. g : lattice rotation, translation, …

  - Problem is computational cost.

Lattice QCD's works

- One of the most successful non-perturbative calculation in the particle physics.
  - Reproduce the hadron spectrum using a few input parameters.

    BMW, PACS-CS, …
  - Monte-Carlo simulation is powerful tool.
  - Precision of lattice computations are getting better year by year thanks to development of algorithm (improved HMC, CAA) and machine (GPGPU, Blue Gene, Kei, …).
  - Flexible methodology to apply other physics concerned with strong interaction (e.g. many flavor, Graphene, …)