Baryon spectrum from $N_f = 2 + 1 + 1$ twisted mass fermions

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Outline

1. Introduction - Motivation
2. Lattice evaluation
   - Wilson twisted mass action for $N_f = 2+1+1$
   - Simulation details
   - Scale setting
   - Tuning of the strange and charm quark mass
   - Effective mass
3. Results
   - Isospin symmetry breaking
   - Chiral and continuum extrapolation
   - Comparison
4. Conclusions

Why we want to calculate baryon masses?

- easy to calculate

  first quantities one calculates before proceeding with more complex observables

- large signal to noise ratio

  reliable way to study lattice effects

- significant for on-going experiments

  observation of doubly-charmed Ξ baryons (SELEX, hep-ex/0208014, hep-ex/0209075, hep-ex/0406033) - interest in charmed baryon spectroscopy

- are the experimentally known masses reproduced?

  safe and reliable predictions for the rest

- doublet of light quarks: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ R. Frezzotti et al. `arXiv:hep-lat/0306014`

Transformation of quark fields:
$$\left.\begin{aligned} \psi(x) &= \frac{1}{\sqrt{2}}\left(\mathbb{1} + i\tau^3\gamma_5\right)\chi(x) \\ \overline{\psi}(x) &= \overline{\chi}(x)\frac{1}{\sqrt{2}}\left(\mathbb{1} + i\tau^3\gamma_5\right) \end{aligned}\right\}$$

mass term
$$\overline{\psi}m\psi \to \overline{\chi}i\gamma_5\tau^3 m\chi$$

$$S_F^{(l)} = a^4 \sum_x \overline{\chi}(x)\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu + m_{0,l} + i\gamma_5\tau^3\mu\right]\chi(x)$$

- heavy quarks: $\chi_h = \begin{pmatrix} s \\ c \end{pmatrix}$ In the sea we use the action: R. Frezzotti et al. `arXiv:hep-lat/0311008`

$$S_F^{(h)} = a^4 \sum_x \overline{\chi}_h(x)\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu + m_{0,h} + i\mu_\sigma\gamma_5\tau^1 + \tau^3\mu_\delta\right]\chi_h(x)$$

presence of τ^1 introduces mixing of the strange and charm flavors
valence sector: use Osterwalder-Seiler valence heavy quarks $\chi^{(s)} = (s^+, s^-)$, $\chi^{(c)} = (c^+, c^-)$

re-tuning of the strange and charm quark masses required

Wilson TM at maximal twist

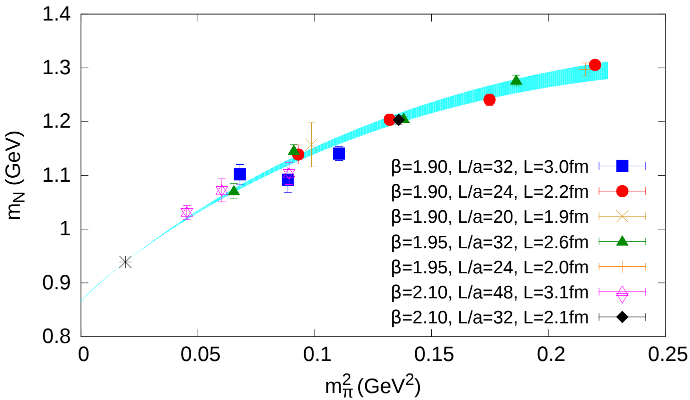
- cut-off effects are automatically $\mathcal{O}(a)$ improved
- no operator improvement is needed (important for nucleon structure)

- Total of 10 $N_f = 2+1+1$ gauge ensembles produced by ETMC (R. Baron et al. (ETMC) arXiV:1004.5284)

	$\beta = 1.90,\ a = 0.0936(13)$ fm				
$32^3 \times 64,\ L = 3.0$ fm	$a\mu$	0.0030	0.0040	0.0050	
	No. of Confs	200	200	200	
	m_π (GeV)	0.2607	0.2975	0.3323	
	$m_\pi L$	3.97	4.53	5.05	
	$\beta = 1.95,\ a = 0.0823(10)$ fm				
$32^3 \times 64,\ L = 2.6$ fm	$a\mu$	0.0025	0.0035	0.0055	0.0075
	No. of Confs	200	200	200	200
	m_π (GeV)	0.2558	0.3018	0.3716	0.4316
	$m_\pi L$	3.42	4.03	4.97	5.77
	$\beta = 2.10,\ a = 0.0646(7)$ fm				
$48^3 \times 96,\ L = 3.1$ fm	$a\mu$	0.0015	0.002	0.003	
	No. of Confs	196	184	200	
	m_π (GeV)	0.2128	0.2455	0.2984	
	$m_\pi L$	3.35	3.86	4.69	

- two lattice volumes
- pion masses from 210-430 MeV $\rightarrow$ chiral extrapolations
- three values of the lattice spacing $\rightarrow$ investigation of finite lattice effects

- for baryon masses $\rightarrow$ physical nucleon mass
- dedicated high statistics analysis on 17 $N_f = 2+1+1$ ensembles
- use HBχPT leading one-loop order result $m_N = m_N^{(0)} - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$
- fit simultaneously for $\beta = 1.90$, $\beta = 1.95$ and $\beta = 2.10$
- systematic error due to the chiral extrapolation $\rightarrow$ use $\mathcal{O}(p^4)$ HBχPT with explicit Δ-degrees of freedom



β	a (fm)
1.90	0.0936(13)(35)
1.95	0.0823(10)(35)
2.10	0.0646(7)(25)

- fitting for each β separately yields consistent values - negligible cut-off effects for the nucleon case
- light σ-term for nucleon $\sigma_{\pi N} = 64.9(1.5)(19.6)$ MeV

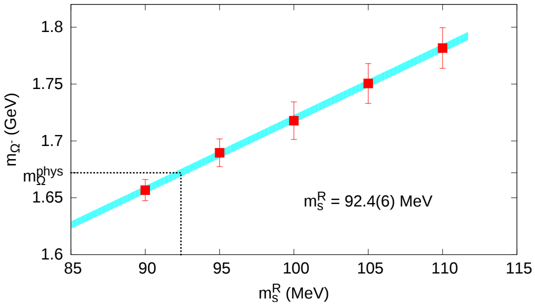
- use $\mathbf{\Omega^-}$ for strange quark and $\mathbf{\Lambda_c^+}$ for charm quark
- fix renormalized strange and charm masses using non-perturbatively determined renormalization constants (N. Carrasco et al. arXiv:1403.4504) in the $\overline{MS}$ scheme at 2 GeV

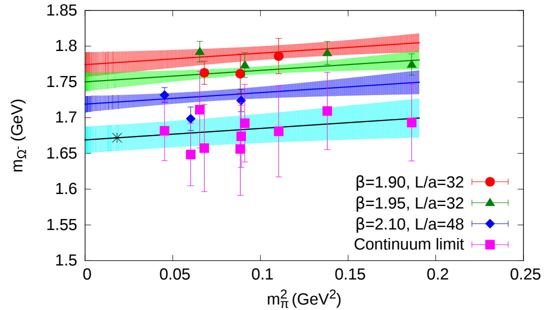
Strange quark mass tuning

- use a set of strange quark masses to interpolate the mass of Ω^- to a given value of m_s^R and extrapolate to the continuum and physical pion mass using

$$m_\Omega = m_\Omega^0 - 4c_\Omega^{(1)} m_\pi^2 + da^2$$

- match with physical mass of Ω^-



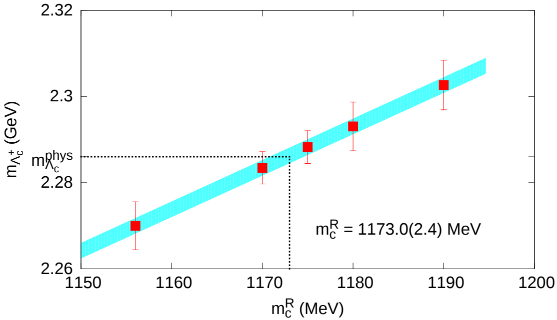


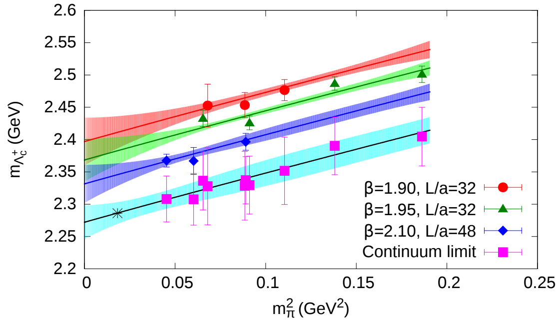
$$\overline{MS} : m_s^R(2\ \text{GeV}) = 92.4(6)(2.0)\ \text{MeV}$$

Charm quark mass tuning

- follow the same procedure using Λ_c^+ and fit using

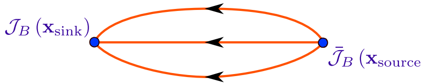
$$m_{\Lambda_c} = m^0_{\Lambda_c} + c_1 m_\pi^2 + c_2 m_\pi^3 + da^2$$





$$\overline{MS} : m_c^R(2\ \text{GeV}) = 1173.0(2.4)(17.0)\ \text{MeV}$$

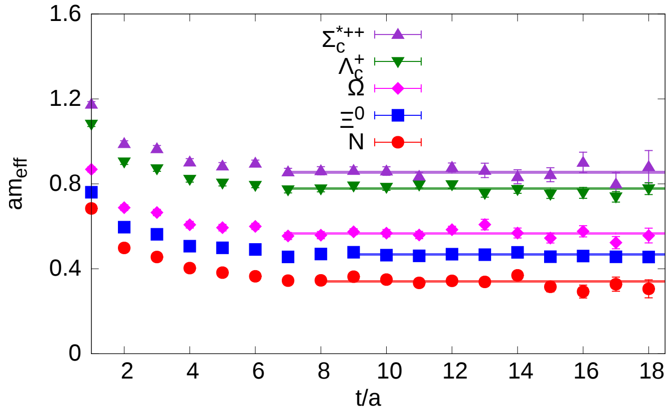
- Effective masses are obtained from two-point correlation functions



$$C_B^{\pm}(t,\vec{p}=\vec{0}) = \sum_{\mathbf{x}_{\text{sink}}} \left[\frac{1}{4}\mathrm{Tr}\left(1\pm\gamma_0\right)\left\langle \mathcal{J}_B\left(\mathbf{x}_{\text{sink}}\right)\bar{\mathcal{J}}_B\left(\mathbf{x}_{\text{source}}\right)\right\rangle\right], \quad t = t_{\text{sink}} - t_{\text{source}}$$

  - Gaussian smearing at source and sink, APE smearing at spatial links
  - source position chosen randomly

$$am_{\text{eff}}^B(t) = \log\left(\frac{C_B(t)}{C_B(t+1)}\right)$$

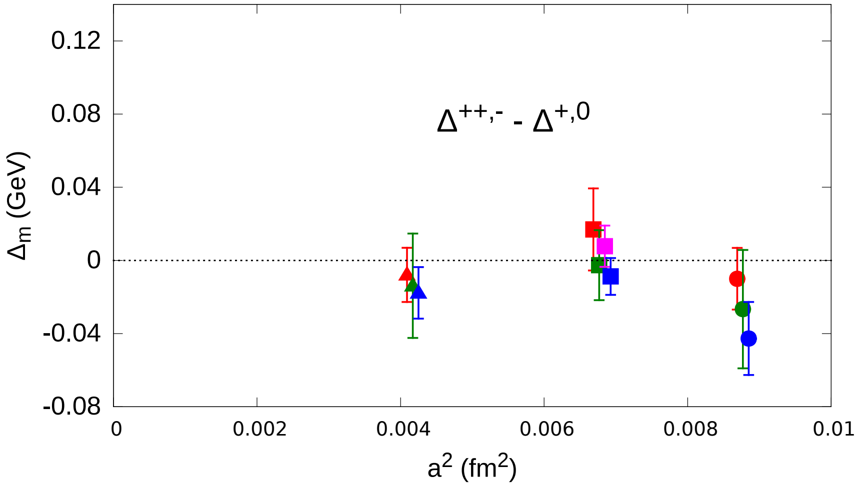


Results I: Isospin symmetry breaking

- Wilson twisted mass action breaks isospin symmetry explicitly to $\mathcal{O}(a^2)$
- it is expected to be zero in the continuum limit
- manifests itself as mass splitting between baryons belonging to the same isospin multiplets due to lattice artifacts
- $u \longleftrightarrow d$ is a symmetry, e.g. Δ^{++}(uuu), Δ^{-}(ddd) and Δ^{+}(uud), Δ^{0}(ddu) are degenerate

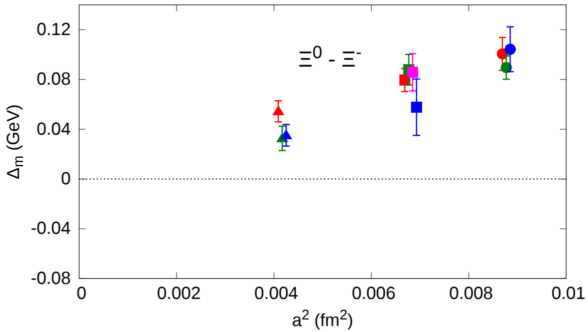
Results I: Isospin symmetry breaking

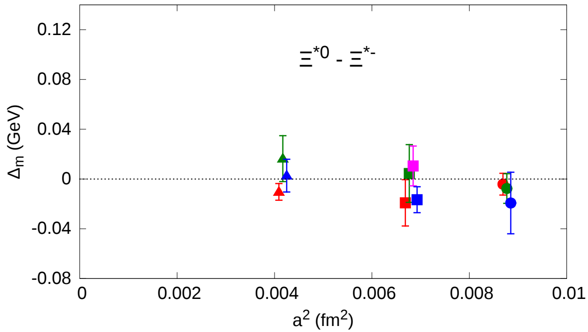
- Δ baryons



- isospin splitting effects are consistent with zero for all lattice spacings and pion masses

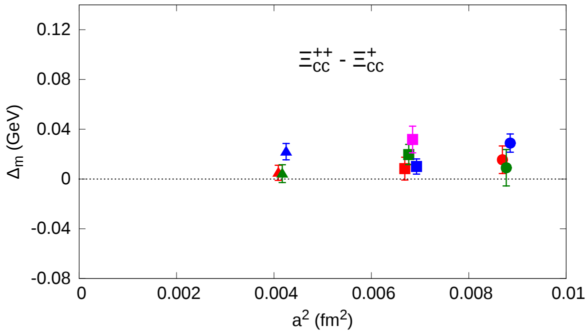
- Hyperons

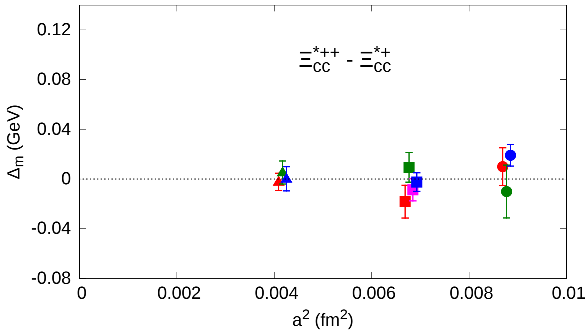




- small mass splittings for the spin-$1/2$ hyperons - decreased as $a \longrightarrow 0$
- isospin splitting consistent with zero for spin-$3/2$ hyperons

- Charmed baryons



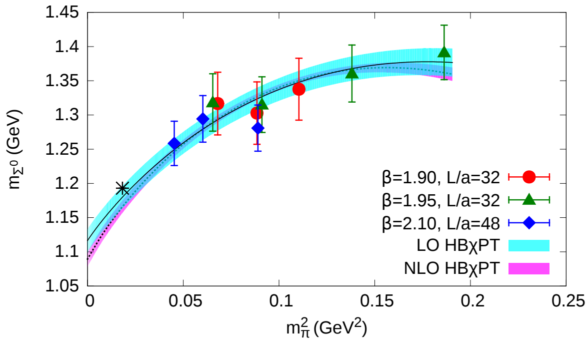


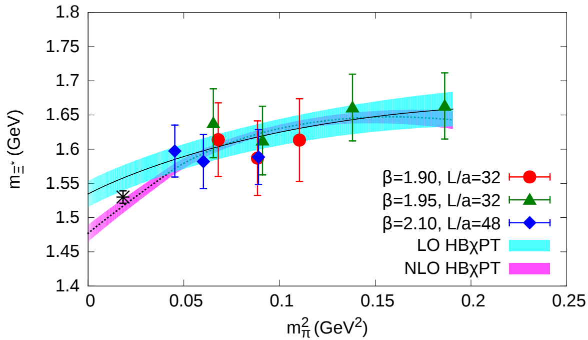
- very small effects for spin-1/2 charmed baryons
- no isospin symmetry breaking for spin-3/2 charmed baryons

- fit in the whole pion mass range 210-430 MeV
- include all β's
- allow for cut-off effects by including a term $\propto a^2$

Hyperons

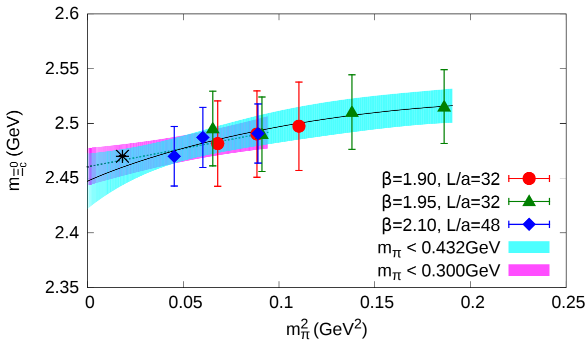
- use leading one-loop order continuum HBχPT
- systematic error due to the chiral extrapolation $\rightarrow$ use $\mathcal{O}(p^4)$ HBχPT

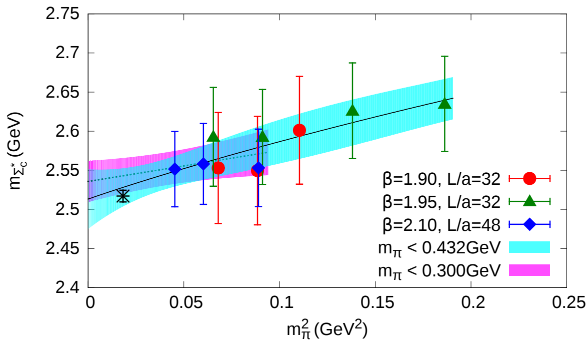




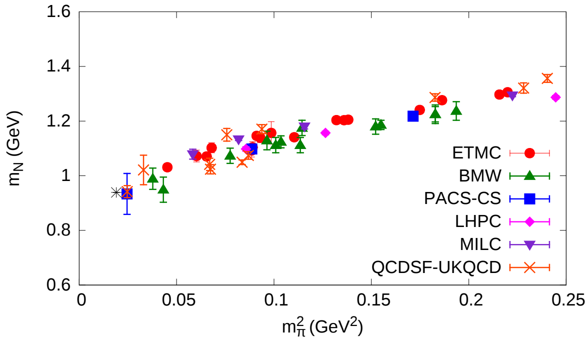
Charmed baryons

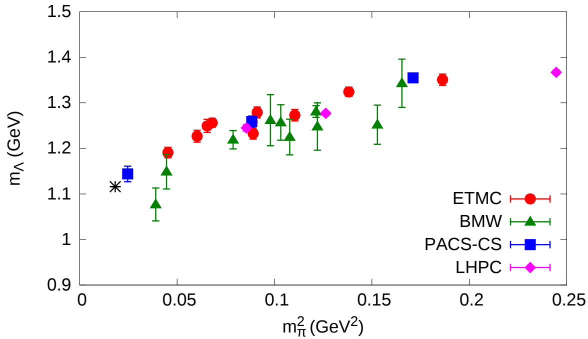
- use Ansatz $m_B = m_B^{(0)} + c_1 m_\pi^2 + c_2 m_\pi^3 + d a^2$
- systematic error due to the chiral extrapolation $\rightarrow$ set $c_2 = 0$ and restrict $m_\pi < 300$ MeV





- systematic error due to the tuning for all baryons
- finite-a corrections $\sim 1\% - 9\%$ - cut-off effects are small
- reproduction of experimentally known baryon masses $\rightarrow$ Predictions!





BMW: $N_f = 2+1$ clover fermions S. Durr et al. arXiV:0906.3599

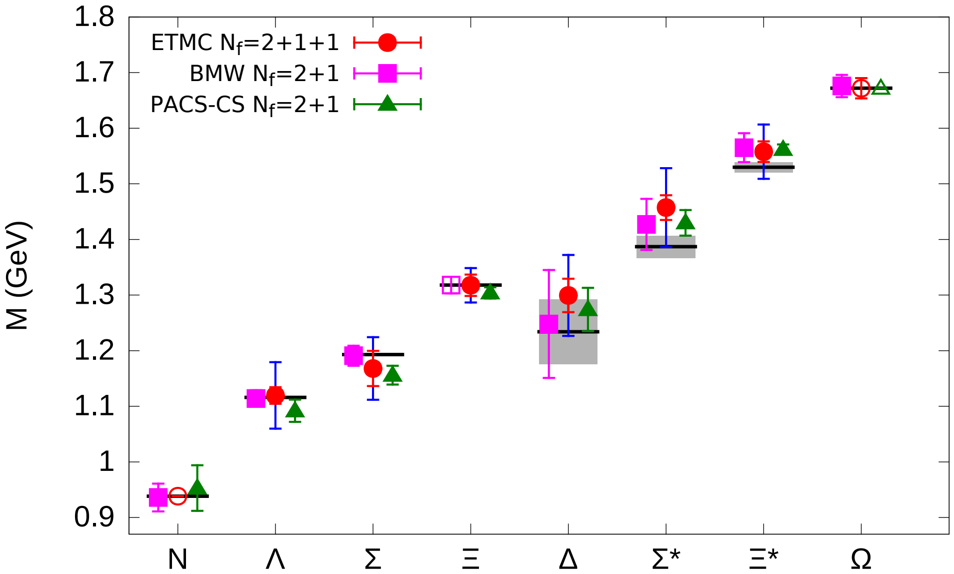
PACS-CS: $N_f = 2+1$ $\mathcal{O}(a)$ improved clover fermions A. Aoki et al. arXiV:0807.1661

LHPC: domain wall valence quarks on a staggered fermions sea (hybrid) A. Walker-Loud et al. arXiV:0806.4549

MILC: $N_f = 2+1+1$ Kogut-Susskind fermion action C.W. Bernard et al. hep/lat 0104002

QCDSF-UKQCD: $N_f = 2$ Wilson fermions G. Bali et al. arXiV:1206.7034

- Octet - Decuplet spectrum



S. Durr et al. `arXiV:0906.3599`,
A. Aoki et al. `arXiV:0807.1661`,
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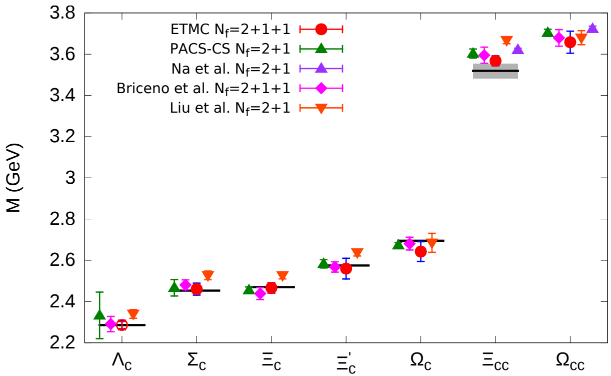
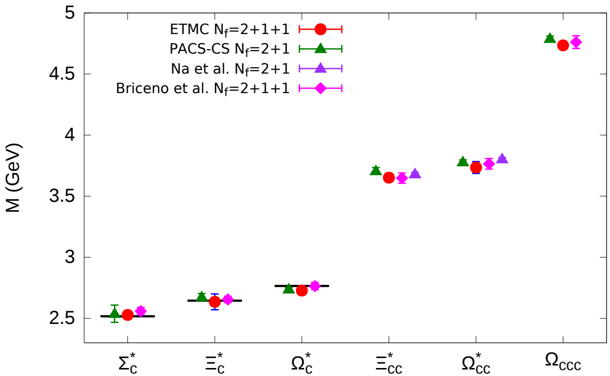
- Charmed baryon spectrum





R. A. Briceno et al. `arXiV:1207.3536`,
H. Na et al. `arXiV:0812.1235`,
H. Na et al. `arXiV:0710.1422`,
L. Liu et al. `arXiV:0909.3294`,
Particle Data Group

Conclusions

- twisted mass formulation with $N_f = 2+1+1$ flavors provides a good framework to study baryon spectrum
- physical nucleon mass appropriate to fix lattice spacing when studying baryon masses
- isospin symmetry breaking effects are small and vanish as the continuum limit is approached
- cut-off effects are small and under control
- good agreement with other lattice calculations and with experiment - reliable predictions of the Ξ^*_{cc}, Ω_{cc}, Ω^*_{cc} and Ω_{ccc} masses

(C. Alexandrou et al. `arXiv:1406.4310`)

What's next

- $N_f = 2$ simulation at the physical pion mass by the ETMC is available

  Talk by Prof. Constantia Alexandrou (Wednesday at 12:30, 301 Pupin)

  Talk by Bartosz Kostrzewa (Friday at 15:35, 428 Pupin)

Thank you!

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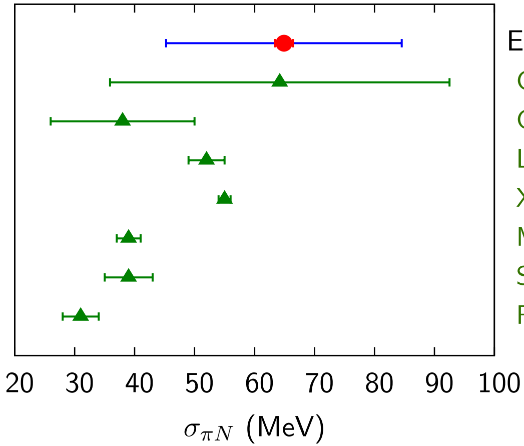


ETMC $N_f = 2+1+1$ (this work)

C. Alexandrou et al. (ETMC) arXiv:0910.2419

G. Bali et al. (QCDSF) arXiv:1111.1600

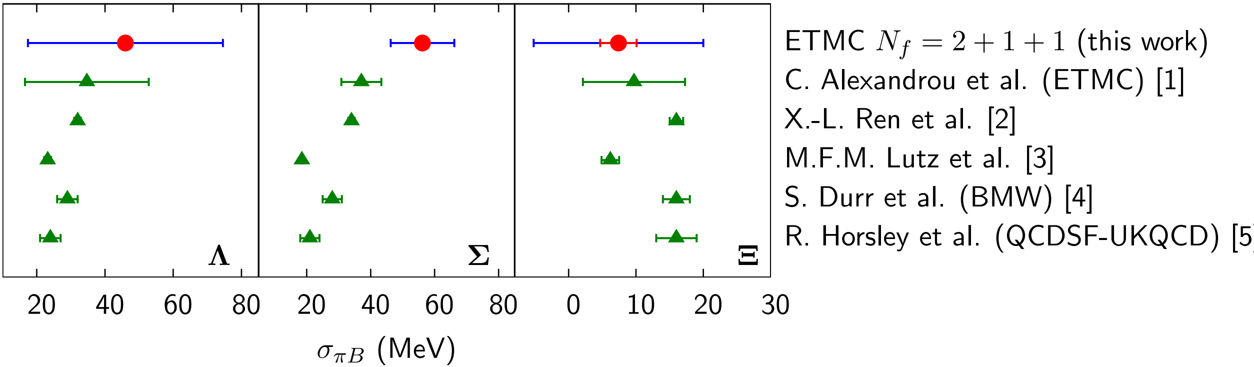
L. Alvarez-Ruso et al. arXiv:1304.0483

X.-L. Ren et al. arXiv:1404.4799

M.F.M. Lutz et al. arXiv:1401.7805

S. Durr et al. (BMW) arXiv:1109.4265

R. Horsley et al. (QCDSF-UKQCD) arXiv:1110.4971



[1] C. Alexandrou et al. (ETMC) arXiv:0910.2419
[2] X.-L. Ren et al. arXiv:1404.4799
[3] M.F.M. Lutz et al. arXiv:1401.7805
[4] S. Durr et al. (BMW) arXiv:1109.4265
[5] R. Horsley et al. (QCDSF-UKQCD) arXiv:1110.4971

Backup slides

Interpolating fields

- constructed such that they have the quantum numbers of the baryon in interest

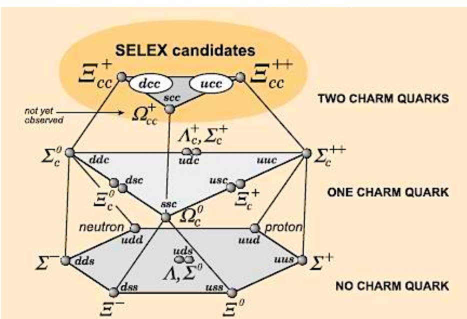
4 quark flavors, baryons (qqq) } SU(3) subgroups of SU(4)

Examples

p (uud)	$\mathcal{J} = \epsilon_{abc} \left(u_a^T C \gamma_5 d_b \right) u_c$
Σ^0 (uds)	$\mathcal{J} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left[\left(u_a^T C \gamma_5 s_b \right) d_c + \left(d_a^T C \gamma_5 s_b \right) u_c \right]$
Ξ_c^+ (usc)	$\mathcal{J} = \epsilon_{abc} \left(u_a^T C \gamma_5 s_b \right) c_c$
$\Xi^{\star 0}$ (uss)	$\mathcal{J}_\mu = \epsilon_{abc} \left(s_a^T C \gamma_\mu u_b \right) s_c$
$\Sigma_c^{\star ++}$ (uuc)	$\mathcal{J}_\mu = \frac{1}{\sqrt{3}} \epsilon_{abc} \left[\left(u_a^T C \gamma_\mu u_b \right) c_c + 2 \left(c_a^T C \gamma_\mu u_b \right) u_c \right]$
$\Omega_c^{\star 0}$ (ssc)	$\mathcal{J}_\mu = \epsilon_{abc} \left(s_a^T C \gamma_\mu c_b \right) s_c$

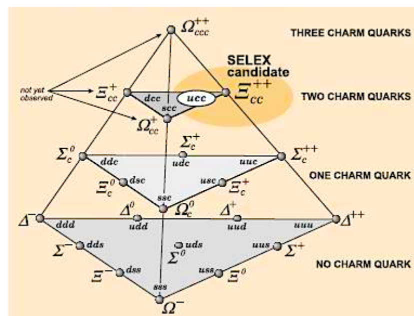
20plet of spin-1/2 baryons

$20 = 8 \oplus 6 \oplus 3 \oplus \bar{3}$



20plet of spin-3/2 baryons

$20 = 10 \oplus 6 \oplus \bar{3} \oplus 1$



$$m_{\text{eff}}^{B}(t) = \log\left(\frac{C_B(t)}{C_B(t+1)}\right) = m_B + \log\left(\frac{1+\sum_{i=1}^{\infty} c_i e^{-\Delta_i t}}{1+\sum_{i=1}^{\infty} c_i e^{-\Delta_i (t+1)}}\right) \xrightarrow[t\to\infty]{} m_B\ , \quad \Delta_i = m_i - m_B$$

$$m_{\text{eff}}^{B}(t) \approx m_B^e + \log\left(\frac{1+c_1 e^{-\Delta_1 t}}{1+c_1 e^{-\Delta_1 (t+1)}}\right)$$

criterion for plateau selection

$$\left|\frac{m_B^c - m_B^e}{\frac{1}{2}(m_B^c + m_B^e)}\right| \leq \frac{1}{2}\sigma_{m_B^c}$$