Light nuclei from quenched lattice QCD

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New Horizons for Lattice Computations with Chiral Fermions

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Outline

1. Introduction
2. Simulation parameters
3. Results
    1. 3- and 4-nucleon channels
    2. NN channels
4. Preliminary results of $N_f = 2 + 1$
5. Summary and future work

1. Introduction

Strong interaction

$$\text{Bind} \begin{cases} \text{protons and neutrons} & \rightarrow \quad \text{nuclei} \\ \text{quarks and gluons} & \rightarrow \quad \text{protons and neutrons} \end{cases}$$

Spectrum of nuclei

success of Shell model since 1949: Jensen and Mayer

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons}}^{\text{Shell model}}}_{\text{(lattice) QCD}} \rightarrow \text{nuclei}$$

1. Introduction

Strong interaction

$$\text{Bind} \begin{cases} \text{protons and neutrons} \rightarrow \text{nuclei} \\ \text{quarks and gluons} \rightarrow \text{protons and neutrons} \end{cases}$$

Spectrum of nuclei

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Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

Motivation: Understand property of nuclei from (lattice) QCD

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}}_{\text{(lattice) QCD}}$$

1. Introduction

Motivation :

Understand property and structure of nuclei from QCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

such as neutron rich nuclei

So far only few works for multi-baryon bound states
Before studying such difficult problems, we should check

$\rightarrow$ Can we calculate known binding energy in a-few-nucleon systems?

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. 3- and 4-nucleon systems
   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
2. H dibaryon in ΛΛ system (S=−2, I=0)
   '11 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV
   '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV
   '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
3. NN systems
   '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV
4. ΞΞ system
   '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV

presented in this talk

Other studies: 2- and 3-nucleon forces HALQCD, ΩΩ channel Buchoff *et al.*

Problems of multi-nucleon bound state

Traditional calculation method of 0th state energy

Example: ^{4}He channel ($J^P = 0^+, I = 0$)

$$\begin{aligned} C_4(t) &= \langle 0|O_{^4\mathrm{He}}(t)\overline{O}_{^4\mathrm{He}}(0)|0\rangle \\ &= \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|\overline{O}_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t} \end{aligned}$$

1. Statistical error
   → Unphysically heavy quark mass + large # of measurements
2. Calculation cost
   → Reduce calculation costs with several methods

   Multi-meson systems: '10 Detmold and Savage
   Multi-baryon systems: '12 Doi and Endres
3. Identification of bound state in finite volume
   → Volume dependence of energy shift of 0th state

2. Simulation parameters

Quenched Iwasaki gauge action at $\beta = 2.416$

$a^{-1} = 1.54$ GeV with $r_0 = 0.49$ fm

Tad-pole improved Wilson fermion action

$m_\pi = 0.8$ GeV and $m_N = 1.62$ GeV

reduce large statistical fluctuation

Finite volume dependence of 0th state

(^{4}He, ^{3}He, $^3S_1$ and $^1S_0$ channels)

- Three volumes: $L = 3.1, 6.1, 12.3$ fm

L	L [fm]	N_{conf}	N_{meas}
24	3.1	2500	2
48	6.1	400	12
96	12.3	200	12

- Two smearing sources: for consistency check

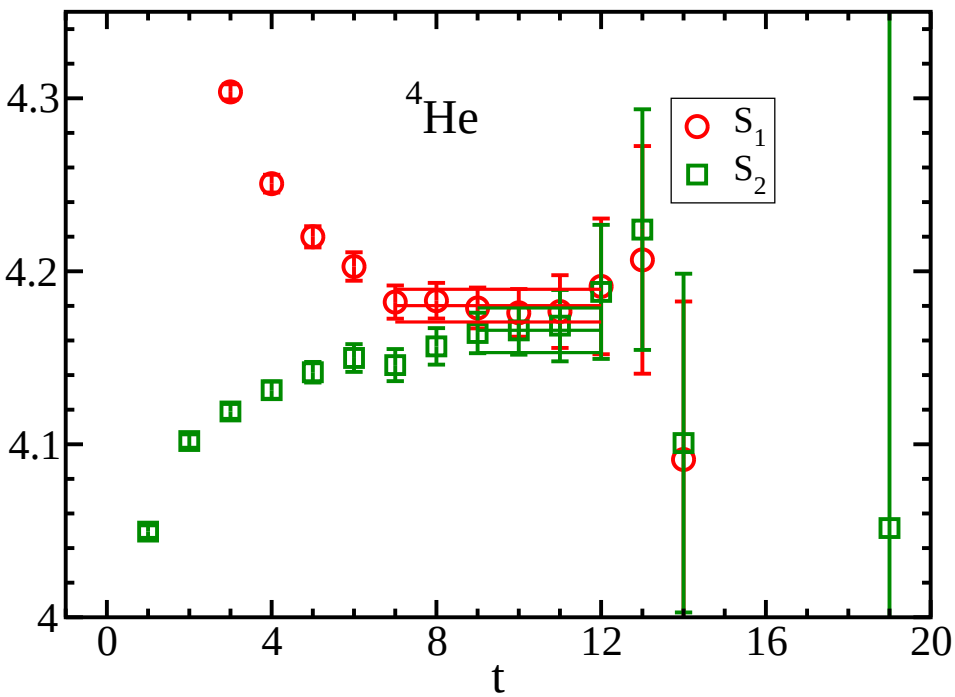
Simulations:

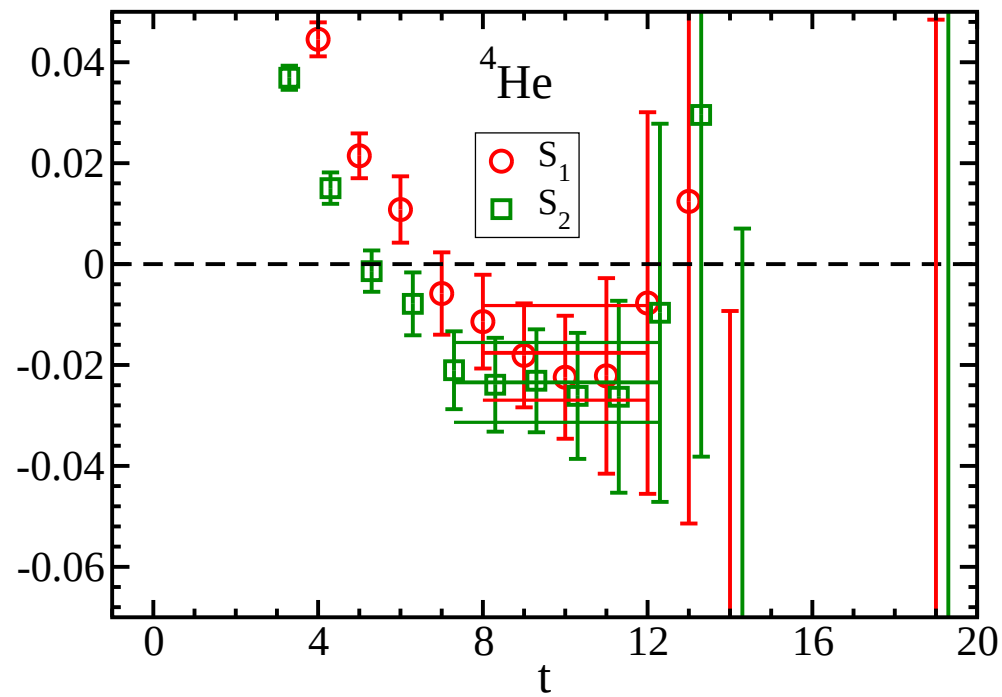
PACS-CS, T2K-Tsukuba at Univ. of Tsukuba, HA8000 at Univ. of Tokyo

4. Results

Effective energy of ^{4}He channel at $L = 48$ $E_0 = \log\left(\frac{C_{\mathrm{He}}(t)}{C_{\mathrm{He}}(t+1)}\right)$

and $\Delta E_L = \log\left(\frac{R_{^4\mathrm{He}}(t)}{R_{^4\mathrm{He}}(t+1)}\right)$ with $R_{^4\mathrm{He}}(t) = \frac{C_{^4\mathrm{He}}(t)}{(C_N(t))^4}$





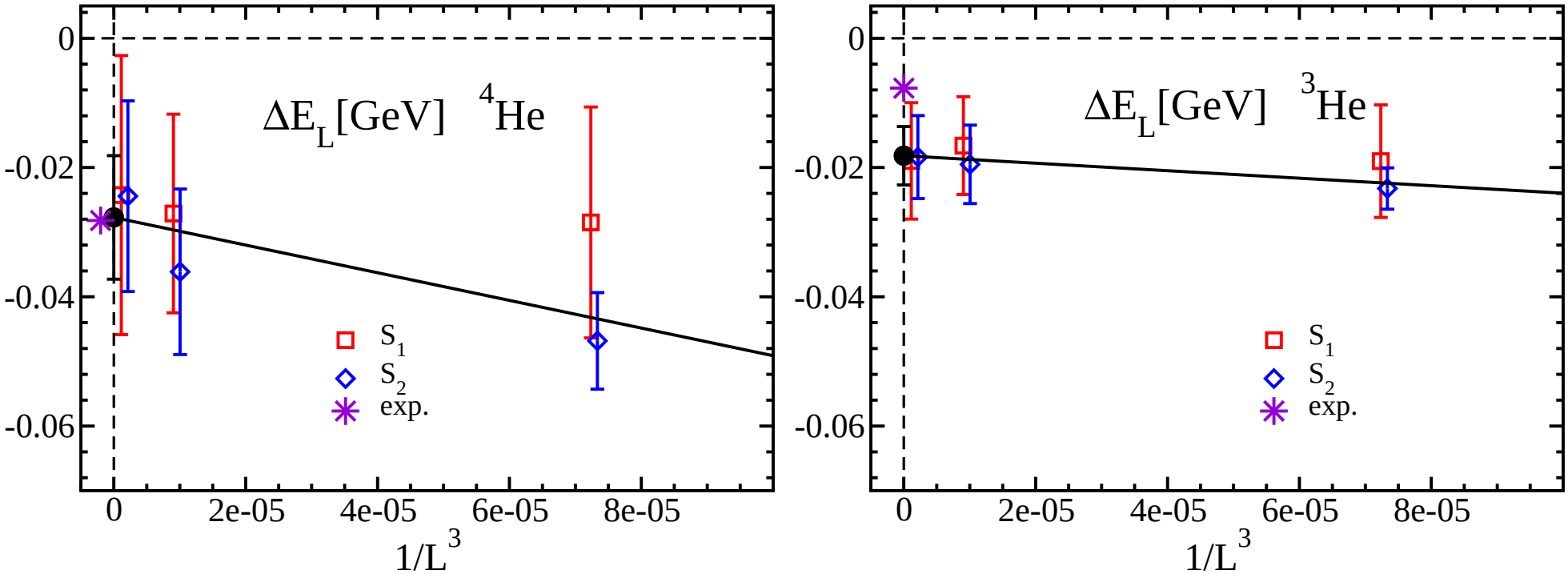
- Clear signal in $t < 12$, but larger error in $t \geq 12$
- consistent plateaus in $8 \lesssim t \leq 12$

3. Results

3- and 4-nucleon channels $\Delta E_L = E_0 - N_N m_N$

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE



- $\Delta E_L < 0$ in three volumes $\Leftarrow$ statistically independent ensembles
- Small volume dependence
- Infinite volume limit with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

3. Results

3- and 4-nucleon channels $\Delta E_L = E_0 - N_N m_N$

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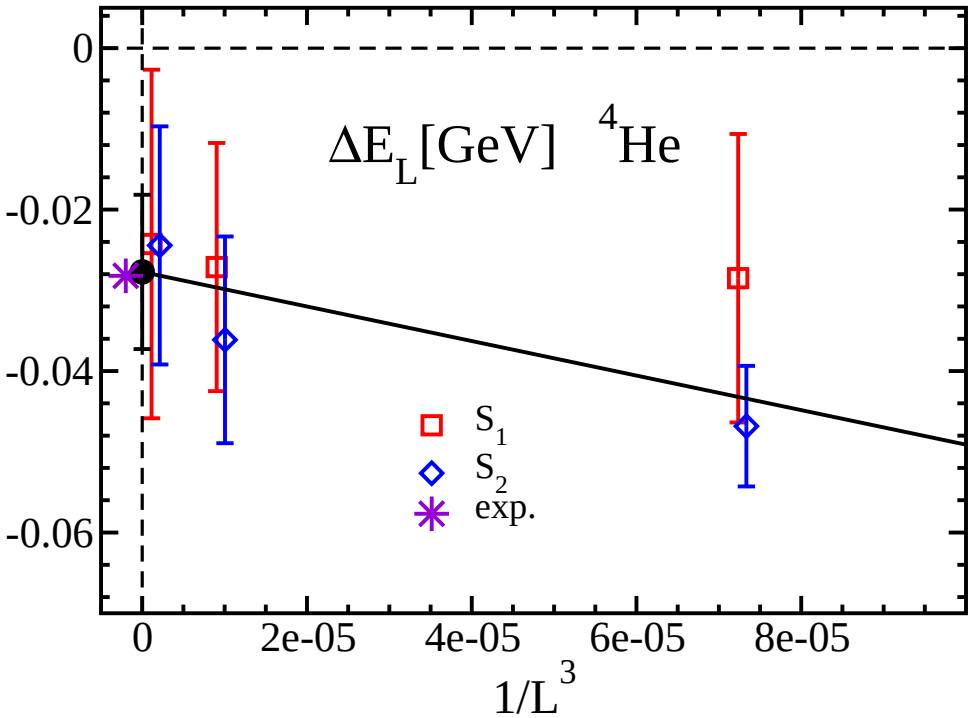
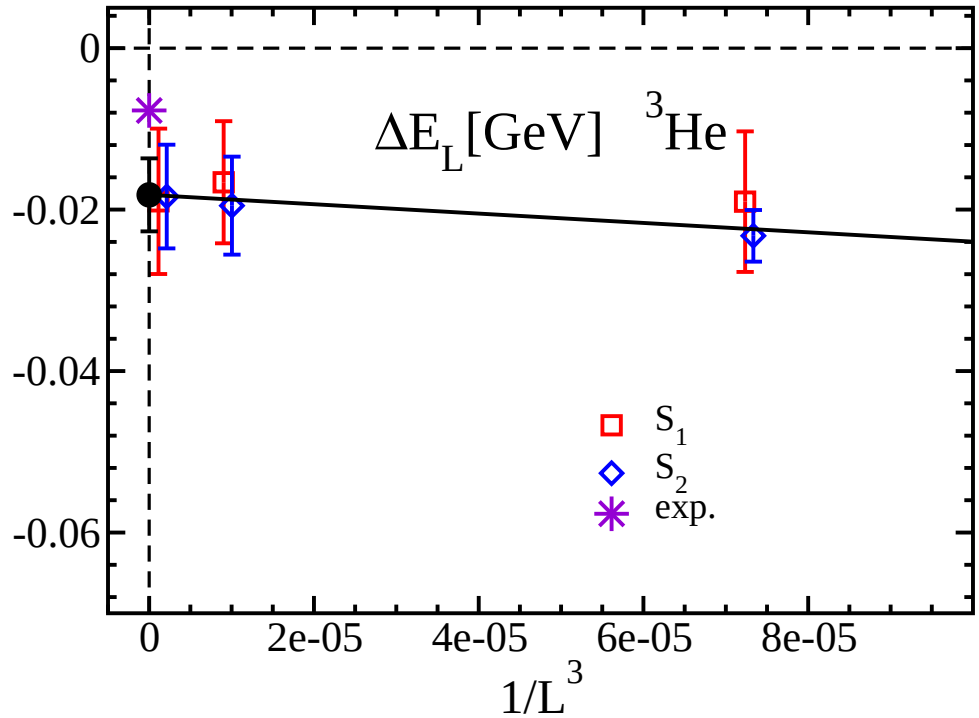
Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\mathrm{He}} = 27.7(7.8)(5.5)$ MeV $\qquad$ $\Delta E_{^3\mathrm{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

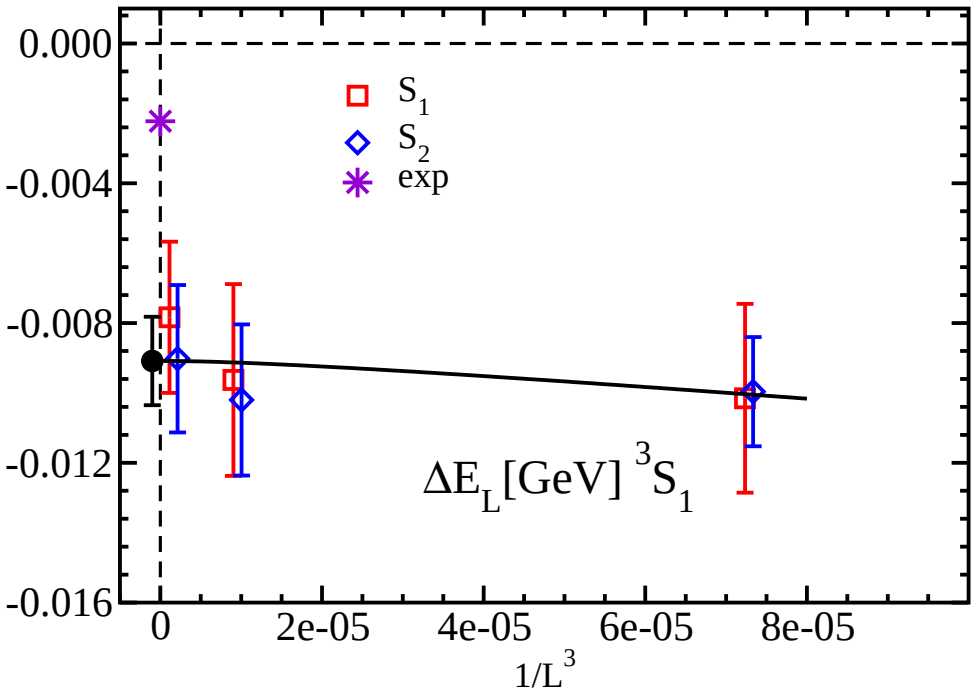
However, large systematic errors, e.g., $m_\pi = 0.8$ GeV

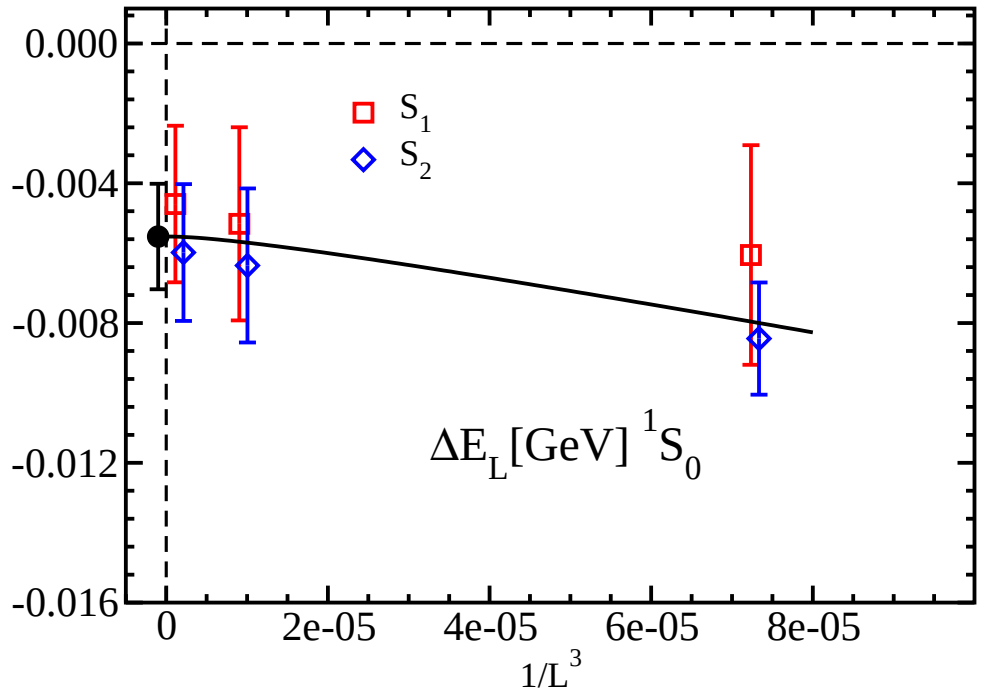
3. Results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

PACS-CS Collaboration, PRD84:054506(2011)

Identification of bound state from volume dependence of ΔE





$\Delta E_L < 0$ in three volumes

Infinite volume extrapolation

'04 Beane *et al.*, '06 Sasaki & TY

$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\},\ \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

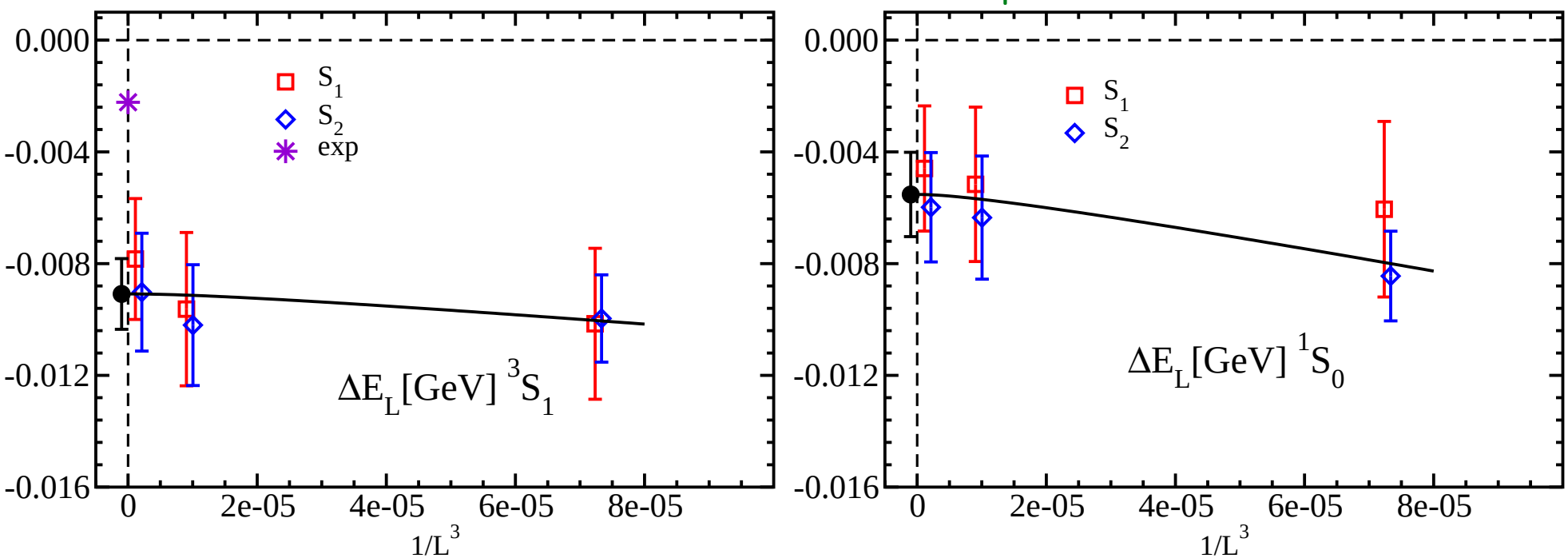
based on Lüscher's finite volume formula

3. Results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

PACS-CS Collaboration, PRD84:054506(2011)

Identification of bound state from volume dependence of ΔE



Bound state in both channels ← different from experiment

$\Delta E_{^3S_1} = 9.1(1.1)(0.5)$ MeV $\qquad$ $\Delta E_{^1S_0} = 5.5(1.1)(1.0)$ MeV

Probably bound states in $N_f = 2+1$ $m_\pi = 0.39$ GeV ('12 NPLQCD)

might be caused by heavy quark mass in calculation

4. Preliminary results of $N_f = 2 + 1$

Reduce systematic errors

Quark mass dependence of ΔE and a_0

Bound state in $^1S_0$ vanishes as quark mass decreases?

Heavy quark mass

$m_\pi = 0.8$ GeV

Quenched effect

Finite lattice spacing

$a^{-1} = 1.54$ GeV

4. Preliminary results of $N_f = 2 + 1$

Reduce systematic errors

Quark mass dependence of ΔE and a_0

Bound state in $^1S_0$ vanishes as quark mass decreases?

Heavy quark mass

$m_\pi = 0.8$ GeV $\rightarrow$ 0.5 (and 0.7) GeV

Quenched effect

$N_f = 2 + 1$ QCD

Finite lattice spacing

$a^{-1} = 1.54$ GeV $\rightarrow a^{-1} \sim 2$ GeV

Calculation of NN, ^{3}He, ^{4}He channels

4. Preliminary results of $N_f = 2 + 1$

Simulation parameters

- Iwasaki gauge action at $\beta = 1.90$

$$a^{-1} = 2.194 \text{ GeV with } m_\Omega$$

- Non-perturbative improved Wilson fermion action

$$m_\pi = 0.5 \text{ GeV and } m_N = 1.3 \text{ GeV}$$

$$m_s \sim \text{physical strange mass}$$

Finite volume dependence of 0th state

Four volumes: $L = 2.9, 3.6, 4.3, 5.8$ fm

Measurements: $\sim$ 200 confs $\times$ $O(100)$/per conf

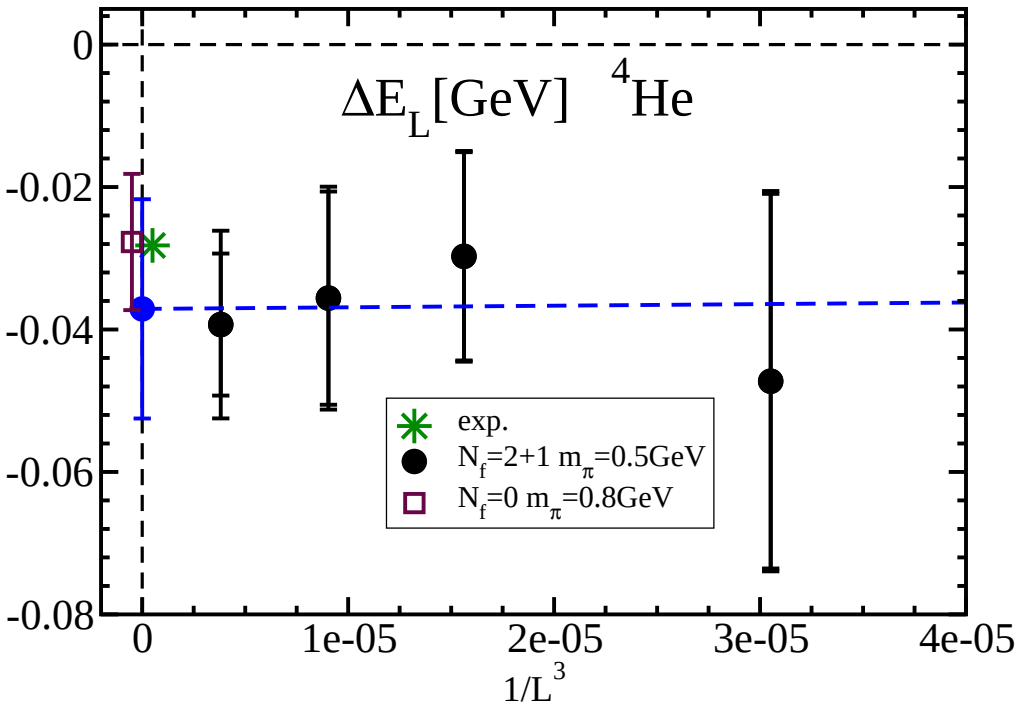
Simulations:

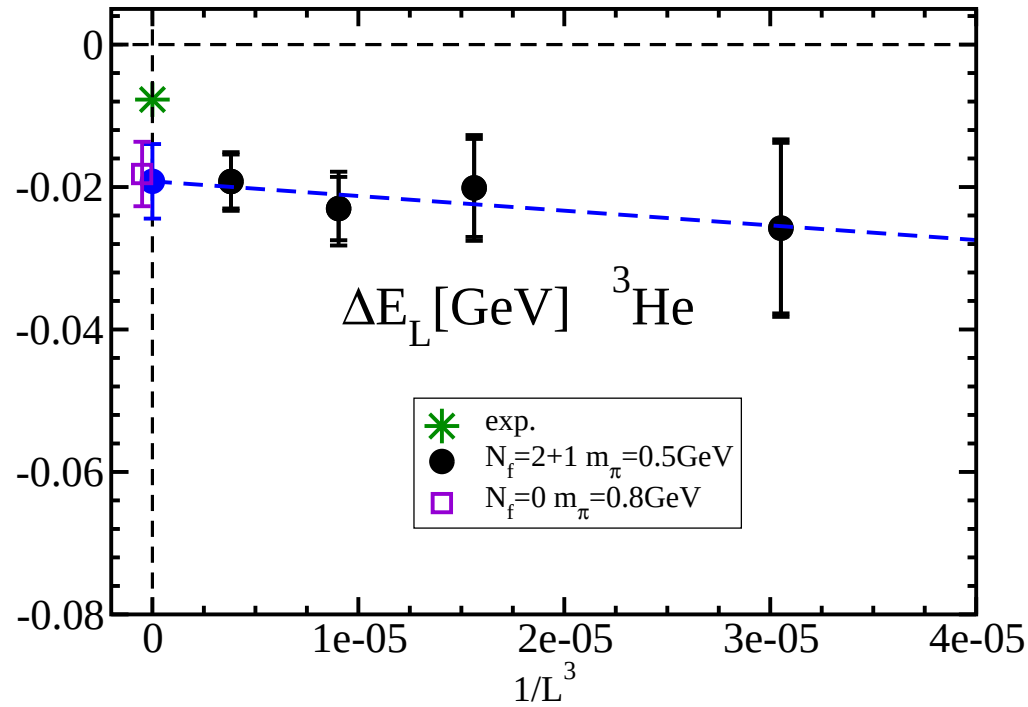
PACS-CS, T2K-Tsukuba, HA-PACS at Univ. of Tsukuba, HA8000 at Univ. of Tokyo and K at AICS

4. Preliminary results of $N_f = 2 + 1$

3- and 4-nucleon channels

Identification of bound state from volume dependence of ΔE



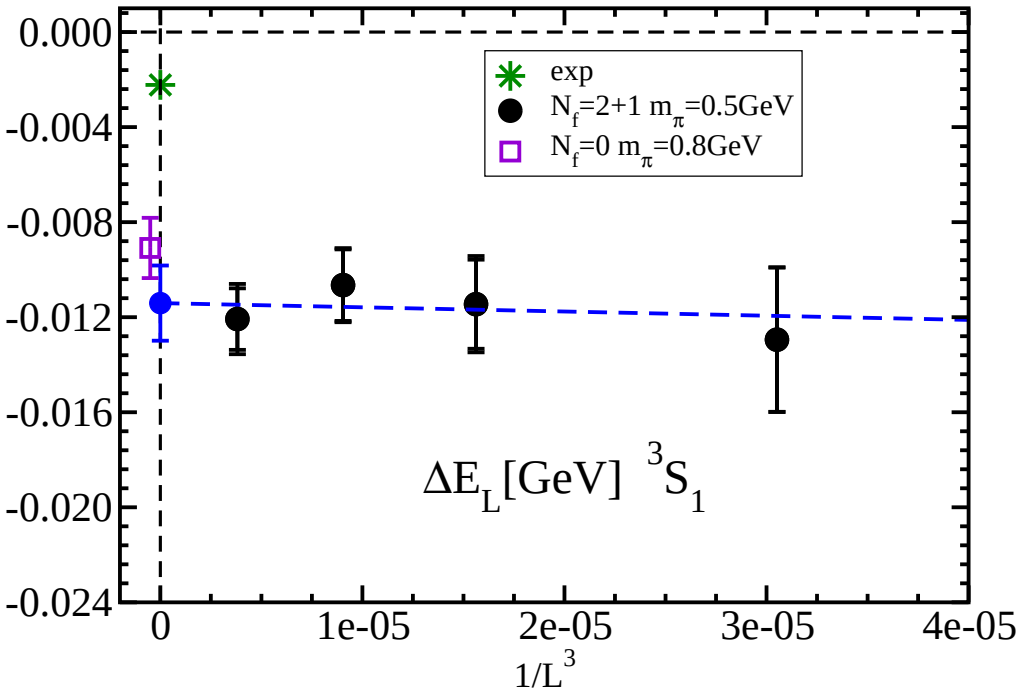


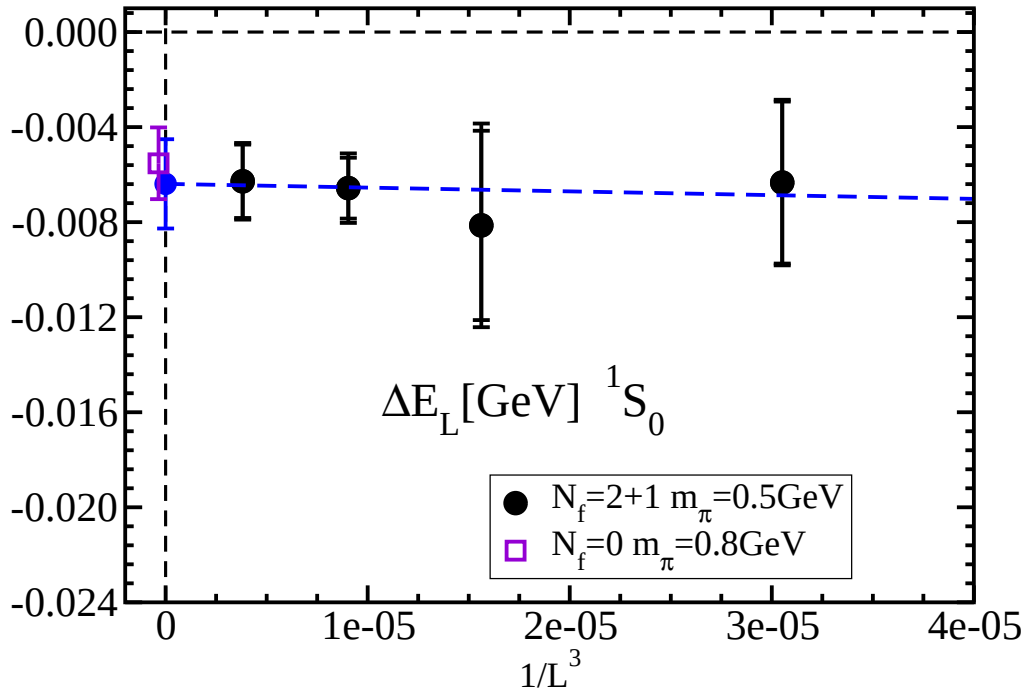
Similar results to $N_f = 0$ $m_\pi = 0.8$ GeV

4. Preliminary results of $N_f = 2 + 1$

NN ($^3S_1$ and $^1S_0$) channels

Identification of bound state from volume dependence of ΔE



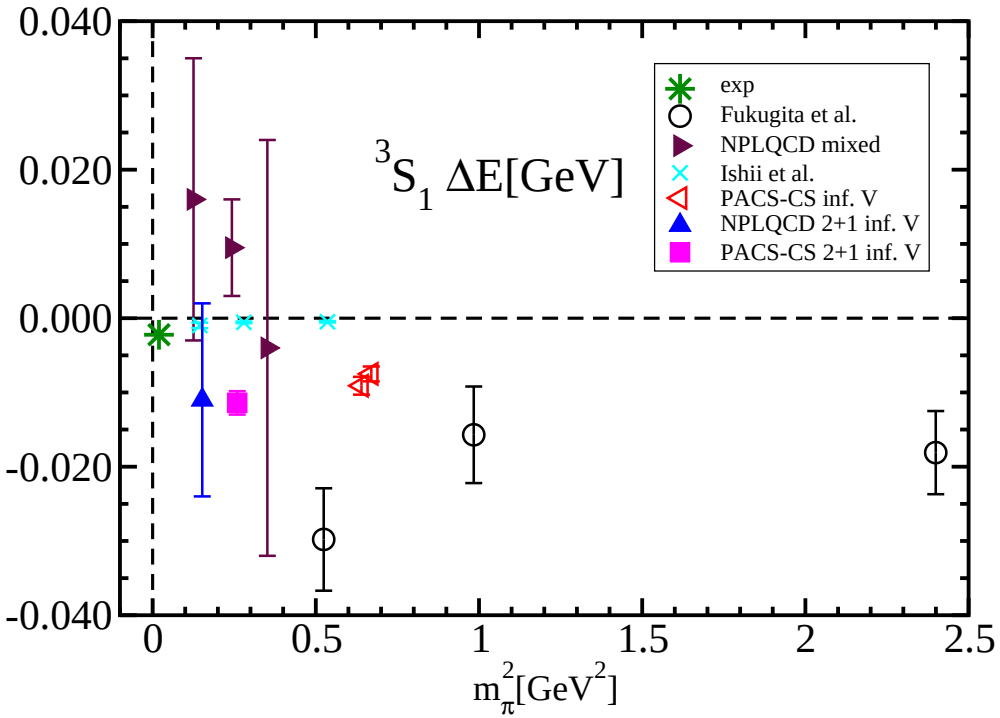


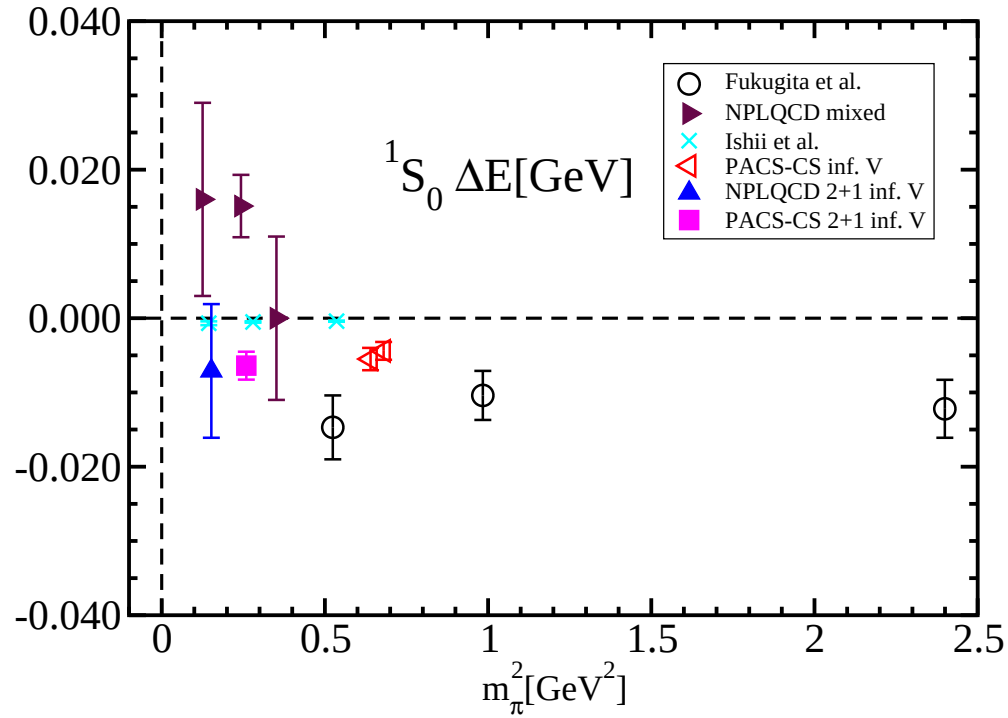
Similar results to $N_f = 0$ $m_\pi = 0.8$ GeV

4. Preliminary results of $N_f = 2 + 1$

NN ($^3S_1$ and $^1S_0$) channels

Current status of ΔE





filled(open) symbols: $N_f = 2 + 1$(quenched) results

Roughly consistent with other results

Still quark mass might be heavy

Need further lighter quark mass for clear quark dependence

5. Summary and future work

Exploratory study of light nuclei in quenched lattice QCD

- Unphysically heavy quark mass
- Volume dependence of energy shift of ground state

1. $\Delta E \neq 0$ of 0th state in infinite volume limit

(2. Expected properties of 1st excited state in NN)

$\rightarrow$ bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ at $m_\pi = 0.8$ GeV

Larger ΔE than experiment

Bound state in ^{1}S$_0$ not observed in experiment

probably observed $N_f = 2 + 1$ at $m_\pi = 0.39$ GeV ('12 NPLQCD)

Need to reduce systematic errors

5. Summary and future work

$N_f = 2 + 1$ calculation is on-going at $m_\pi = 0.5$ GeV

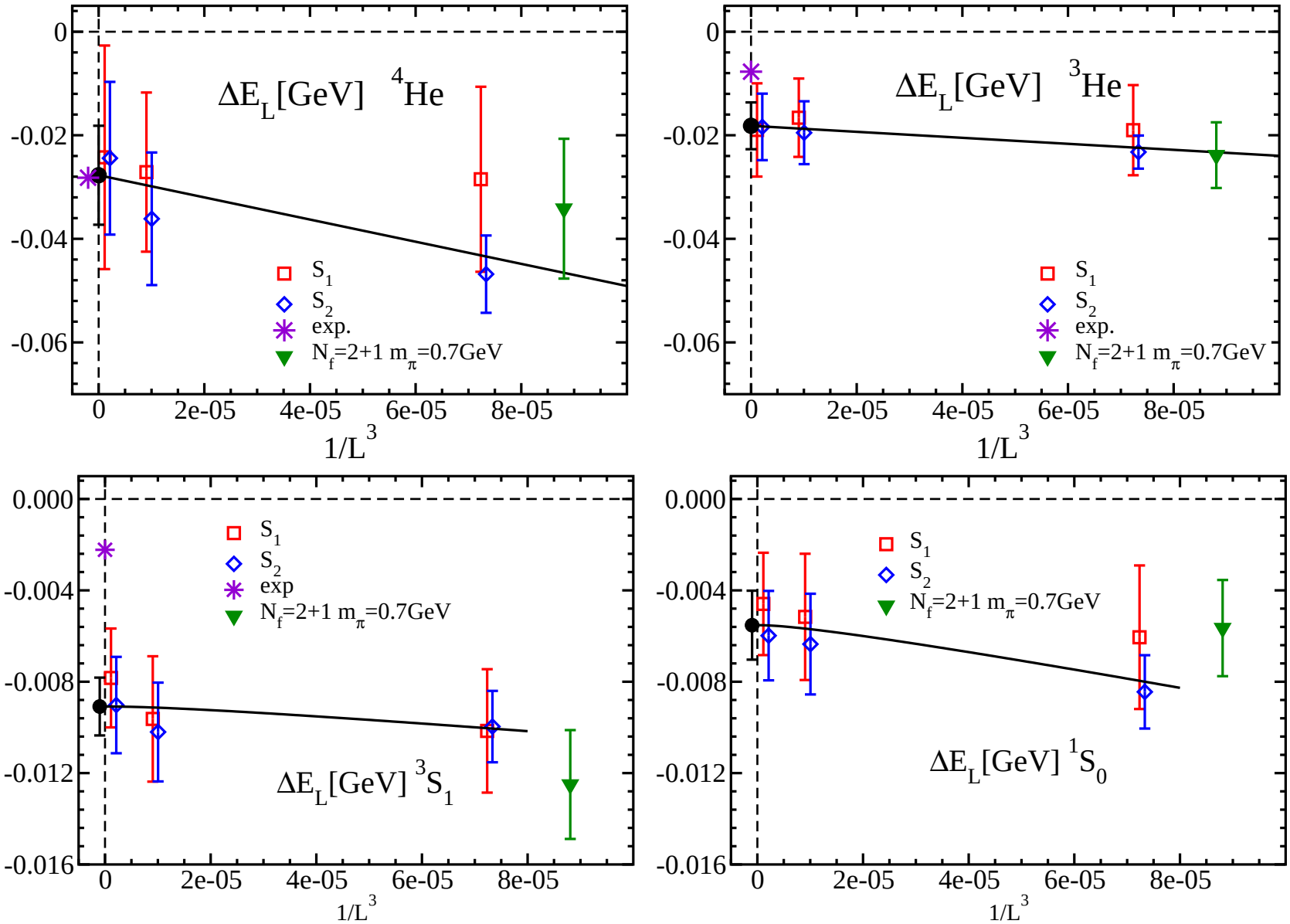
Similar result to quenched case

$\rightarrow$ bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ at $m_\pi = 0.5$ GeV

Need further lighter quark mass calculation

Back up

Very preliminary $N_f = 2 + 1$ results at $m_\pi = 0.7$ GeV

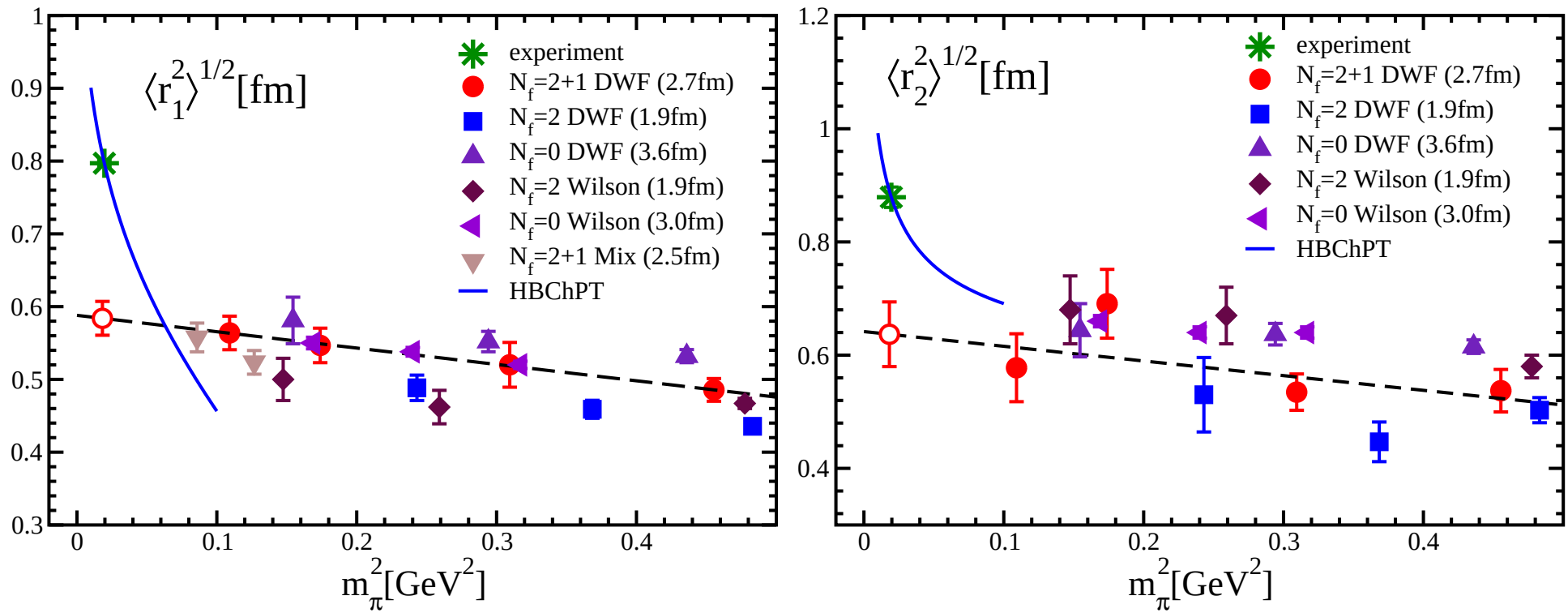


Larger volumes at $m_\pi = 0.5$ and 0.7 GeV underway

Example of large quark mass dependence

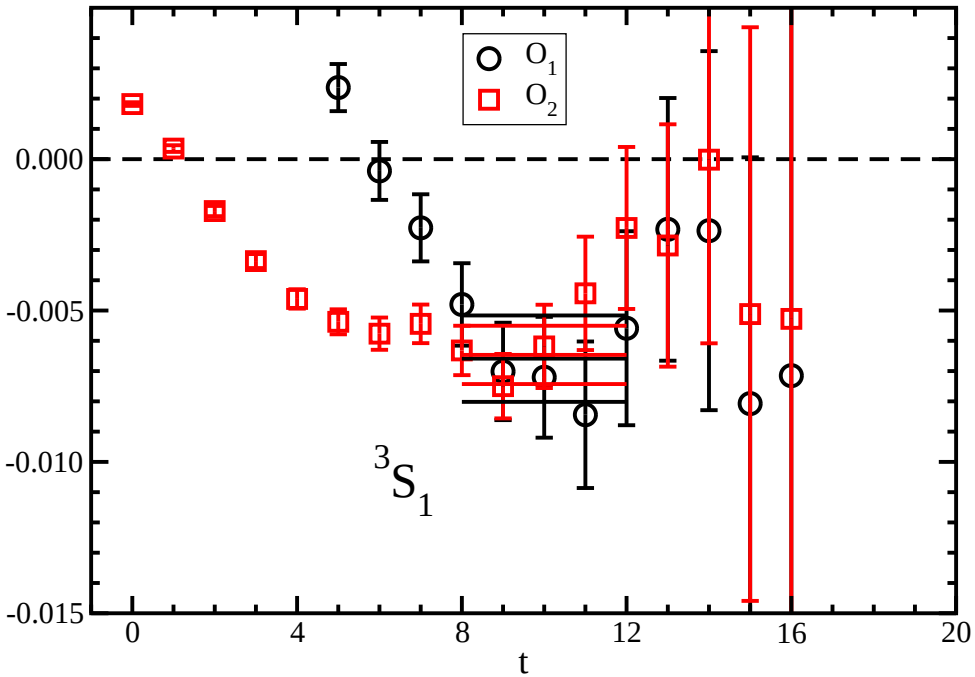
rms radii from form factors F_1 and F_2

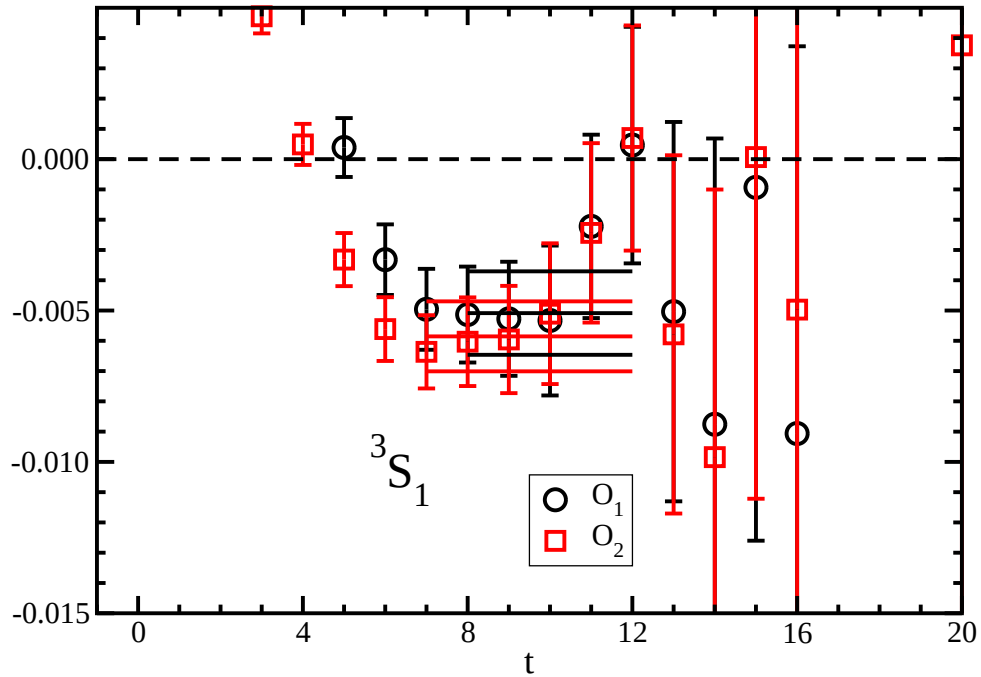
'09 RBC + UKQCD



Effective energy shift $\Delta E_L = E_{\mathrm{NN}} - 2m_N$ of $^3\mathrm{S}_1$ at $L = 24$ and $L = 96$

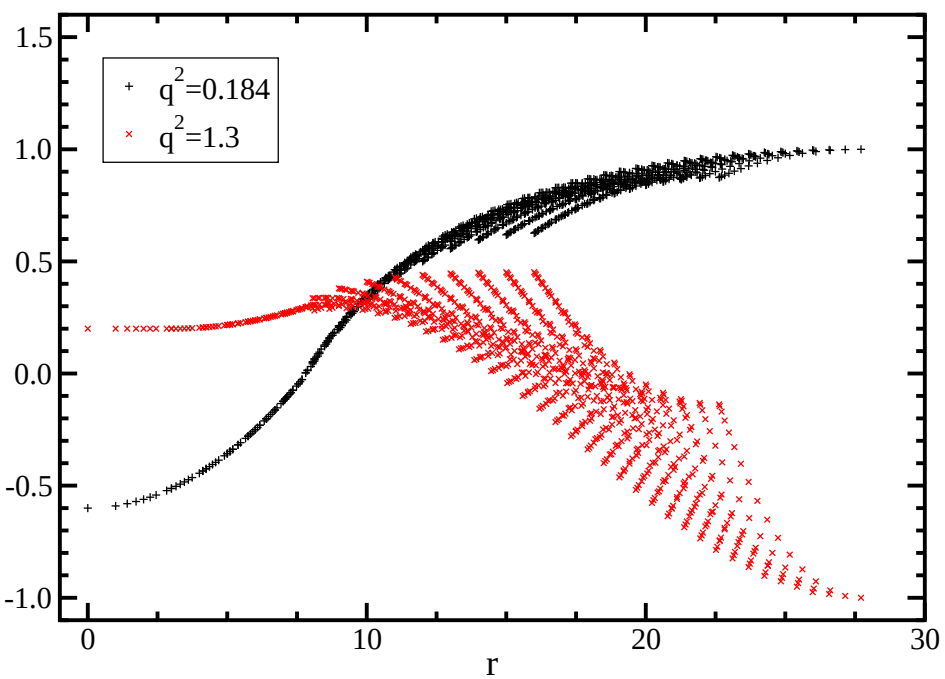
$$\Delta E_L(t) = \log\left(\frac{R(t)}{R(t+1)}\right), \quad R(t) = \frac{C_{\mathrm{NN}}(t)}{(C_N(t))^2}$$

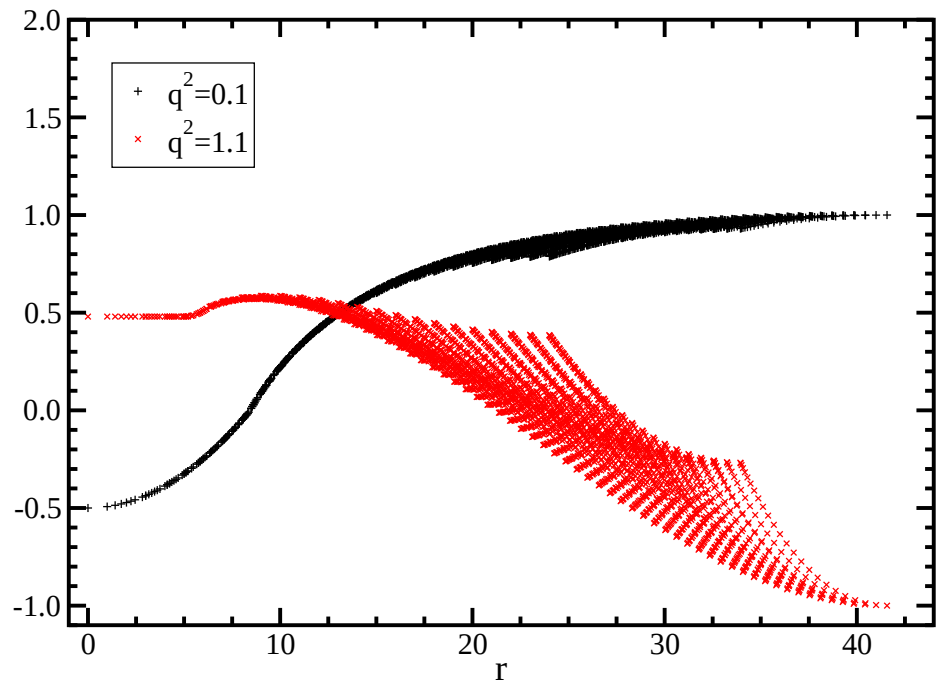




Smearing functions of sink operator at $L = 32$ and $L = 48$

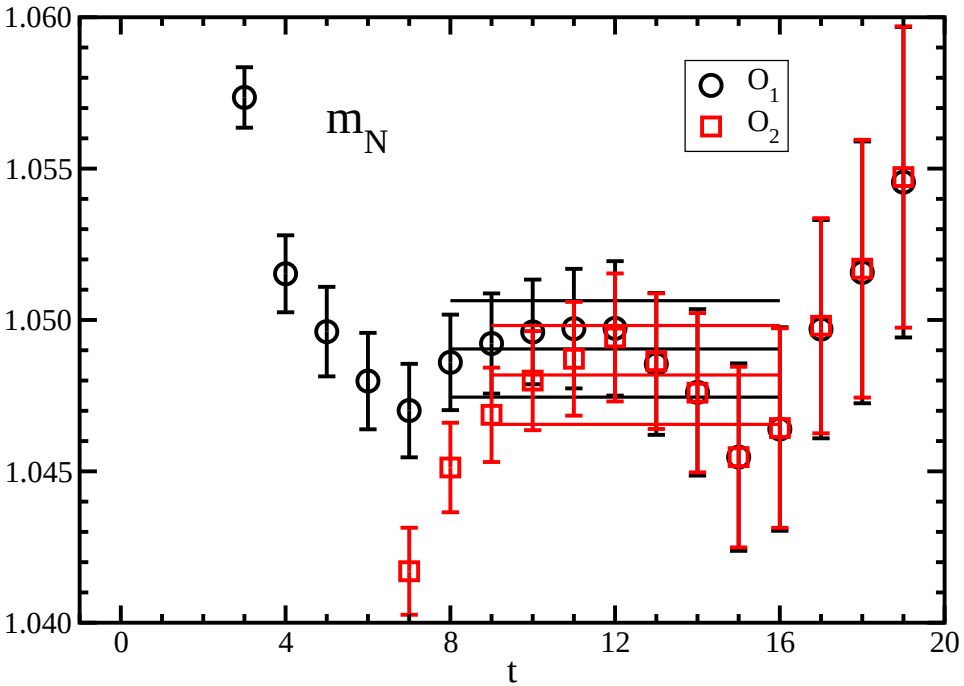
$$N(\mathbf{x})W_{q^2}(|\mathbf{x}-\mathbf{y}|)N(\mathbf{y})$$

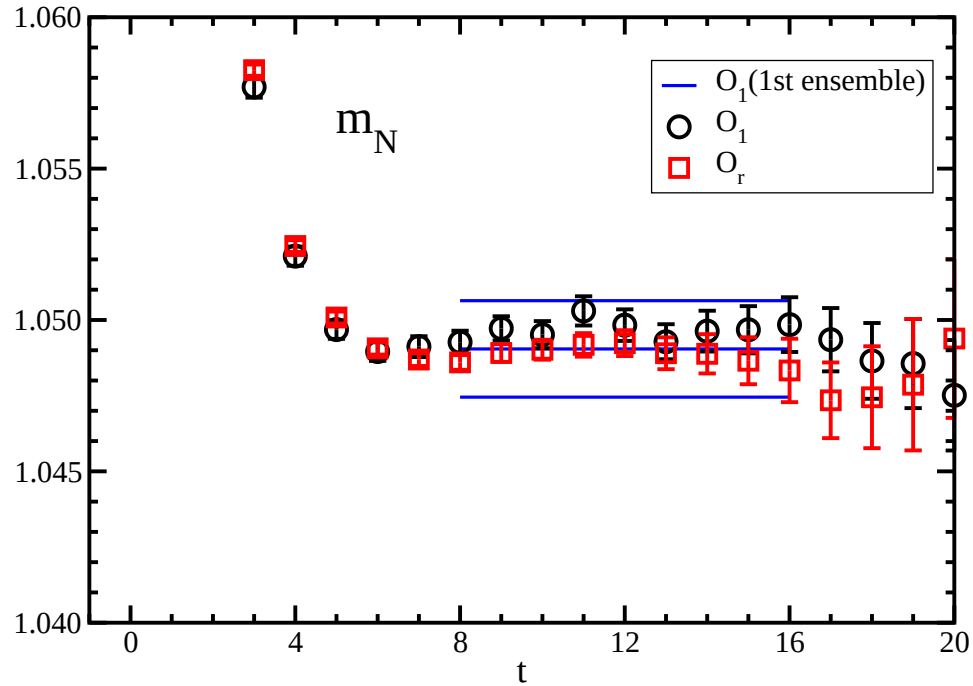




Effective nucleon mass at $L = 48$

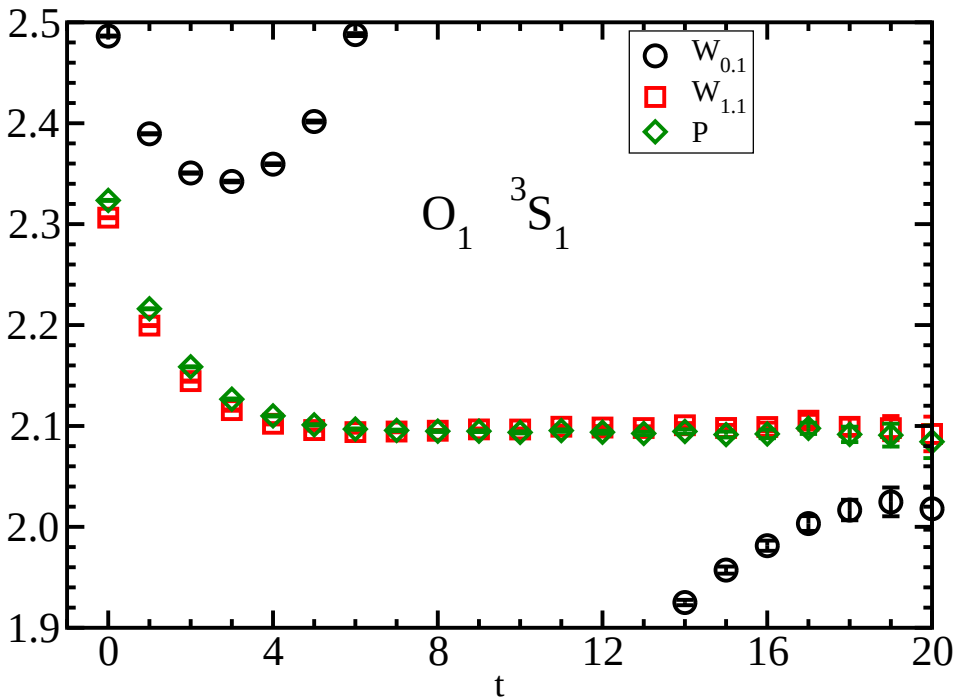
$$M_N(t) = \log\left(\frac{C_N(t)}{C_N(t+1)}\right)$$

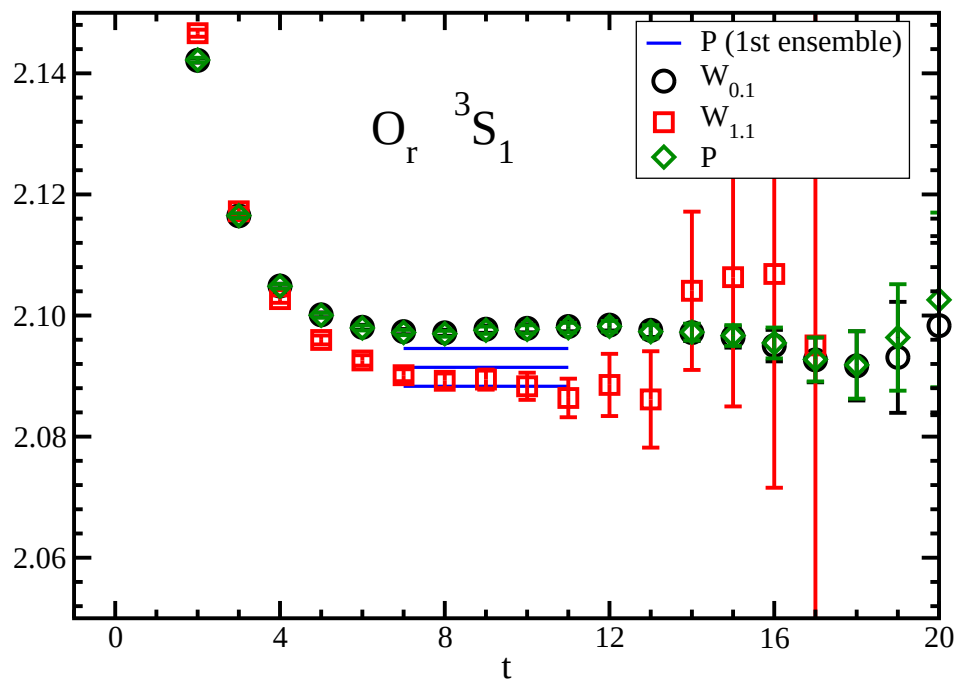




Effective energy before and after diagonalization $L = 48$

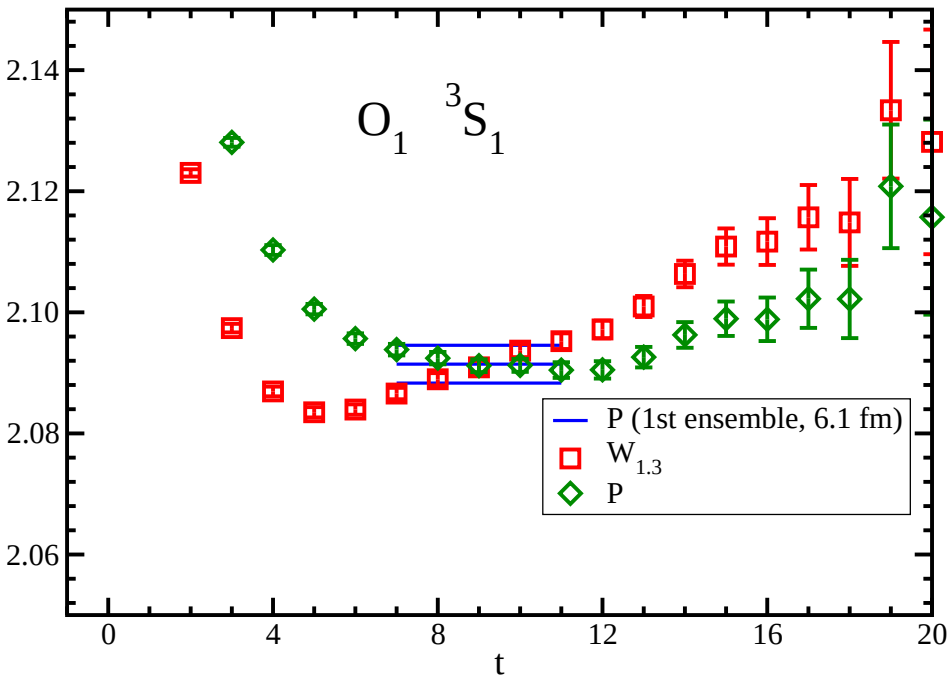
$$E_{ij}(t) = \log\left(\frac{G_{ij}(t)}{G_{ij}(t+1)}\right)$$

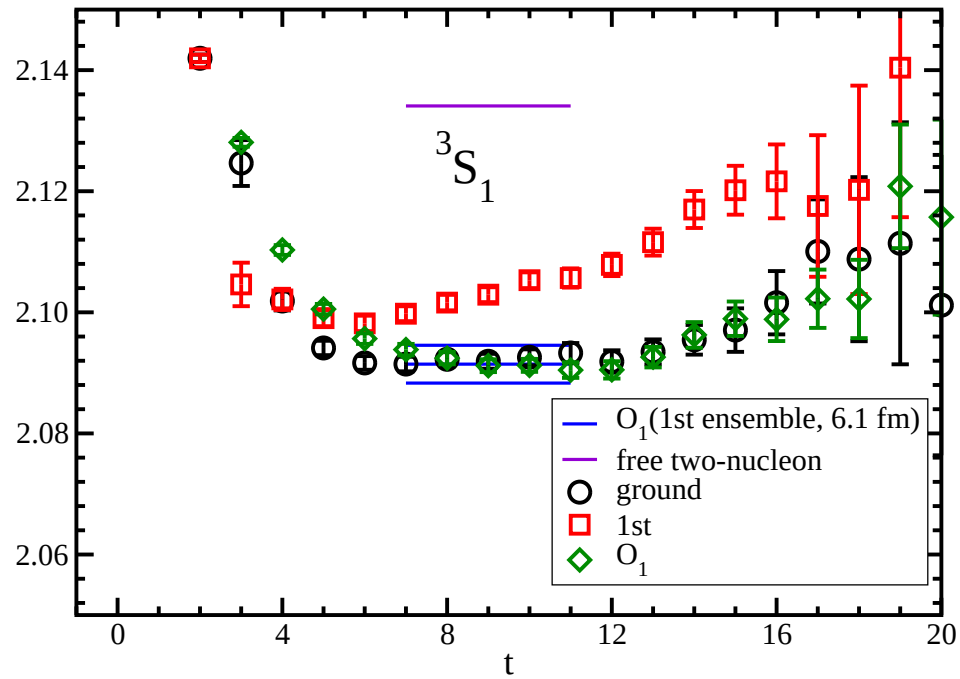




Effective energy before and after diagonalization $L = 32$

$$E_{ij}(t) = \log\left(\frac{G_{ij}(t)}{G_{ij}(t+1)}\right)$$

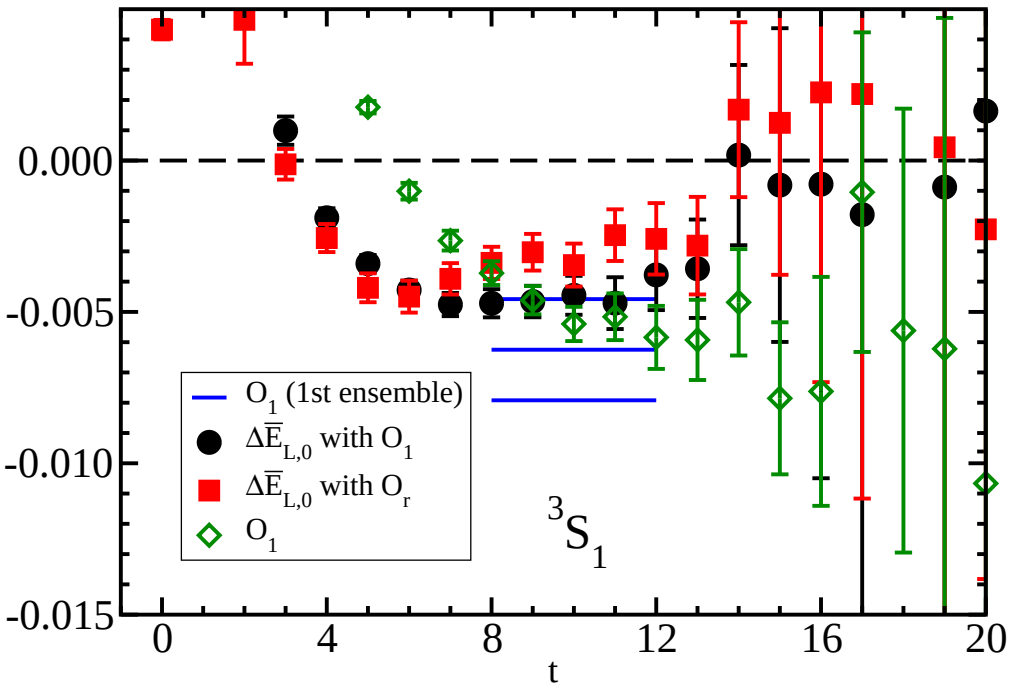


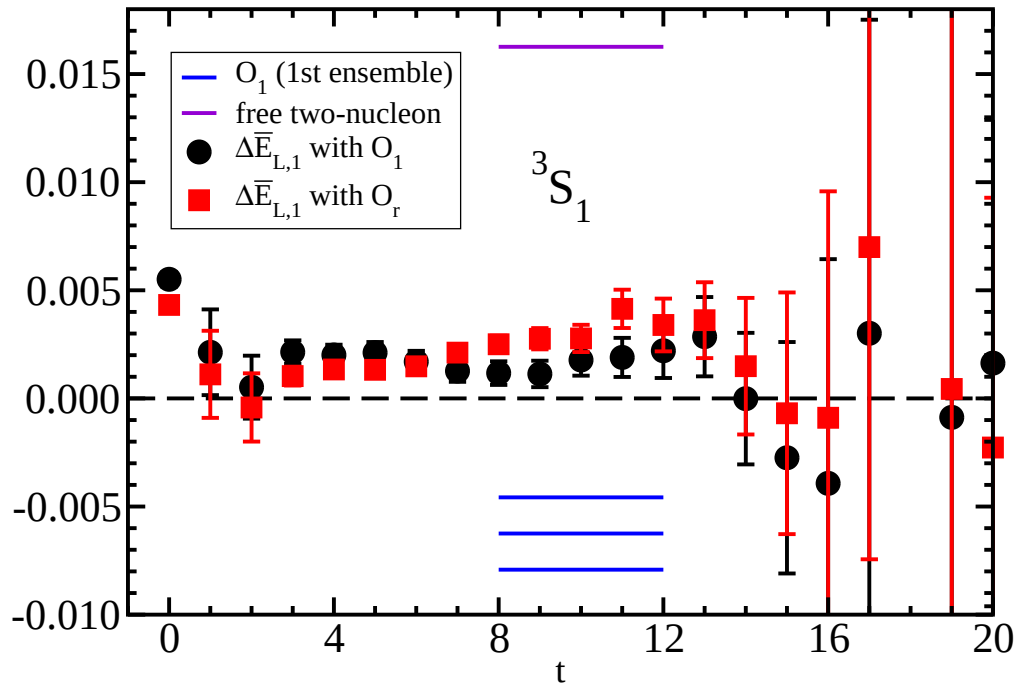


Effective energy shift $\Delta\overline{E}_{L,\alpha} = \overline{E}_\alpha - 2m_N$

for ground and 1st excited states of $^3S_1$ at $L = 48$

$$\Delta\overline{E}_{L,\alpha}(t) = \log\left(\frac{\overline{R}_\alpha(t)}{\overline{R}_\alpha(t+1)}\right), \quad \overline{R}_\alpha(t) = \frac{\lambda_\alpha(t)}{(C_N(t))^2}$$

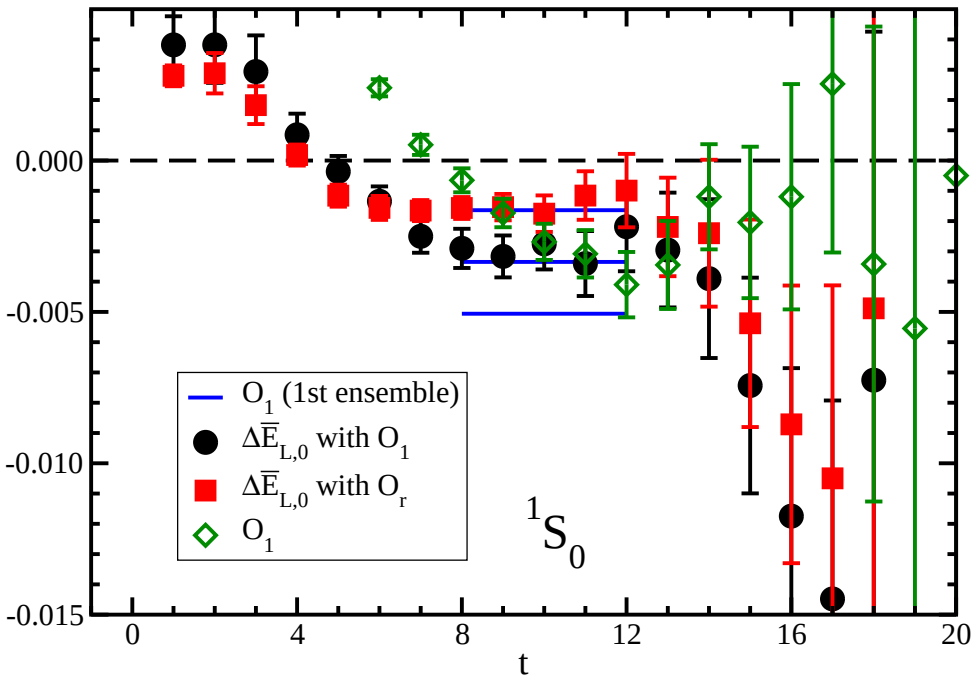


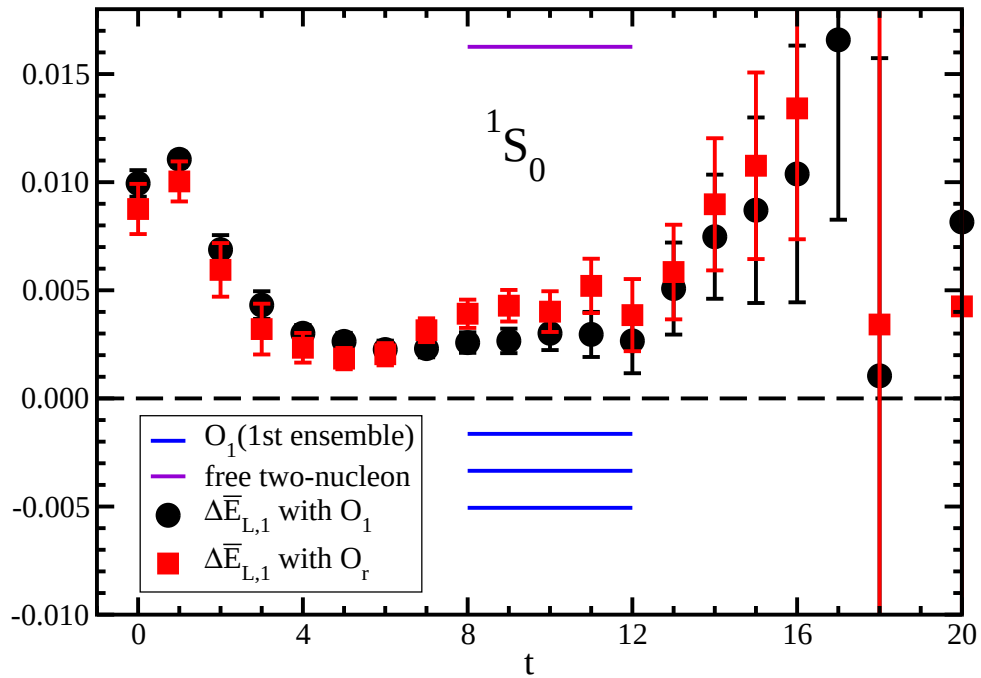


Effective energy shift $\Delta\overline{E}_{L,\alpha} = \overline{E}_\alpha - 2m_N$

for ground and 1st excited states of $^1S_0$ at $L = 48$

$$\Delta\overline{E}_{L,\alpha}(t) = \log\left(\frac{\overline{R}_\alpha(t)}{\overline{R}_\alpha(t+1)}\right), \quad \overline{R}_\alpha(t) = \frac{\lambda_\alpha(t)}{(C_N(t))^2}$$

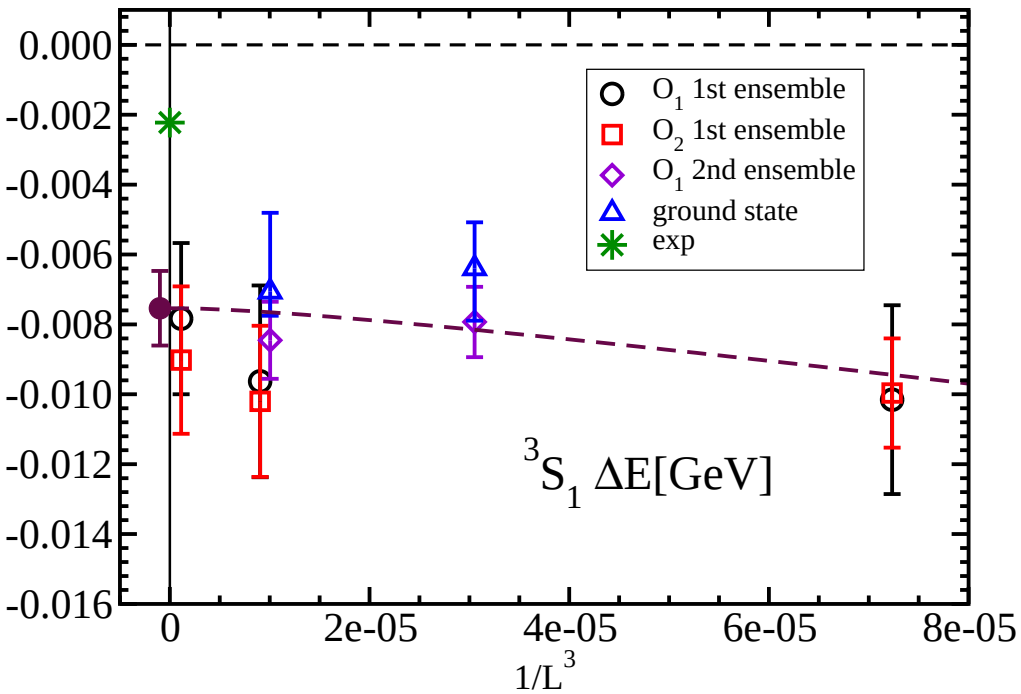


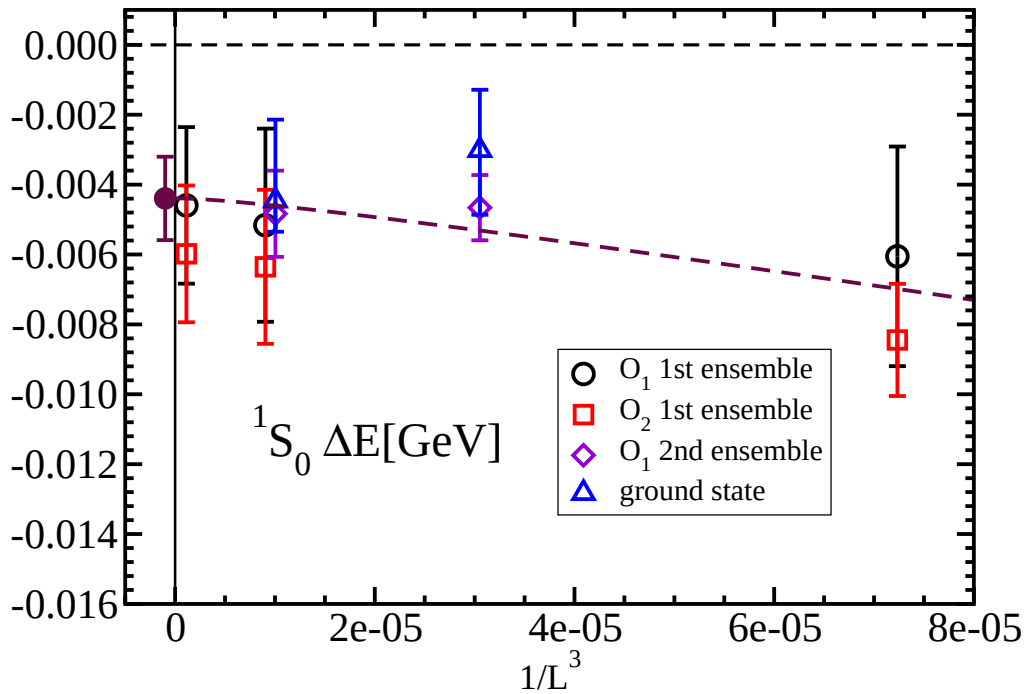


3. Results (cont'd)

2. Two-state analysis

Volume dependence of ΔE_L and $\Delta \overline{E}_{L,0}$





$\Delta E_{^3S_1} = 7.5(0.5)(0.9)$ MeV $\qquad$ $\Delta E_{^1S_0} = 4.4(0.6)(1.0)$ MeV

$^3S_1$: almost four times larger than experimental values

$^1S_0$: bound state exists different from experiment

mainly caused by heavy quark mass in calculation, probably

Future work quark mass dependence

Assume: our results smoothly continue to physical one

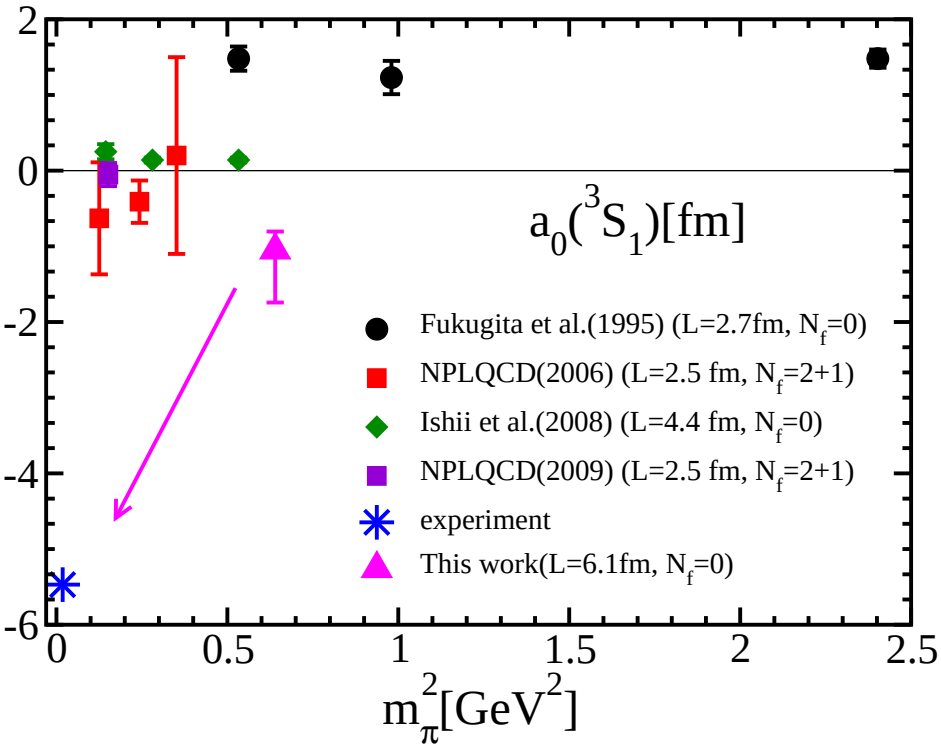
Quark mass $\searrow$

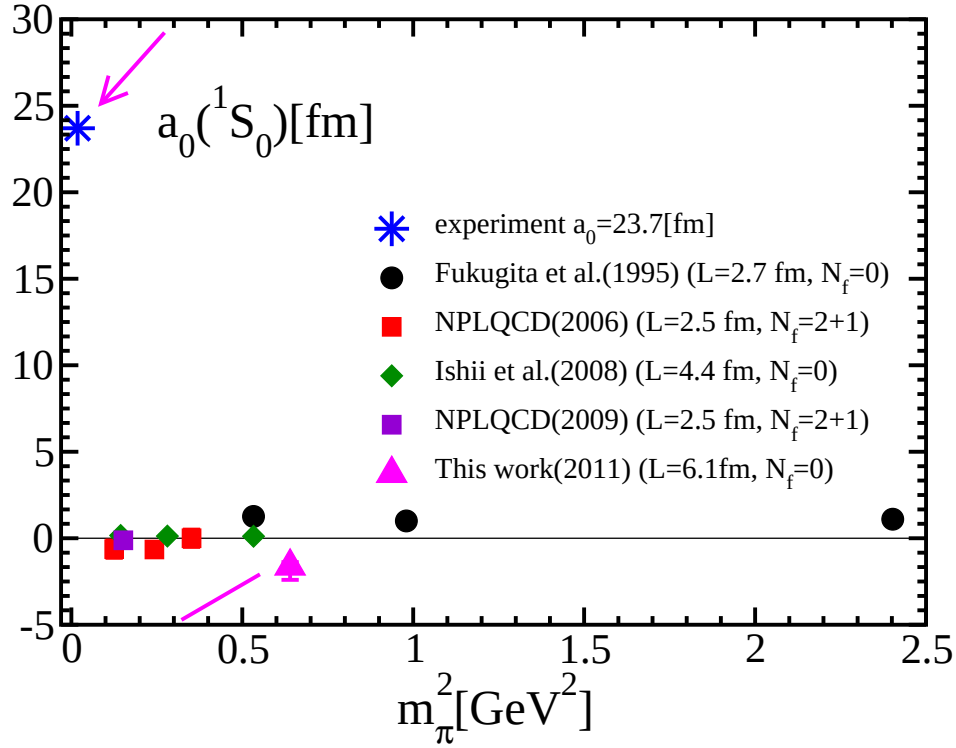
$^3S_1$: $\Delta E \searrow$ and $|a_0| \nearrow$

$^1S_0$: $\Delta E \searrow$ and $|a_0| \nearrow$

$\rightarrow$ bound state vanishes, $|a_0| = \infty$ and $\mathrm{sign}(a_0) = +$

$\rightarrow a_0 \searrow$





Certainly need to study quark mass dependence

3. Simulation parameters

- Quenched Iwasaki gauge action at $\beta = 2.416$

  $a^{-1} = 1.54$ GeV with $r_0 = 0.49$ fm
- Tad-pole improved Wilson fermion action

  $m_\pi = 0.8$ GeV and $m_N = 1.62$ GeV

  reduce large statistical fluctuation

1. Single state analysis

- Three volumes

L	L [fm]	N_{conf}	N_{meas}
24	3.1	2500	2
48	6.1	400	12
96	12.3	200	12

- Exponential smearing sources $q(\vec{x}) = A\ \exp(-B|\vec{x}|)$

  O_1 O_2

  $(A, B) = (0.5, 0.5),\ (0.5,\ 0.1)$ for $L = 24$

  $(A, B) = (0.5, 0.5),\ (1.0,\ 0.4)$ for $L = 48, 96$
- quark operator with non-relativistic projection in nucleon operator

3. Simulation parameters

2. Two-state analysis with 2×2 diagonalization

- Two volumes

L	L [fm]	N_{conf}	N_{meas}	N_r	q_1^2	q_2^2
32	4.1	300	192	40	0.184	1.3
48	6.1	300	144	32	0.1	1.1

- Exponential smearing sources $q(\vec{x}) = A \exp(-B|\vec{x}|)$

  O_1: $(A, B) = (0.5, 0.5)$

  O_r: spread random source with $(A, B) = (0.5, 0.5)$

- Two Wavefunction smearing sinks $N(\mathbf{x}) W_{q^2}(|\mathbf{x} - \mathbf{y}|) N(\mathbf{y})$

  $W_{q^2}(|\mathbf{r}|)$: Based on solution of 3-dimension Helmholtz equation

  parameters: q^2 and overall normalization

- quark operator with non-relativistic projection in nucleon operator

Simulations:

PACS-CS, T2K-Tsukuba at Univ. of Tsukuba

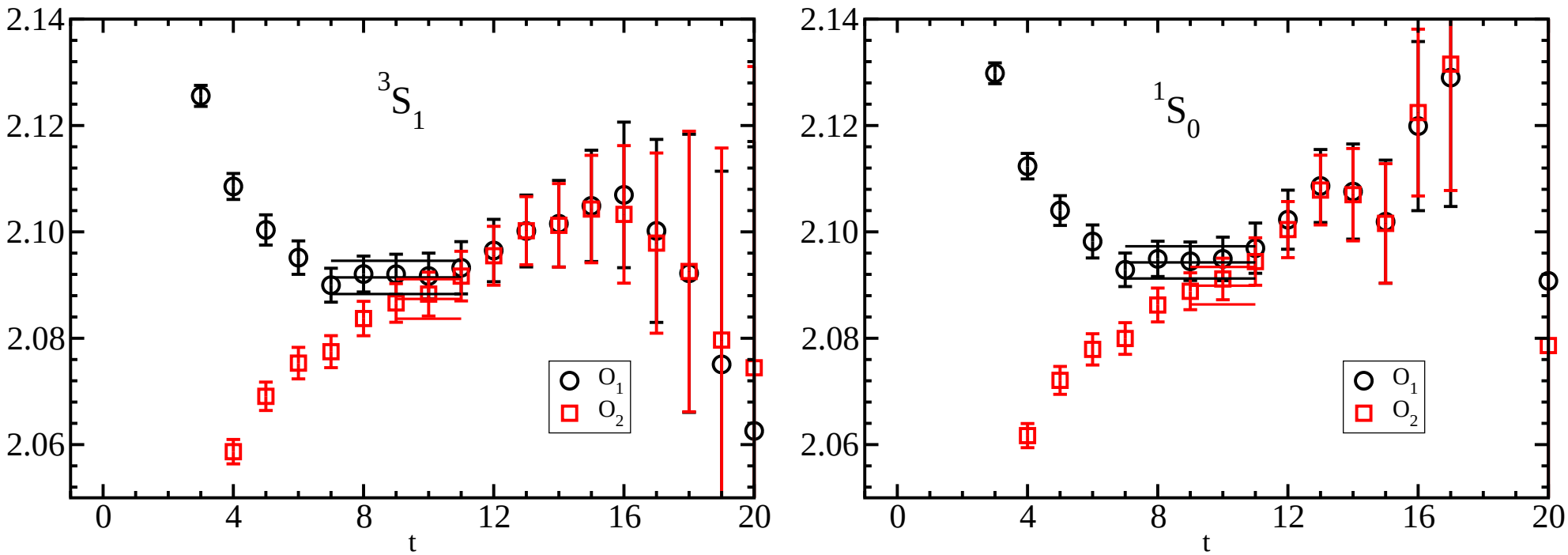
HA8000 at Univ. of Tokyo

4. Results

1. Single state analysis

Effective two-nucleon energy at $L = 6.1$ fm

$$E_{\mathrm{NN}}(t) = \log\left(\frac{C_{\mathrm{NN}}(t)}{C_{\mathrm{NN}}(t+1)}\right)$$



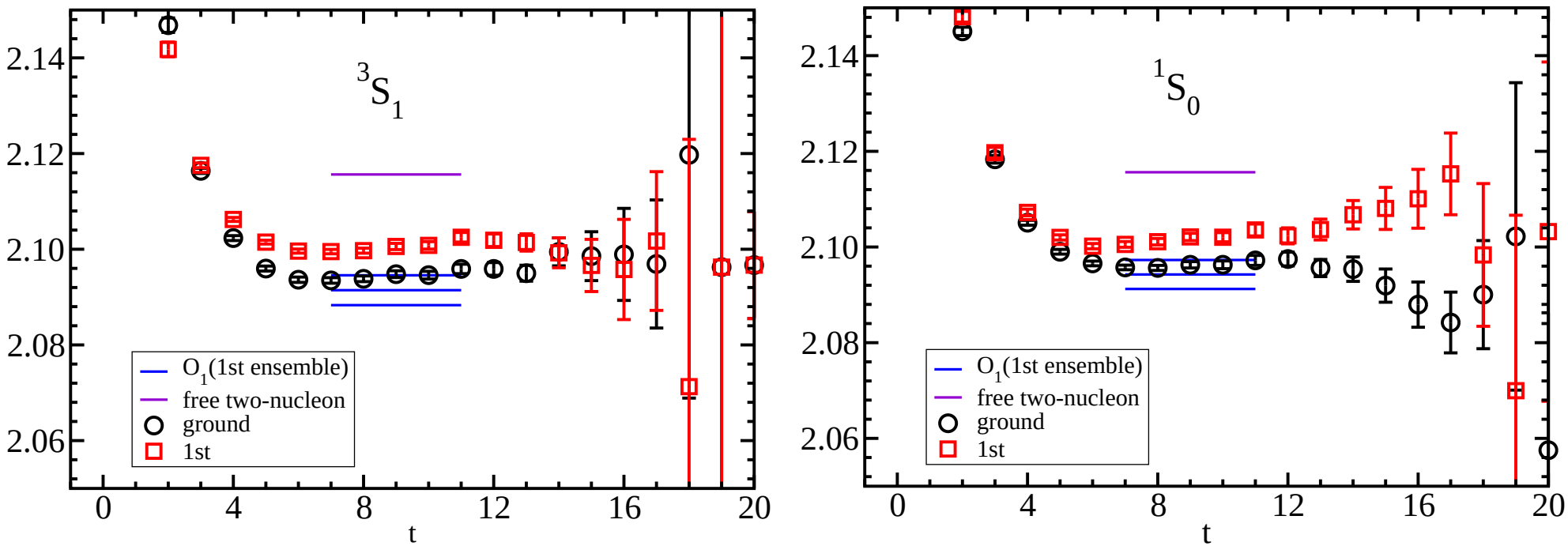
- Clear signal in $t < 12$, but larger error in $t \geq 11$
- consistent plateaus in $8 \lesssim t \leq 11$

2. Two-state analysis

Effective two-nucleon energy after diagonalization at $L = 6.1$ fm

$$\overline{E}_\alpha(t) = \log\left(\frac{\lambda_\alpha(t)}{\lambda_\alpha(t+1)}\right), \quad \text{Diag}\left[G^{-1}(t_0)G(t)\right] = \lambda(t) \text{ with } t_0 = 6$$

$$G_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle$$



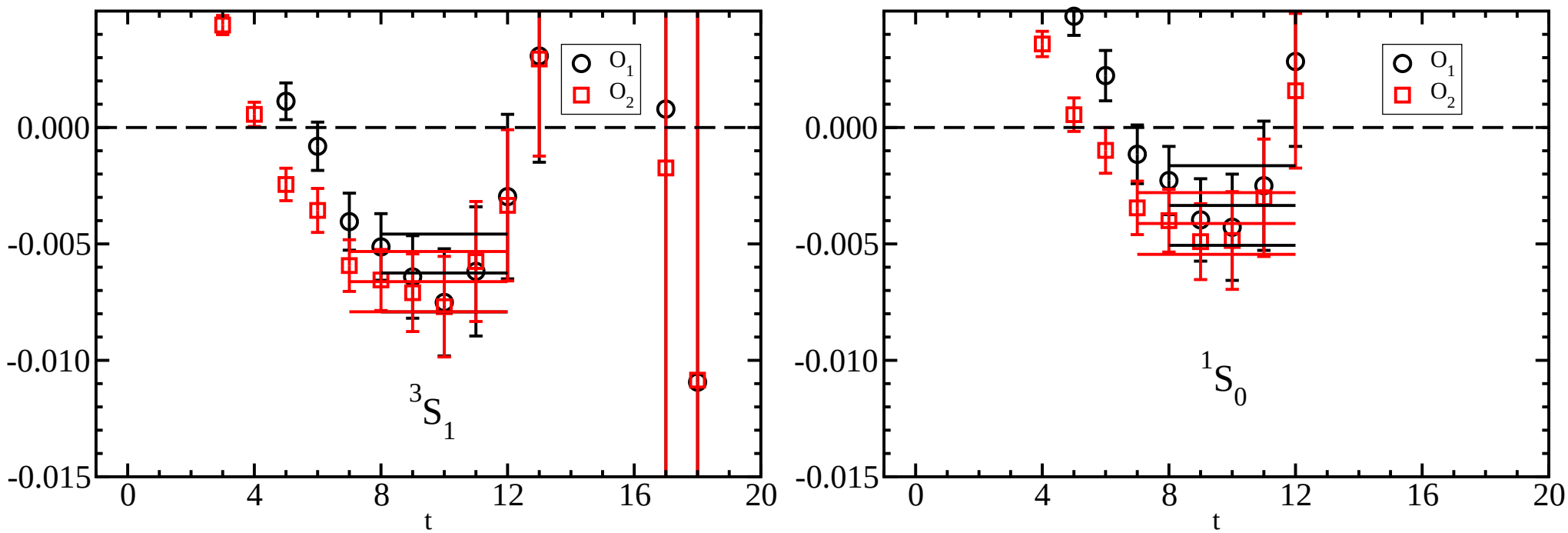
- Close two states, but clearly split
- ground state agree with one in single state analysis
- 1st excited state below free state $2\sqrt{M_N^2 + (2\pi/L)^2}$

4. Results

1. Single state analysis

Effective energy shift $\Delta E_L = E_{\mathrm{NN}} - 2m_N$ at $L = 6.1$ fm

$$\Delta E_L = \log\left(\frac{R(t)}{R(t+1)}\right), \quad R(t) = \frac{C_{\mathrm{NN}}(t)}{(C_N(t))^2}$$



- $\Delta E_L < 0$ in $8 \lesssim t \leq 11$
- consistent plateaus in $8 \lesssim t \leq 11$

Existence of bound state for a_0

System	w/ bound state	w/o bound state
0th	bound state	scattering state
1st	scattering state	scattering state
a_0	< 0 from 1st	> 0 from 0th

Bound state exists $\rightarrow$ a_0 never obtained from 0th state

by Lüscher's finite volume method

$$\Delta E_L = E^0_{NN} - 2m_N = -\frac{4\pi a_0}{m_N L^3} + \cdots$$ ('86, '91 Lüsher)

Need to check existence of bound state to calculate a_0

Two properties in w/ bound state system

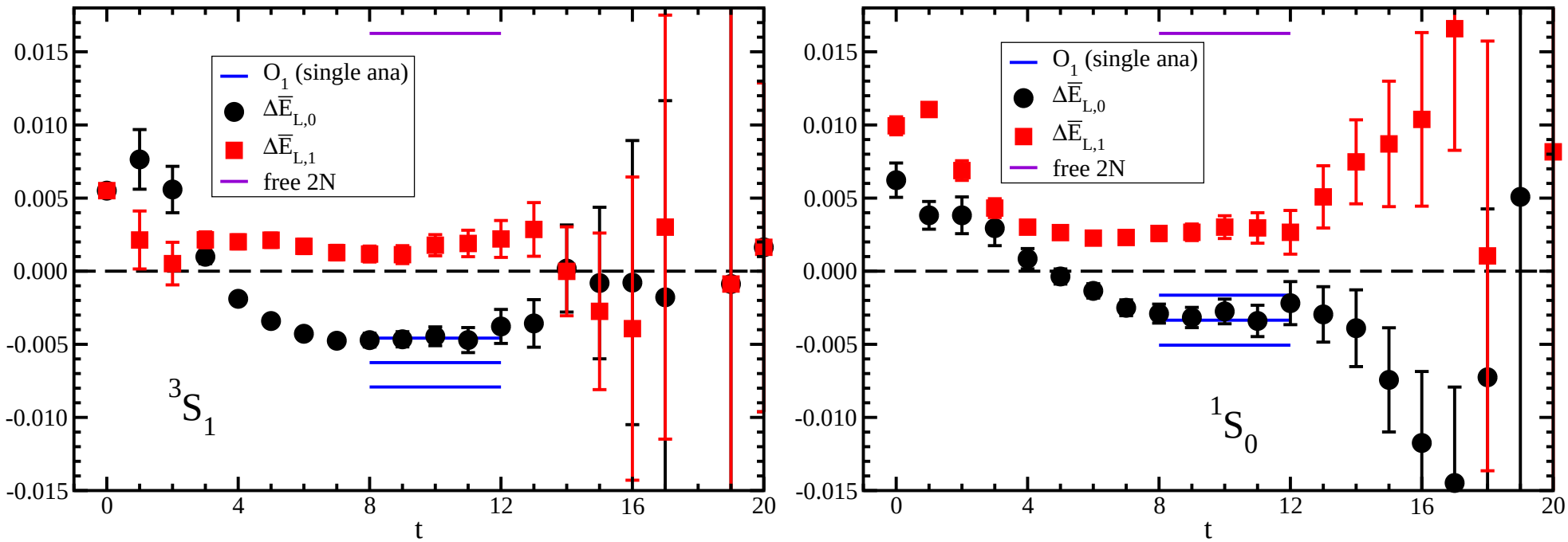
1. 0th state energy
2. a_0 (1st state energy)

2. 1st excited state energy

Effective energy shift for ground and 1st excited states at $L = 6.1$ fm

$$\text{Diag}\left[G^{-1}(t_0)G(t)\right] = \lambda(t) \text{ with } G_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle$$

$$\Delta\overline{E}_{L,\alpha} = E_\alpha - 2m_N = \log\left(\frac{\overline{R}_\alpha(t)}{\overline{R}_\alpha(t+1)}\right), \quad \overline{R}_\alpha(t) = \frac{\lambda_\alpha(t)}{(C_N(t))^2}$$



- $\Delta\overline{E}_{L,0} < 0$ and consistent with ΔE_L
- small, but $\Delta\overline{E}_{L,1} > 0$ as expected

Exploratory study of three- and four-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

3. Identification of bound state on finite volume

Finite volume effect in energy E_0

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Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark mass corresponding to $m_\pi = 0.8$ GeV

c.f. physical pion mass $m_\pi = 0.135$ GeV

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

$\rightarrow$ 1107 using symmetries

$p \leftrightarrow p$, $n \leftrightarrow n$, $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

3. Identification of bound state on finite volume

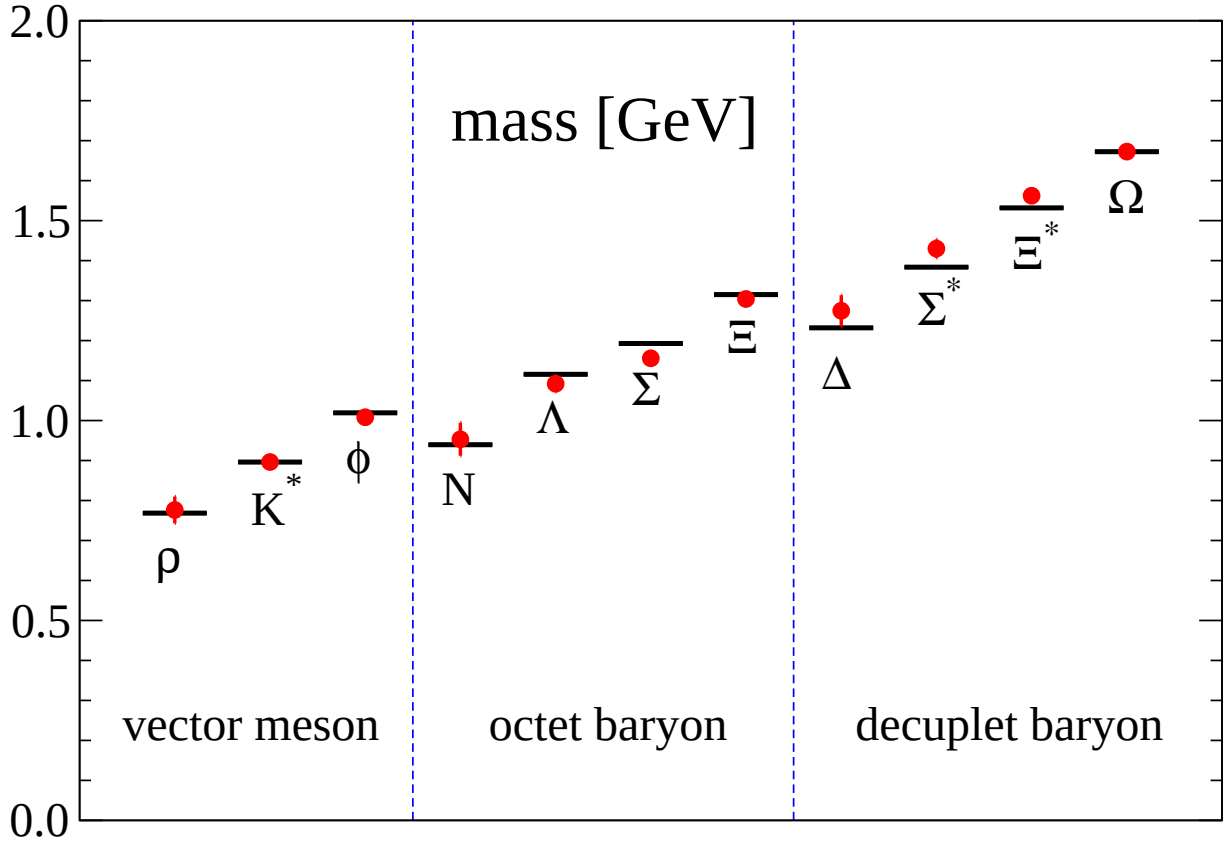
Finite volume effect in energy E_0

$\rightarrow$ Finite volume dependence of $\Delta E = E_0 - N_N m_N$

1. Introduction

Hadron spectrum from $N_f = 2 + 1$ QCD

'09 PACS-CS Collaboration



Input: m_π, m_K and m_Ω

Multi-baryon system from lattice QCD at '09

1. $\Lambda\Lambda$ system (Quenched QCD)

'85 Mackenzie & Thacker
'88 Iwasaki *et al.*
'99 Pochinsky *et al.*
'00 Wetzorke *et al.*
'02 Wetzorke & Karsch
'09 NPLQCD ($N_f = 2 + 1$)

H dibaryon: unbound

2. NN system $^3S_1$ and $^1S_0$

'95 Fukugita *et al.* : Quenched QCD

'06 NPLQCD : $N_f = 2 + 1$ QCD

'08 Ishii *et al.* : Quenched and $N_f = 2 + 1$ QCD

'09 NPLQCD : $N_f = 2 + 1$ QCD

Deuteron: unbound due to $m_\pi \gtrsim 0.3$ GeV

3. NNN system

'09 NPLQCD : $N_f = 2 + 1$ QCD

Triton: likely unbound

1. Statistical error

$$C_4(t) = \langle 0|O_{^4\mathrm{He}}(t)\overline{O}_{^4\mathrm{He}}|0\rangle \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Rough estimate of noise-to-signal ratio for N_N-nucleon channel

$$\frac{\delta C_{N_N}(t)}{C_{N_N}(t)} \propto \frac{1}{\sqrt{N_{\mathrm{meas}}}} \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$$

m_π: proportional to (quark mass)$^{1/2}$

$$\begin{array}{ccc} \text{quark mass} & \to & \text{small} \\ N_N & \to & \text{large} \\ N_{\mathrm{meas}} & \to & \text{small} \end{array} \Rightarrow \frac{\delta C_{N_N}(t)}{C_{N_N}(t)} \to \text{large} \Rightarrow \frac{\delta E_0}{E_0} \to \text{large}$$

Avoid large statistical fluctuation

- unphysically heavy quark mass $m_\pi = 0.8$ GeV and $m_N = 1.62$ GeV
- larger number of measurements than typical calculation

Lighter quark mass

$\to$ new method or much larger number of measurements

2. Calculation cost

$C_4(t) = \langle 0|O_{^4\mathrm{He}}(t)\overline{O}_{^4\mathrm{He}}(0)|0\rangle$ with $O_{^4\mathrm{He}} = p^2n^2 = [udu]^2[dud]^2$

Number of Wick contraction $N_u! \times N_d! = (2N_p + N_n)! \times (2N_n + N_p)!$

^{4}He: $6! \times 6! = 518400$

^{3}He: $5! \times 4! = 2880$

NN: $4! \times 2! = 48$

but contain identical contractions

Summation of $\sim$ (color and dirac)^{2N_N} in each contraction

Reduction of contractions

- Symmetries
  - $p \leftrightarrow p$, $n \leftrightarrow n$ in ^{4}He operator
  - Isospin all $p \leftrightarrow$ all n
- Calculate two contractions simultaneously
  - $u \leftrightarrow u$ in p or $d \leftrightarrow d$ in n

2. Calculation cost (cont'd)

$C_4(t) = \langle 0|O_{^4\mathrm{He}}(t)\overline{O}_{^4\mathrm{He}}(0)|0\rangle$ with $O_{^4\mathrm{He}} = p^2n^2 = [udu]^2[dud]^2$

Number of Wick contraction $N_u! \times N_d! = (2N_p + N_n)! \times (2N_n + N_p)!$

^{4}He: $6! \times 6! = 518400 \longrightarrow 1107$

^{3}He: $5! \times 4! = 2880 \longrightarrow 93$

Summation of $\sim$ (color and dirac)^{2N_N} in each contraction

Further reduction: avoid same calculations of dirac and color indices

Block of three quark propagators B_3

zero momentum nucleon operator in sink time slice

Blocks of two B_3

1, 2, 3 dirac contractions carried out

Multi-meson systems : '10 Detmold and Savage

Recursion relations: $R_{n+1} = \langle R_n \rangle \cdot R_1 - nR_n \cdot R_1$

Multi-baryon systems : '12 Doi and Endres

Unified contraction algorithm

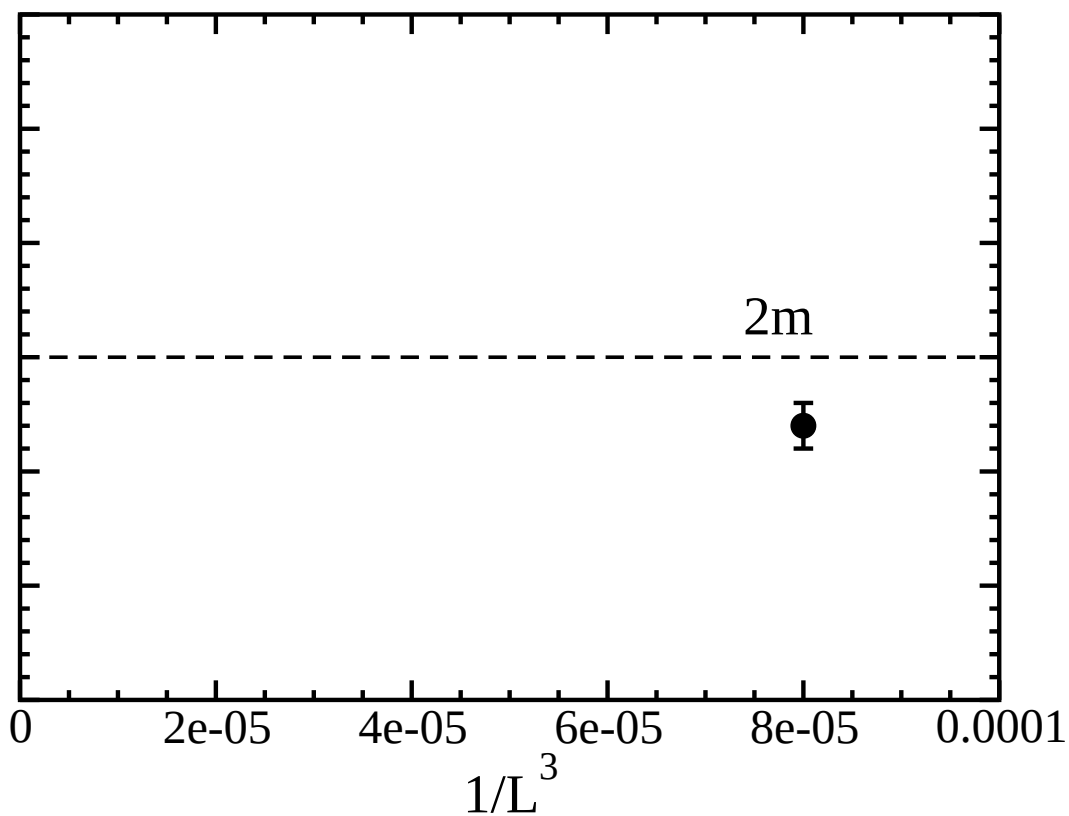
3. Identification of bound state in finite volume

$$C_4(t) = \langle 0|O_{^4\mathrm{He}}(t)\overline{O}_{^4\mathrm{He}}|0\rangle \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Observed state is whether bound state or not?

Example) Two-particle system

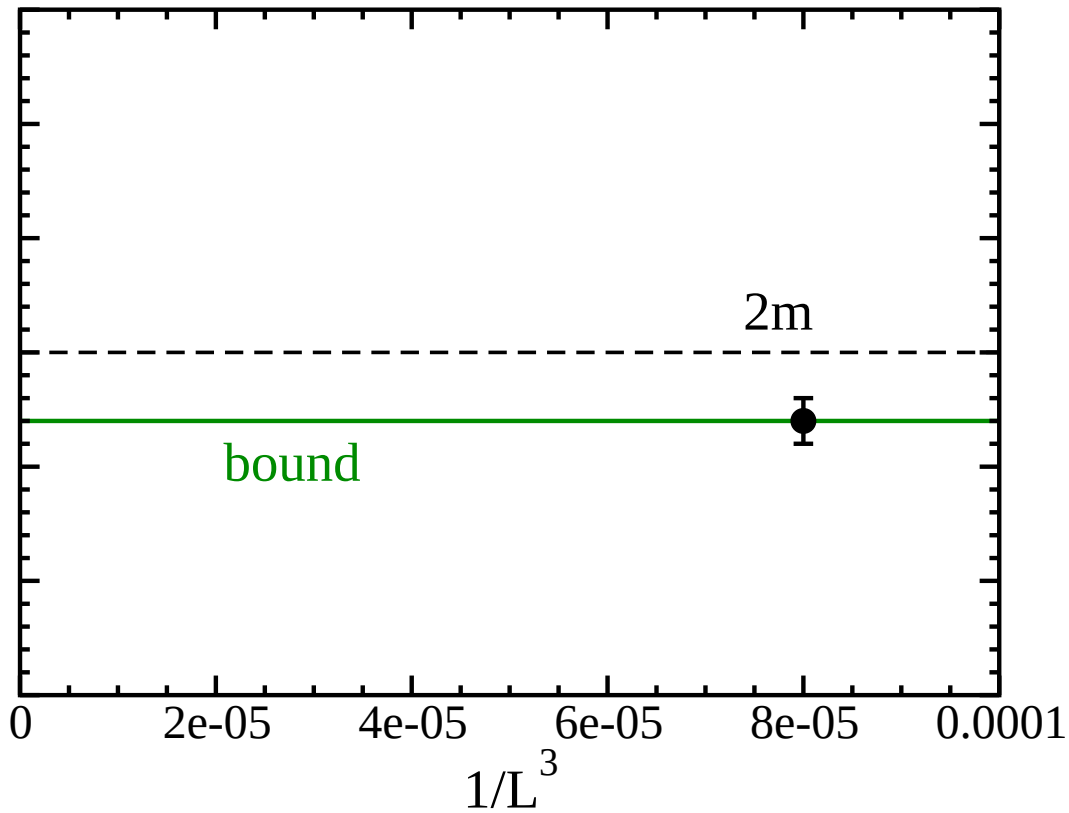
observe small $\Delta E = E - 2m < 0$ at single L



3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at single L



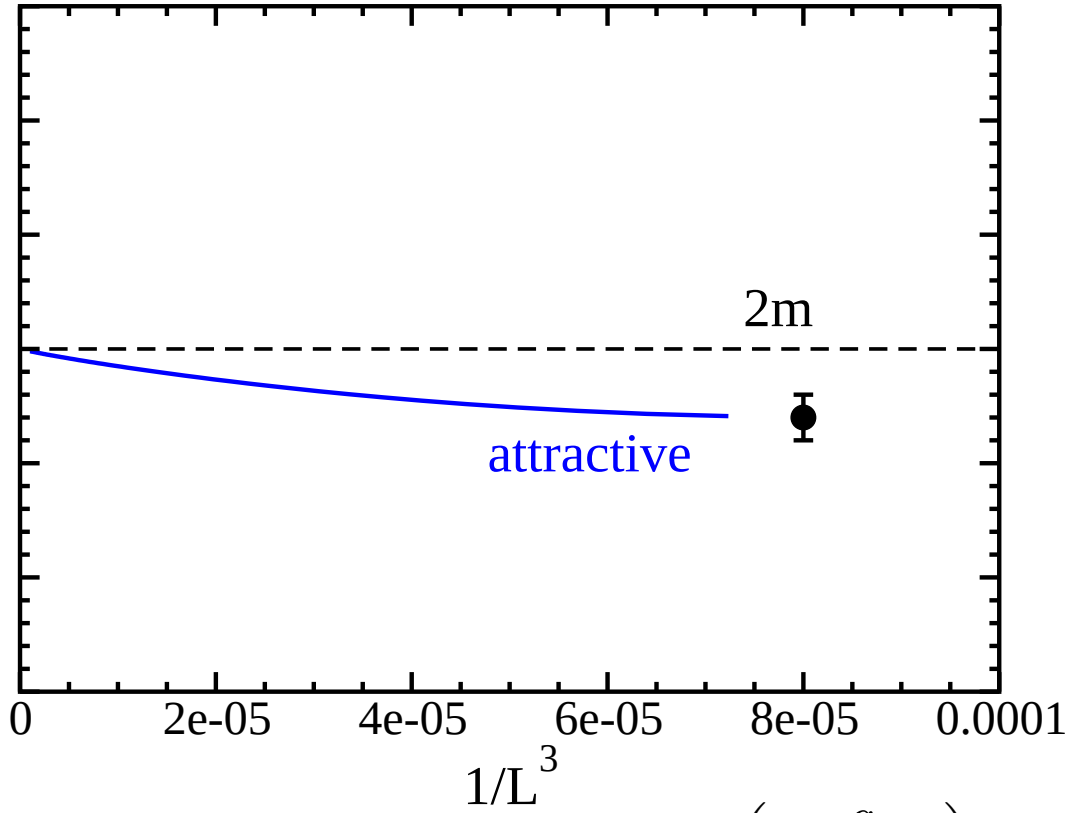
Bound state : $\Delta E = -\Delta E_{\text{bind}} + O(\mathrm{e}^{-\gamma L}) < 0$

Beane *et al.*, PLB585:106(2004), Sasaki & TY, PRD74:114507(2006)

3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at single L



Attractive scattering state : $\Delta E = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

Lüscher, CMP105:153(1986), NPB354:531(1991)

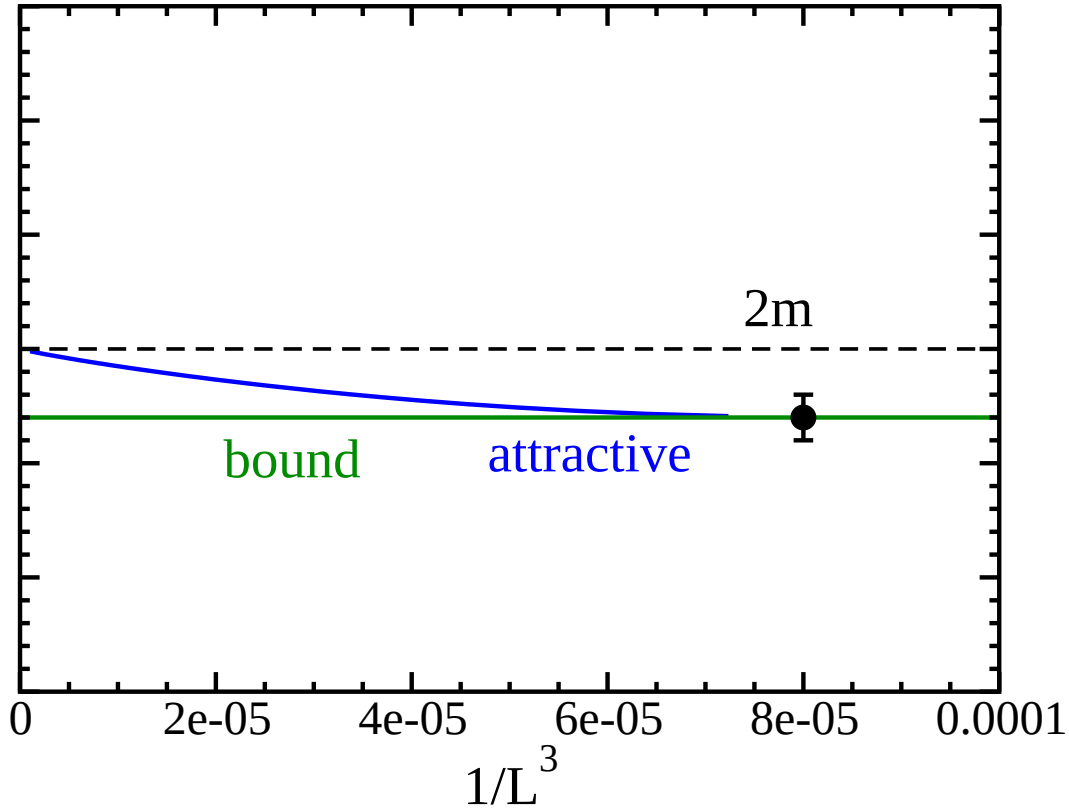
c.f.) N-particle scattering state : $\Delta E = E_{\text{scat}} - Nm = O\left(-\frac{{}_N C_2\, a_0}{ML^3}\right)$

Beane *et al.*, PRD76:074507(2007)

3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at single L



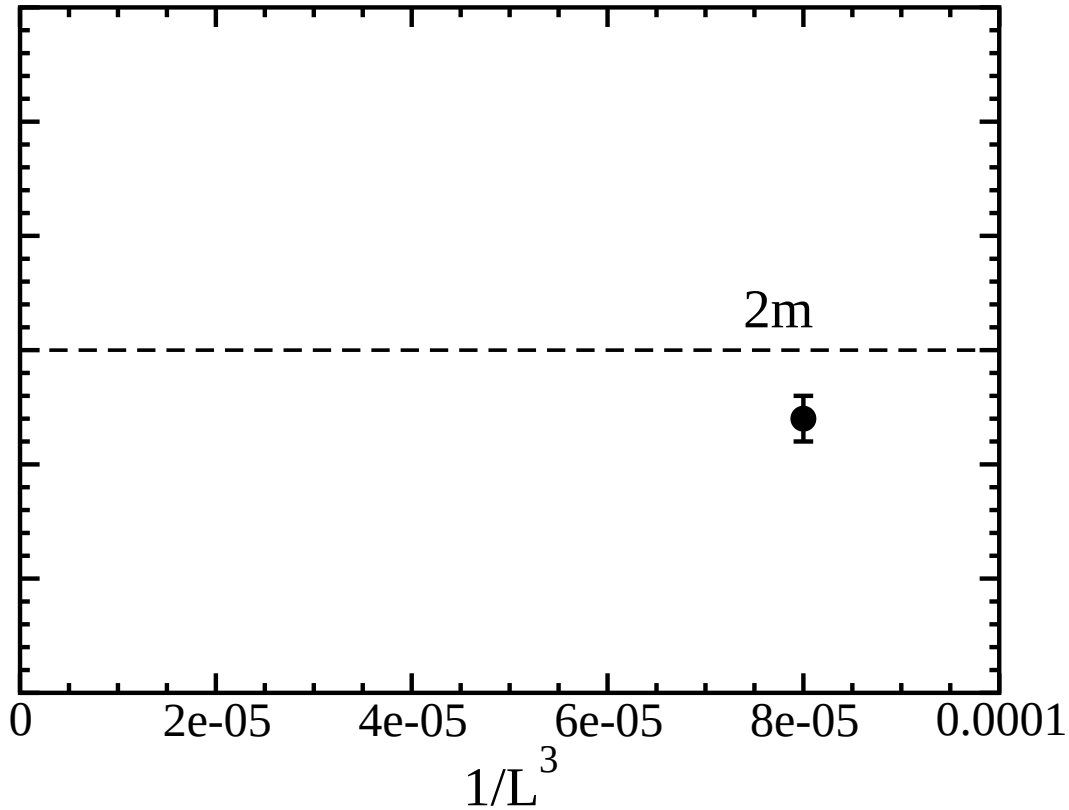
Hard to distinguish at single L

Bound state and Attractive scattering state

3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

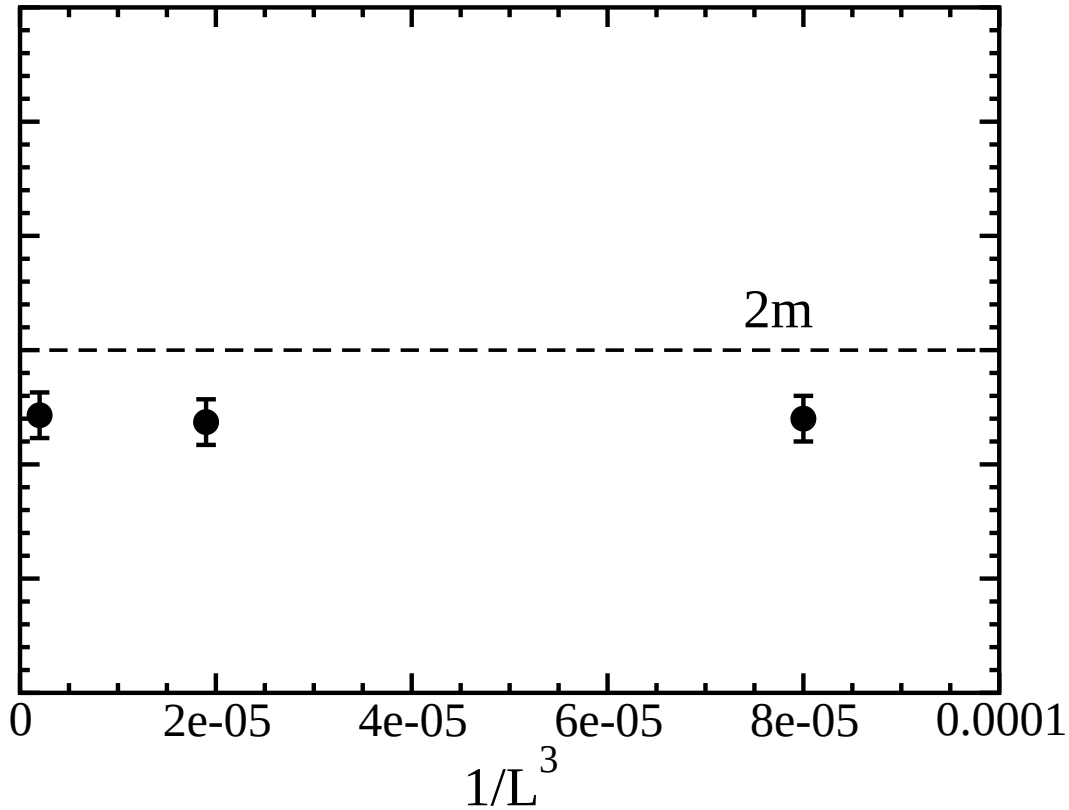
observe small $\Delta E = E - 2m < 0$ at several L



3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

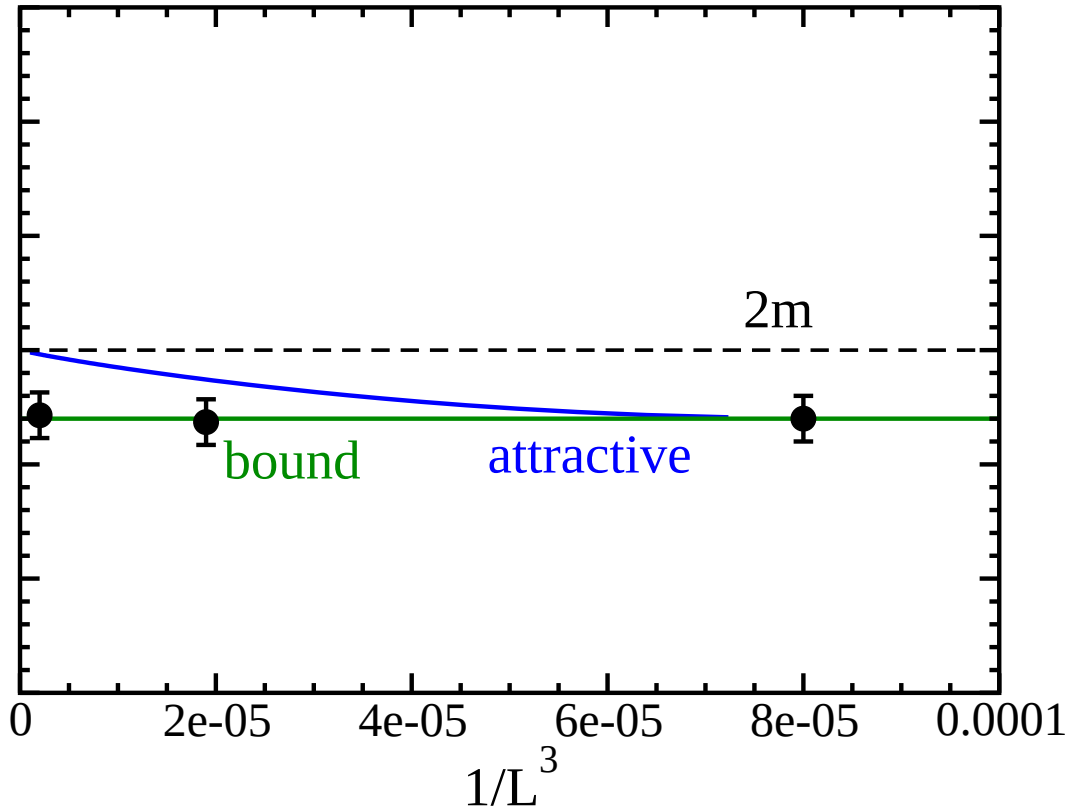
observe small $\Delta E = E - 2m < 0$ at several L



3. Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at several L



Identify bound state from volume dependence of ΔE

observe constant in infinite volume limit with $L = 3.1, 6.1, 12.3$ fm

Other methods: spectral weight: Mathur *et al.*, PRD70:074508(2004)

anti-periodic boundary.: Ishii *et al.*, PRD71:034001(2005)

2. Problems of multi-nucleon bound state

NN channels

1. Statistical error

   Not so severer than 3- and 4-nucleon channels

$$\frac{\delta C_{N_N}(t)}{C_{N_N}(t)} \propto \frac{1}{\sqrt{N_{\text{meas}}}} \exp\left(N_N \left[m_N - \frac{3}{2} m_\pi\right] t\right)$$

2. Calculation cost

   Not so severer than 3- and 4-nucleon channels

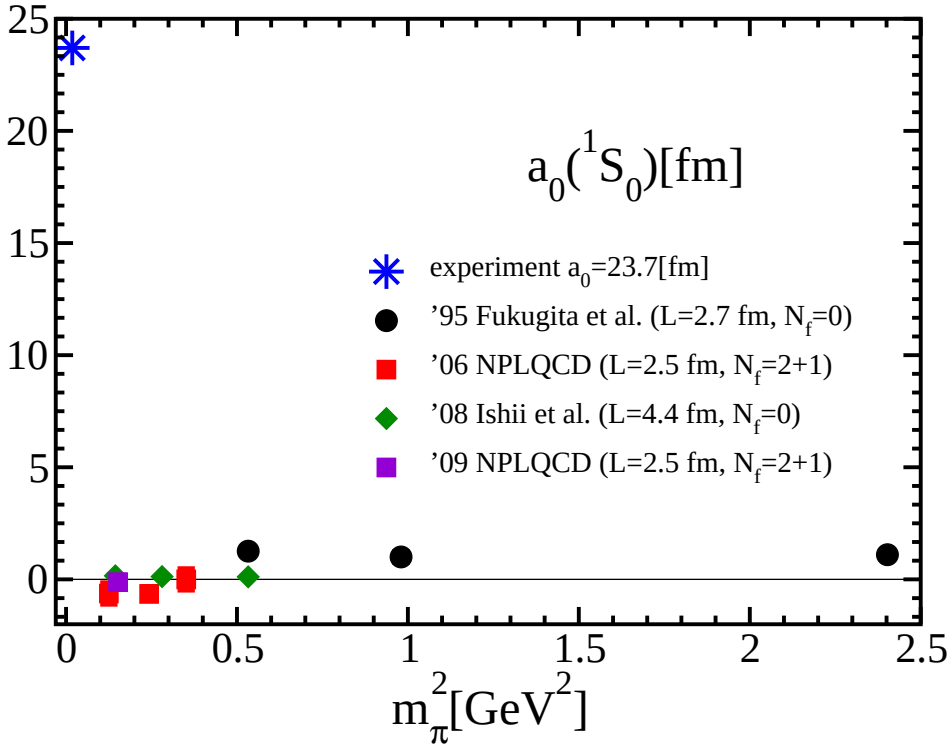
   # of contraction < 10 after reduction

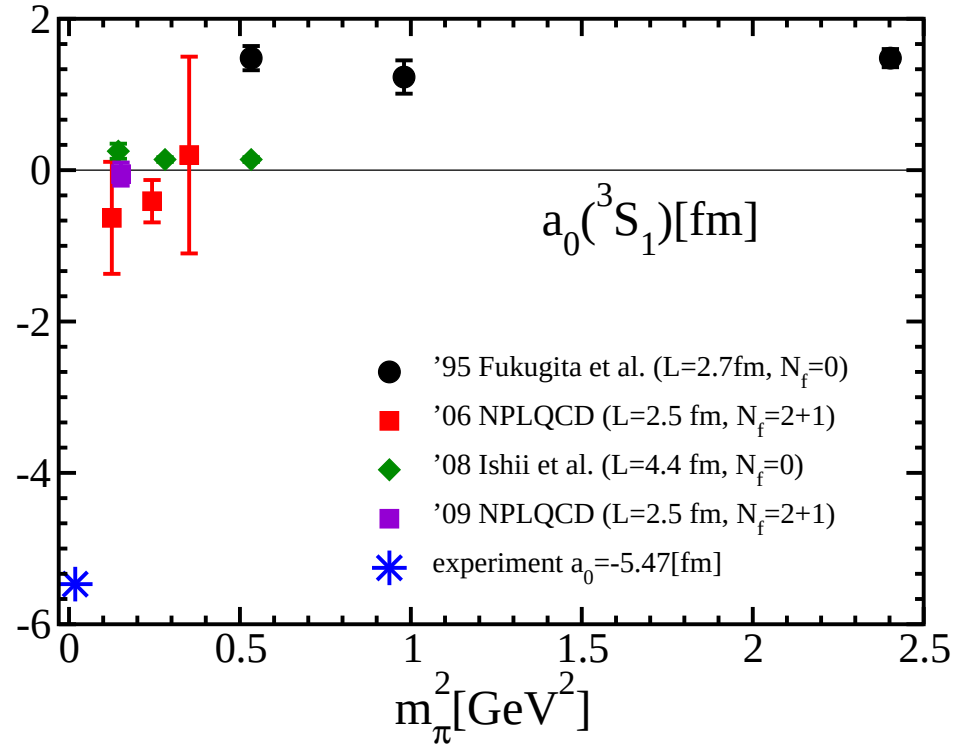
3. Identification of bound state in finite volume

   Similar to 3- and 4-nucleon channels

Bound state in simplest multi-nucleon system, NN system?

Scattering length a_0 in NN system at '09





a_0: Far from experiments due to $m_\pi \gtrsim 0.3$ GeV

Assumption: Deuteron in $^3S_1$ channel unbound due to $m_\pi \gtrsim 0.3$ GeV

Aim of this work: check assumption by simpler method

c.f. using nuclear force '09 HALQCD

Existence of bound state for a_0

System	w/ bound state	w/o bound state
0th	bound state	scattering state
1st	scattering state	scattering state
a_0	< 0 from 1st	> 0 from 0th

Bound state exists $\rightarrow$ a_0 never obtained from 0th state

by Lüscher's finite volume method

$$\Delta E_L = E^0_{NN} - 2M = -\frac{4\pi a_0}{ML^3} + \cdots$$ ('86, '91 Lüsher)

Need to check existence of bound state to calculate a_0

Two methods

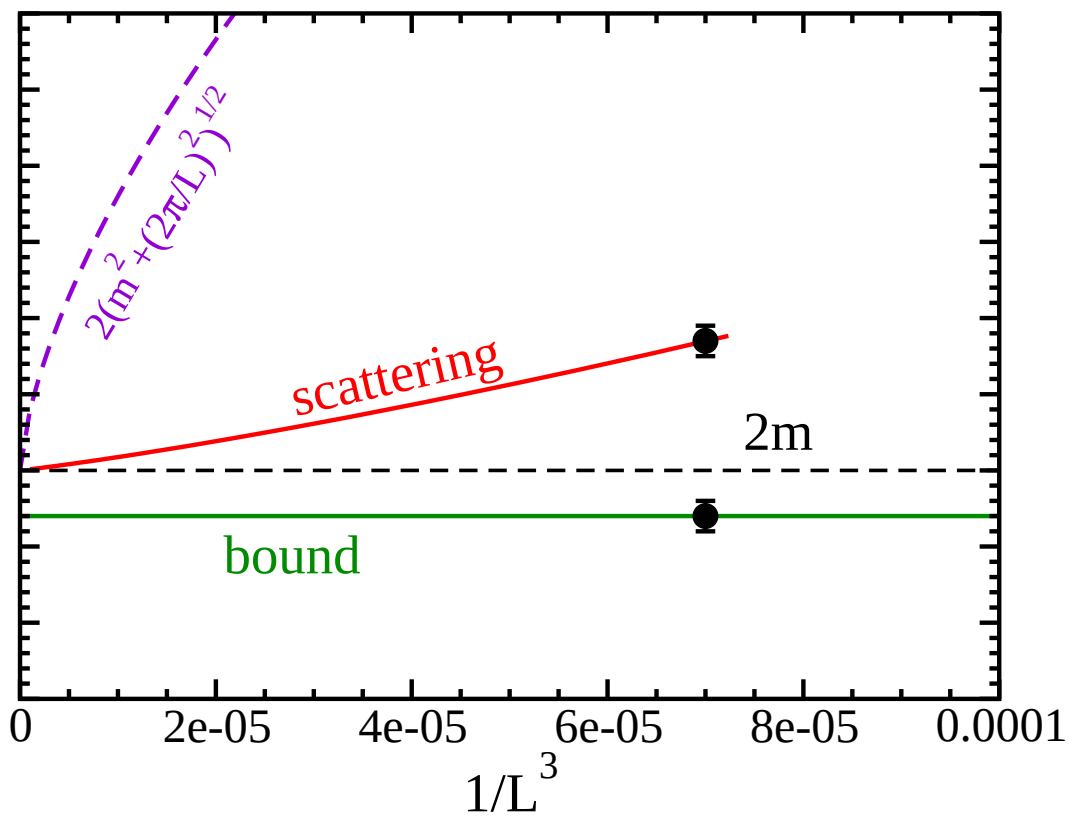
1. Volume dependence of 0th state
2. Properties of 1st state energy

Properties of 1st excited state energy in finite volume

'06 Sasaki & TY

bound state exists $\rightarrow$ 1st excited state = scattering state with $a_0 < 0$

$\rightarrow \Delta E_{L,1} > 0$ at finite L



Scattering state : $\Delta E_L = O\left(-\frac{a_0}{mL^3}\right) > 0 \quad (a_0 < 0)$ '86, '91 Lüscher

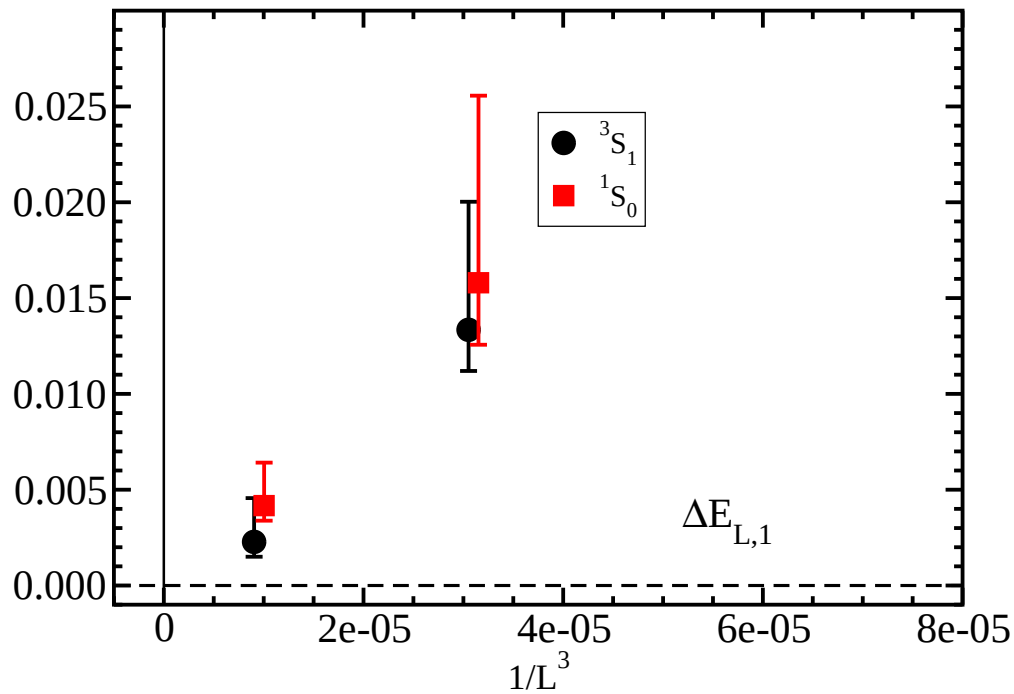
1st excited state $\leftarrow$ diagonalization method '90 Lüscher & Wolff

4. Results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_{L,1} = E_1 - 2m_N$

PACS-CS Collaboration, PRD84:054506(2011)

Volume dependence of $\Delta E_{L,1}$



$\Delta E_{L,1} > 0$ and $1/L^3$ tendency

Scattering length a_0 fm

L[fm]	$^3S_1$
3.1	$-1.5(0.2)\left(^{+0.2}_{-1.4}\right)$
6.1	$-1.05(24)\left(^{+0.05}_{-0.65}\right)$
L[fm]	$^1S_0$
3.1	$-1.8(0.3)\left(^{+0.4}_{-12.9}\right)$
6.1	$-1.62(24)\left(^{+0.01}_{-0.75}\right)$

Observe expected properties of 1st excited state