Phase diagram of QCD at finite isospin chemical potential with Wilson fermions

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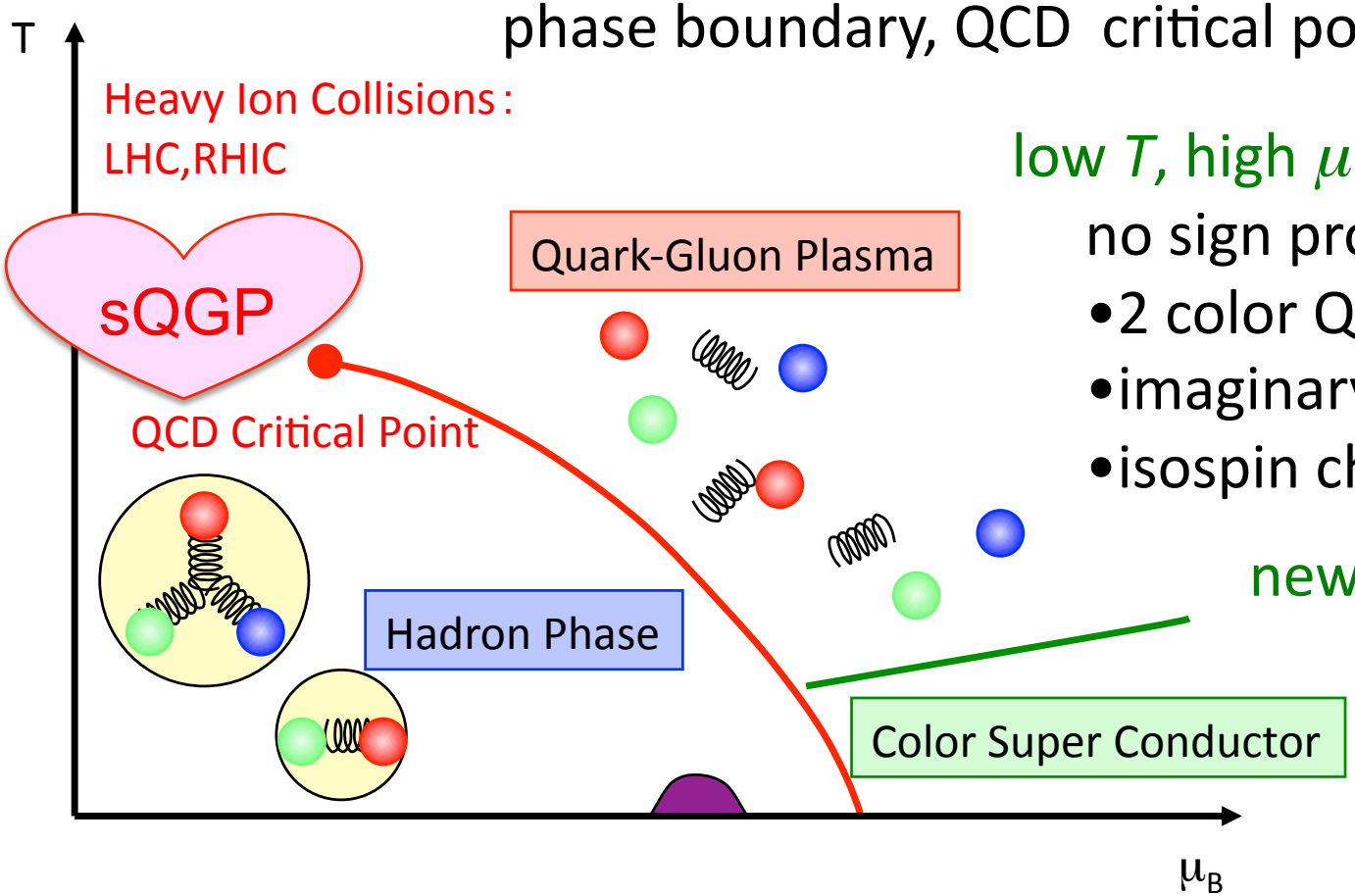
Introduction

- Finite density lattice QCD - sign problem

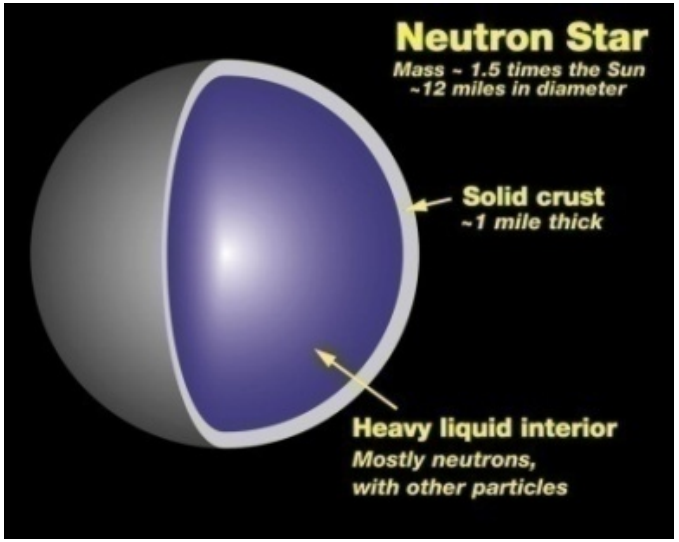
high T, low μ : reweighting method, Taylor expansion…
phase boundary, QCD critical point



low T, high μ: QCD like theories
no sign problem
- 2 color QCD
- imaginary chemical potential
- isospin chemical potential

new development :
complex langevin
etc.

Finite Isospin Chemical Potential



- Core of neutron stars ?

$$\mu_u = \mu + \mu_I$$

$$\mu_d = \mu - \mu_I$$

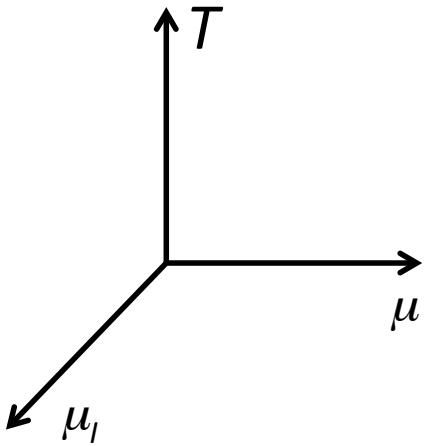
$\mu_I > 0$: $\mu_u > \mu_d$, positive charge
$u_I < 0$: $\mu_u < \mu_d$, negative charge

pion condensation occurs at $\mu_{IC} = \frac{1}{2} m_\pi$ (lowest meson mass)
rho condensation ?
strangeness: kaon condensation? hyperons?

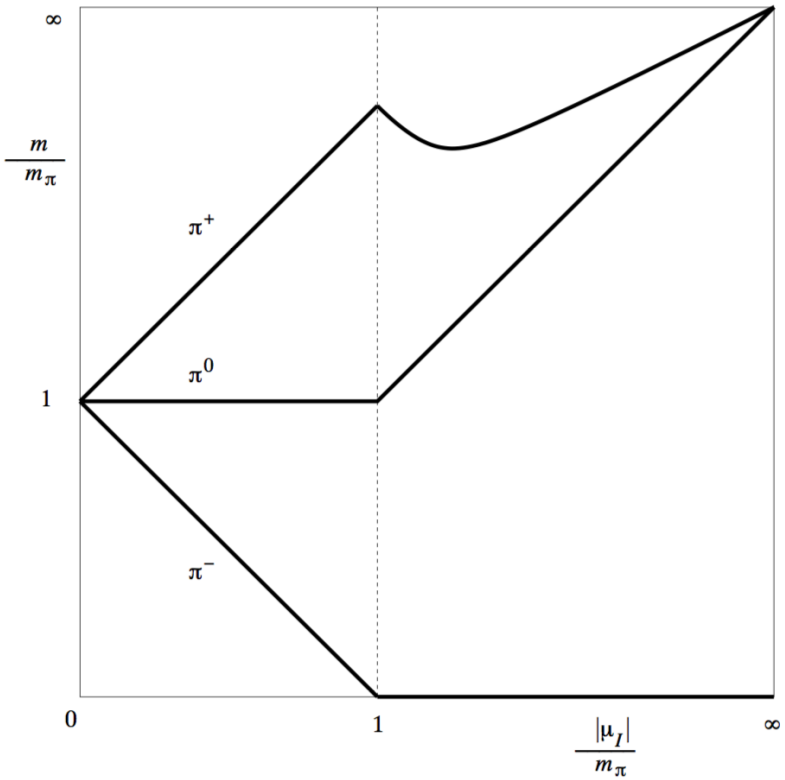
– radius and mass ? ⟵ Equation of state

- Insight of finite chemical potential

– Phase diagram as a function of T, μ and μ_I

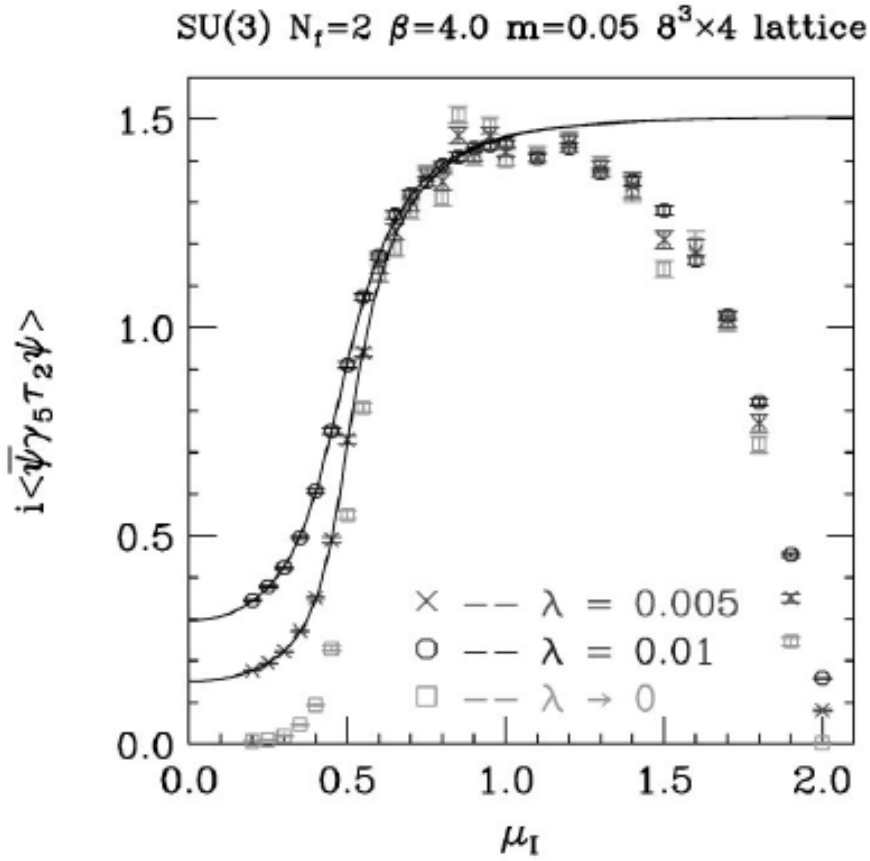


Pion Condensation



Chiral perturbation theory
Son, Stephanov
hep-ph/0011365

$\mu_I \ll m_\rho$

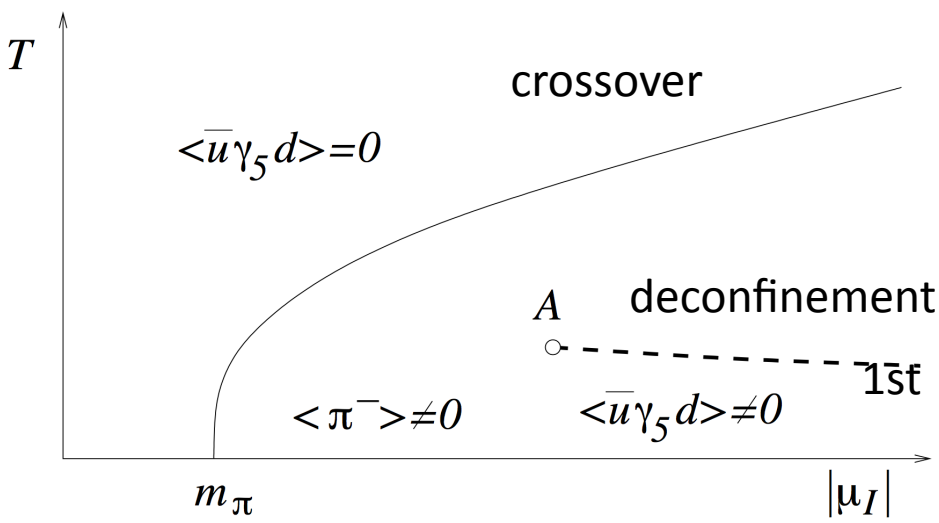


KS fermion
Kogut,Sinclair
Phys rev.D66,034505(2002)

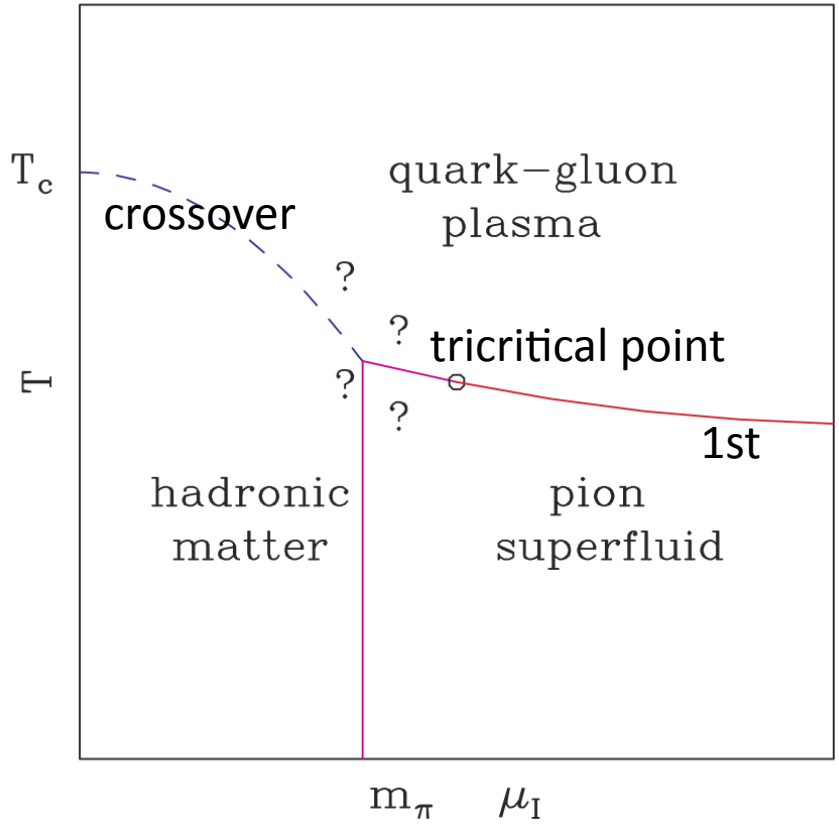
Condensation happens in other channels?

Phase Diagram of QCD at $\mu_I \neq 0$

- Condensation and confinement



Chiral perturbation theory
Son, Stephanov, hep-ph/0011365



KS fermion
Proposed phase diagram in
Kogut, Sinclair, PRD70, 094501(2004)

Introduction of μ_I

- 2 flavor fermion action (Wilson fermion)

$$
\begin{aligned}
S_F &= \bar{\Psi}[\gamma_\mu D_\mu + m_q + \mu\gamma_4 \frac{\tau^3}{2} + i\lambda\gamma_5 \frac{\tau^2}{2}]\Psi \\
&= \bar{\Psi}\begin{pmatrix} D(\mu) & \lambda\gamma_5 \\ -\lambda\gamma_5 & D(-\mu) \end{pmatrix}\Psi \qquad D(\mu) = \gamma_\mu D_\mu + m_q + \frac{\mu}{2}\gamma_4 \\
&= \bar{\Psi} D(U) \Psi
\end{aligned}
$$

τ^2, τ^3: Pauli matrix

- $\bar{\Psi}[\, i\lambda\gamma_5 \frac{\tau^2}{2}]\Psi$
  - λ: explicit I_3 breaking parameter
  - positivity of $\det D(U)$ ⟺ sign problem at finite μ

    $\det D(U) = \det\left[D^\dagger(\mu)D(\mu) + \lambda^2\right]$

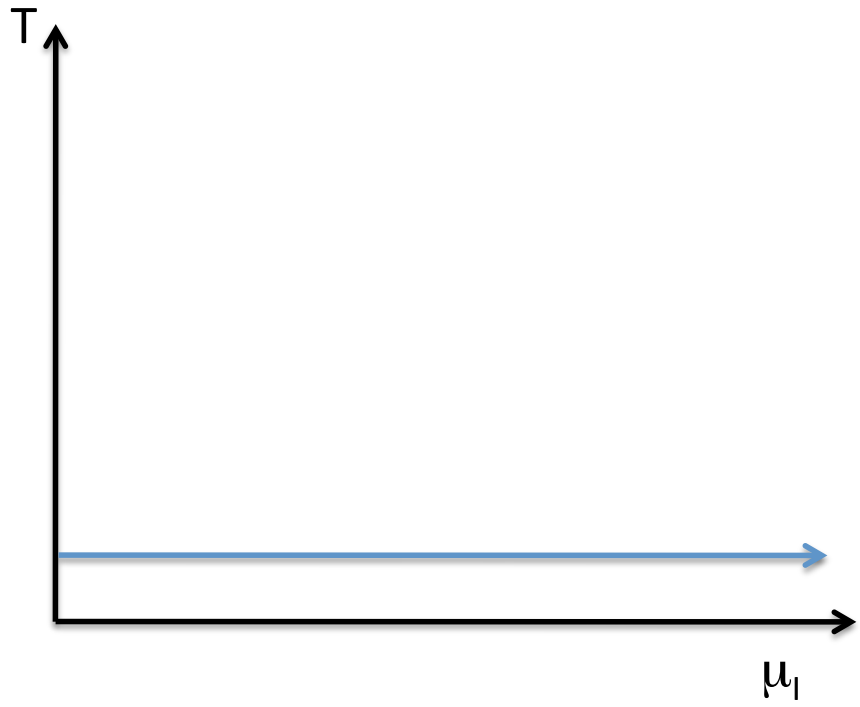
    Hybrid Monte Carlo method
  - observables: $\lambda \longrightarrow 0$

Parameters

- Wilson fermion

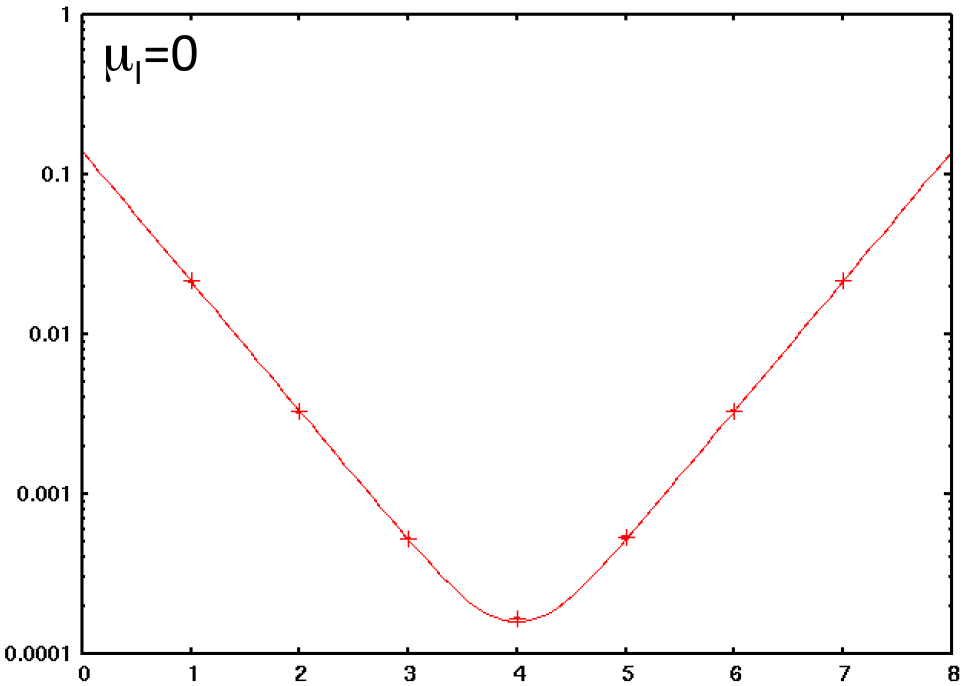
T=0

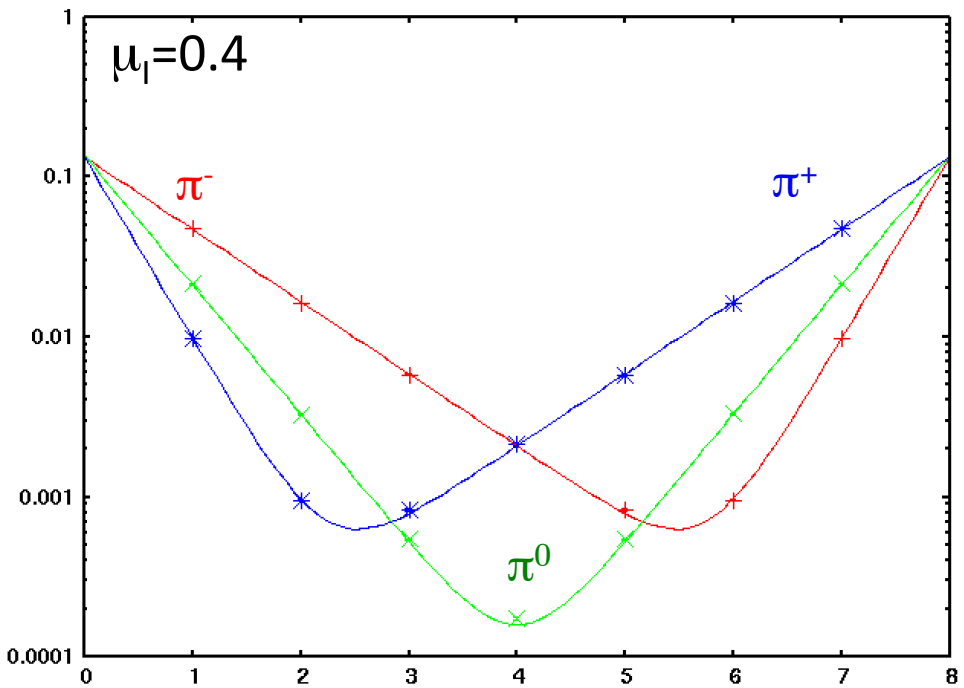
- lattice size:$4^3 \times 8$
- β=5.0
- κ=1.50
- μ_I= 0 ~1.3
- λ= 0
- m_π=1.869(8), m_ρ=1.916(9)



Correlators

- 4^3X8





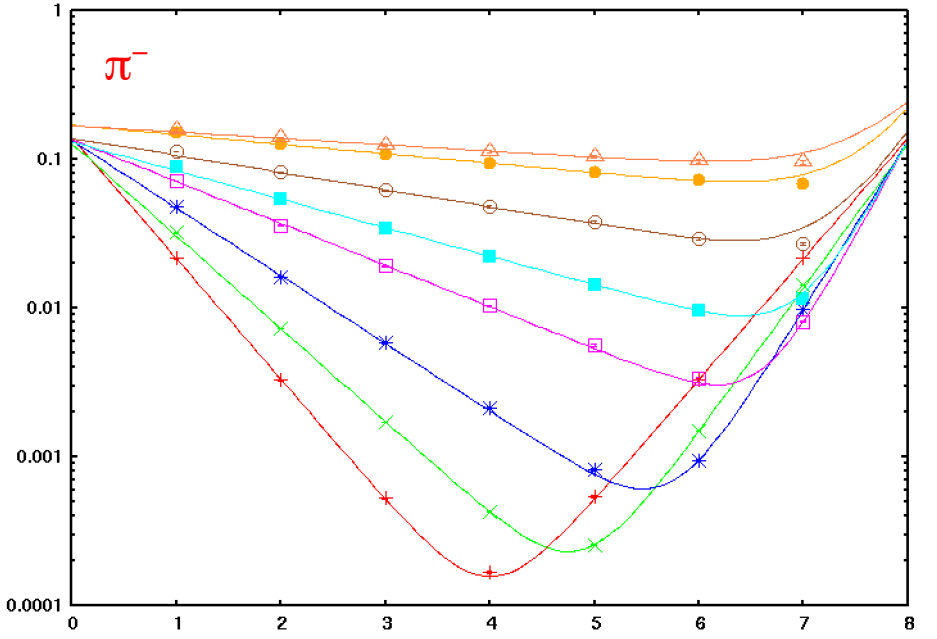
$$f(t) = a\exp(-m_1 t) + a\exp(-m_2(8-t))$$

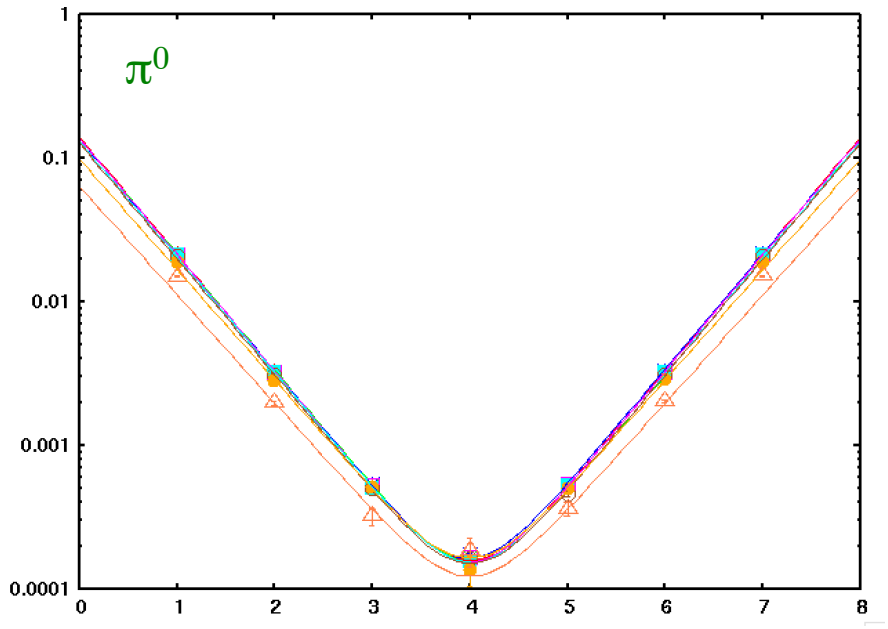
$\mu_l=0$
π^+, π^0, π^- degenerate

$\mu_l\neq0$
μ_l affets π^+, π^0, π^-

Correlators (π)

μ_l=0 ~ 1.0





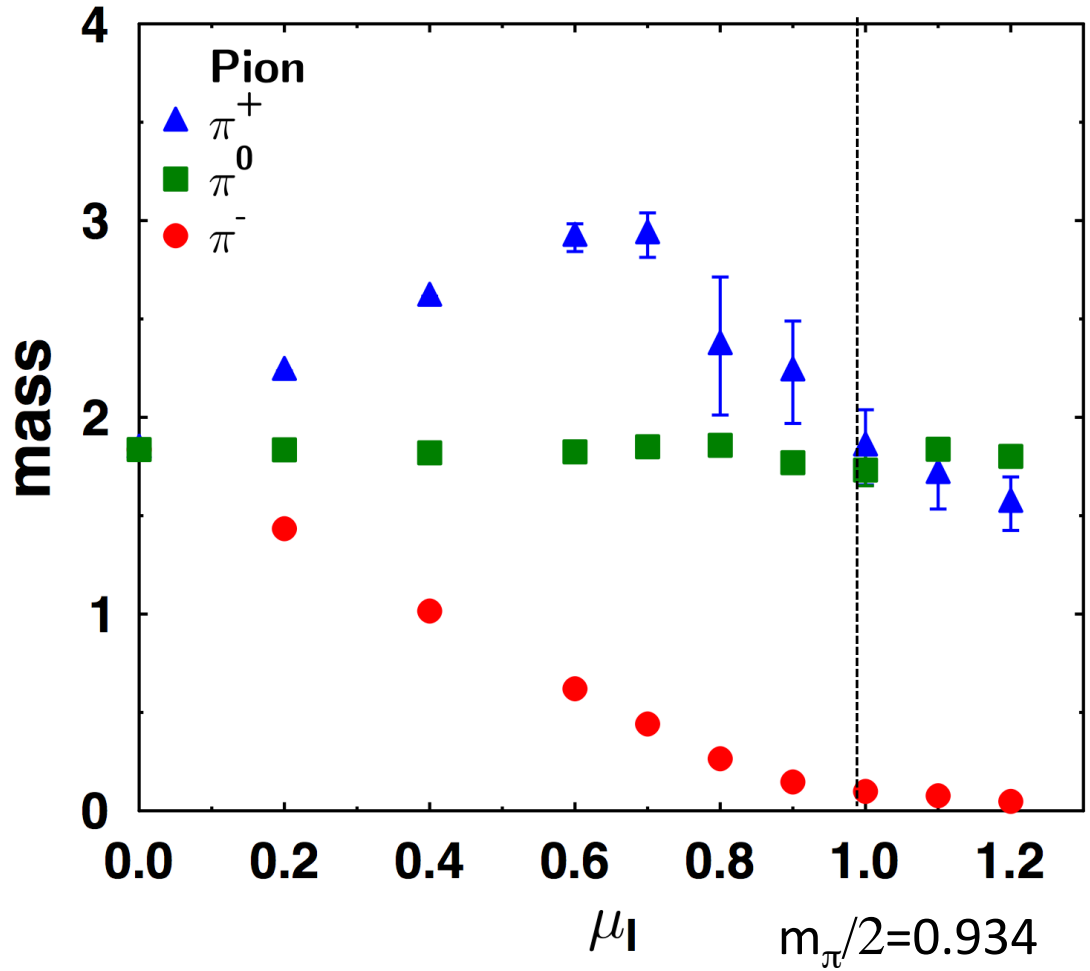
π^-

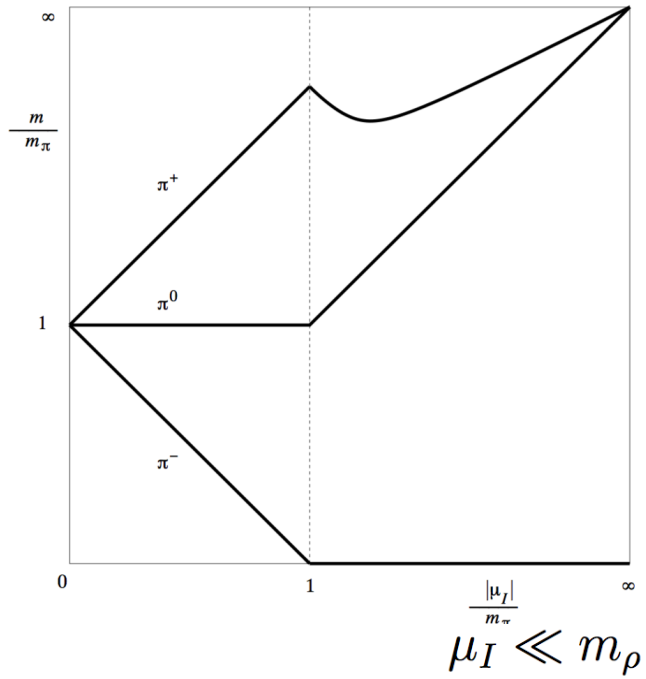
- The behavior of correlator changes dramatically.
- Mass becomes lighter.

π^0

- The behavior of correlator does not change.
- μ_l does not affect π^0

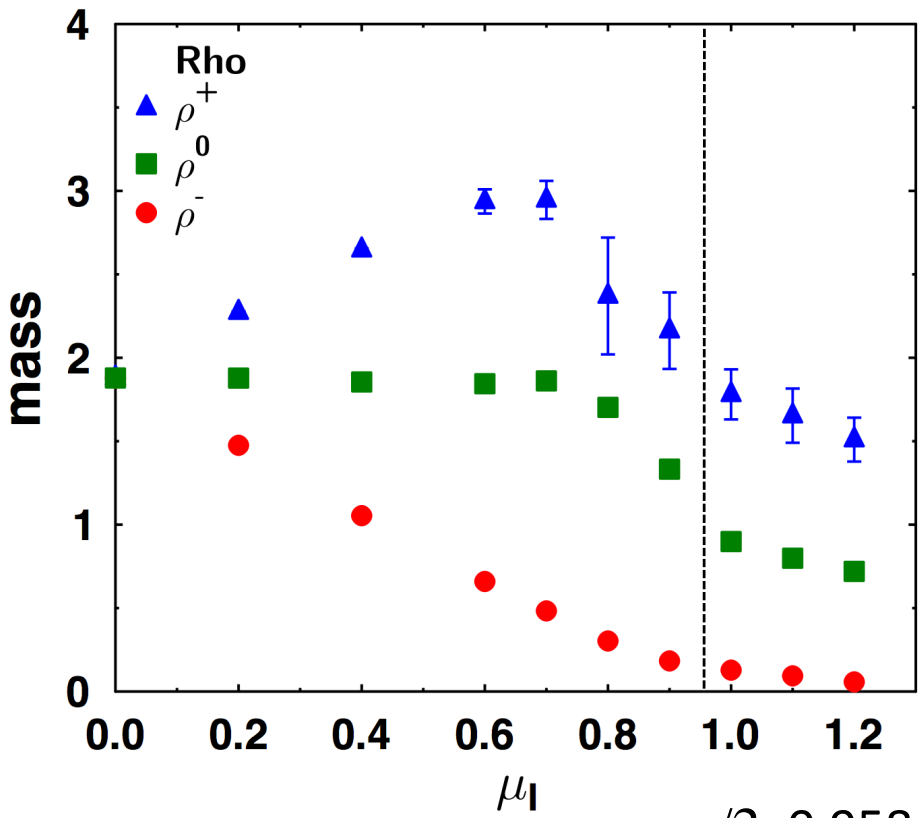
Pion Masses vs μ_I



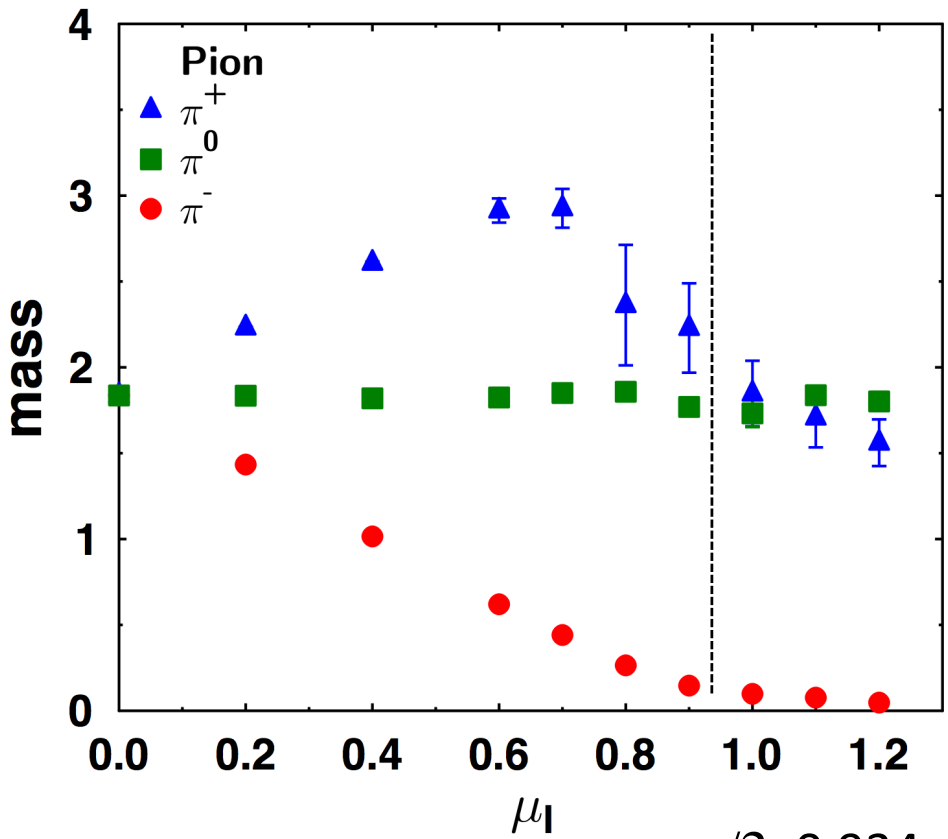


- Consistent with chiral perturbation theory
  - π^- :condensates
  - π^0 :constant
  - π^+ :mass increases.

Rho Masses vs μ_I



$m_\rho/2=0.958$



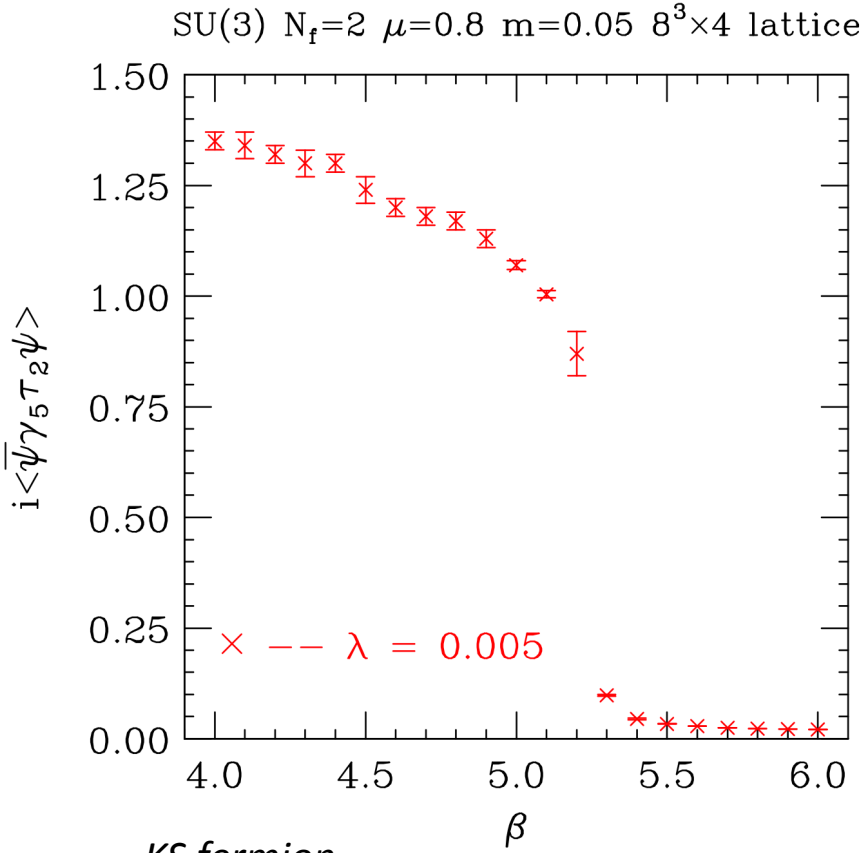
$m_\pi/2=0.934$

- Charged π and ρ
  The behavior of mass is the same.
- Neutral π and ρ
  π^0: mass does not change.
  ρ^0: mass becomes lighter.

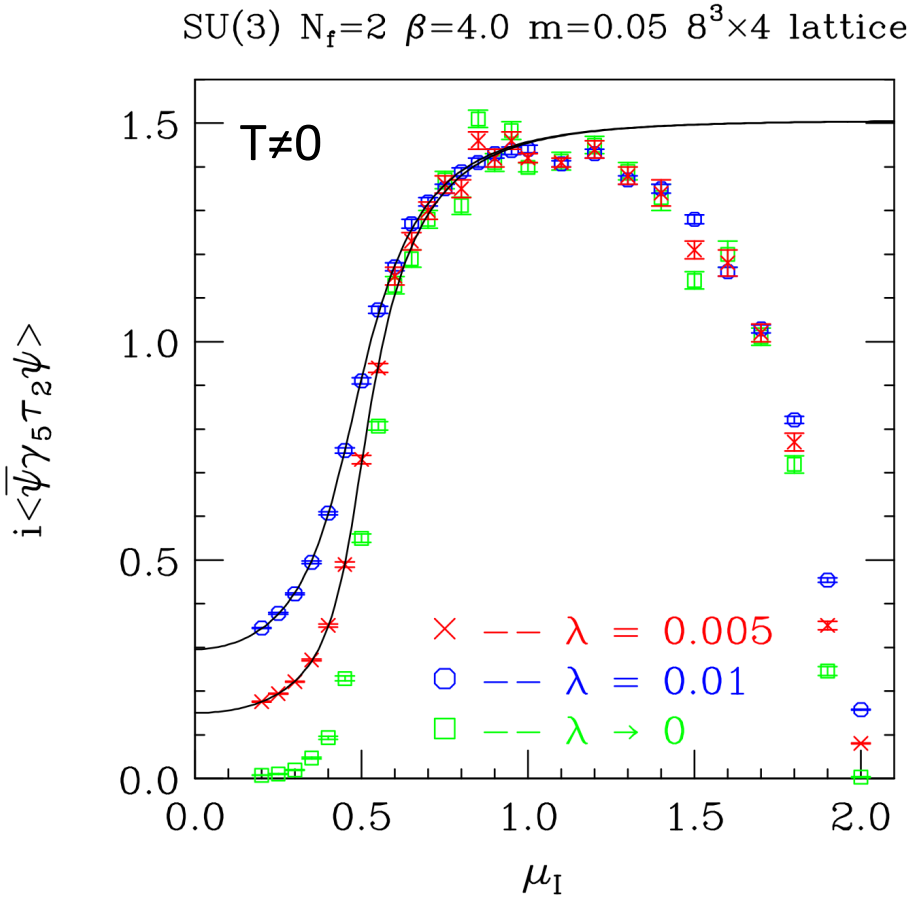
- Caveat
  The quark mass is heavy.
  The interesting window is very small.

C. NONAKA

Finite Temperature



KS fermion
Kogut,Sinclair
arXiv:hap-lat/0202028



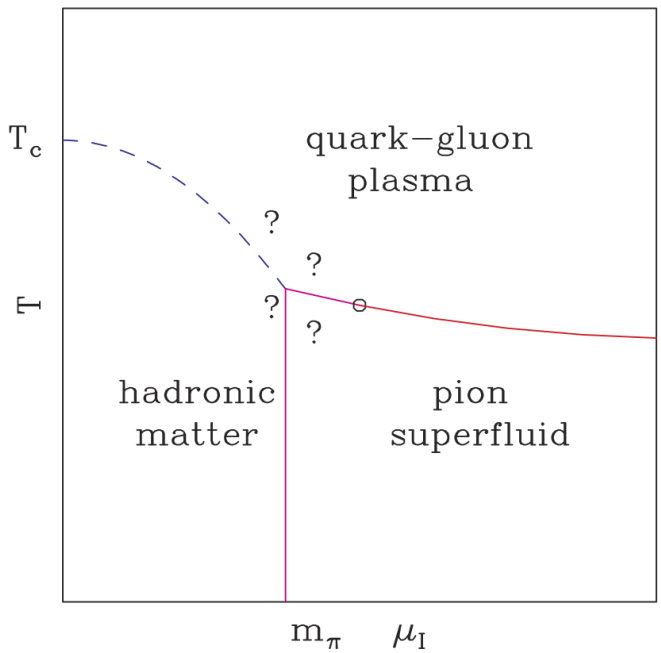
- T≠0
  They observe finite pion condensation.

Parameters for T≠0

- Wilson fermion

- lattice size:$4^3 \times Nt$
- β=5.0
- κ=1.50
- μ_I= 0 ~1.3
- m_π=1.869(8), m_ρ=1.916(9)

- We changed N_T, but we do not observe the phase transition on the small lattice.
- π condensation seems to evaporate.
- Parameter choices are important
  Larger lattice volume, β



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Summary

- Revisit finite isospin chemical potential with Wilson ferminon
  - We observe pion condensation as well as rho condensation
  - π: Consistent with chiral perturbation theory
    - π^- condensate, π^0: mass is constant, π^+: mass increases
  - ρ: mass behavior is almost the same as that of
    - ρ^0: mass decreases.
  - Finite temperature
    - We do not observe the phase transition

      -> lager volume? β should be changed.