N=1 supersymmetric Yang-Mills theory on the lattice

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in the City of New York

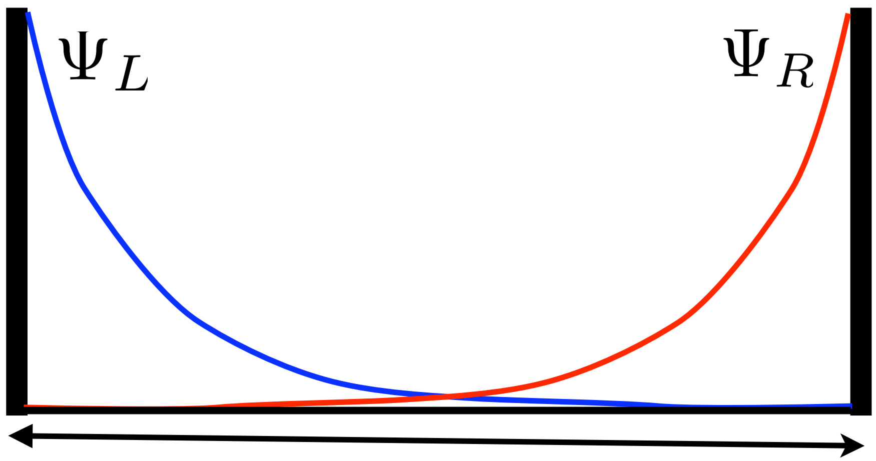
Supersymmetry (SUSY) on the lattice

- Much effort toward formulating supersymmetric lattice theories, motivated by
  - the interesting theoretical challenge
  - potential role of SUSY in beyond the standard model physics
  - understand nonperturbative aspects of SUSY theories
    - dynamical symmetry breaking
    - spectrum
- Naive lattice formulations explicitly break SUSY
  - fine tuning of operators to reach SUSY point

N=1 super Yang-Mills (SYM) on the lattice

- 1 gauge field + 1 adjoint Majorana fermion (gluino)
- Lattice explicitly breaks SUSY, however for this theory, SUSY restored accidentally in continuum and chiral limits D. B. Kaplan (1984)
- Domain wall formulation of lattice fermions ideal for N=1 SYM
  - good chiral properties, no fine tuning D. B. Kaplan and M. Schmaltz (1999)
  - positive definite fermion determinant

N=1 SYM with domain-wall fermions (DWFs)



$$m_{res} \sim \# \frac{e^{-\# L_s}}{L_s} + \# \frac{\rho(0)}{L_s}$$

- Gaugino appears as chiral modes bound to the fifth-dimension boundaries of a 5-dimensional theory
- Overlap of wave-functions give rise to non-zero residual Majorana mass (m_{res}) at finite L_s
- Vanishing residual mass corresponds to chiral limit and the supersymmetric point (assuming vanishing input quark mass)

N=1 SYM on the lattice

- Test theoretical predictions
  - gluino condensate
  - discrete chiral symmetry breaking, domain walls
  - spectrum
  - sorry, no SUSY breaking in this theory

N=1 SYM on the lattice

- Test theoretical predictions
  - gluino condensate — Vranas, Kogut and Fleming (2001)
  - discrete chiral symmetry breaking, domain walls
  - spectrum
  - sorry, no SUSY breaking in this theory...

N=1 SYM with DWF

- Questions not addressed by Vranas, et. al.
  - lattice scale
  - what is the residual mass (how close to SUSY point)?
  - continuum limit and renormalization of gluino condensate
- Since then
  - improved algorithms (e.g. RHMC algorithm)
  - much faster computers
  - better understanding of DWFs (e.g. L_s-dependence of m_{res}; can do better job extrapolating to chiral limit)

N=1 SYM on the lattice

- Two recent independent studies using DWF:
  - M. G. Endres (2008) arXiv:0810.0431
  - J. Geidt, et. al. (2008) arXiv:0810.5746

N=1 SYM on the lattice

- Test theoretical predictions
  - gluino condensate
  - discrete chiral symmetry breaking, domain walls
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Numerical simulations

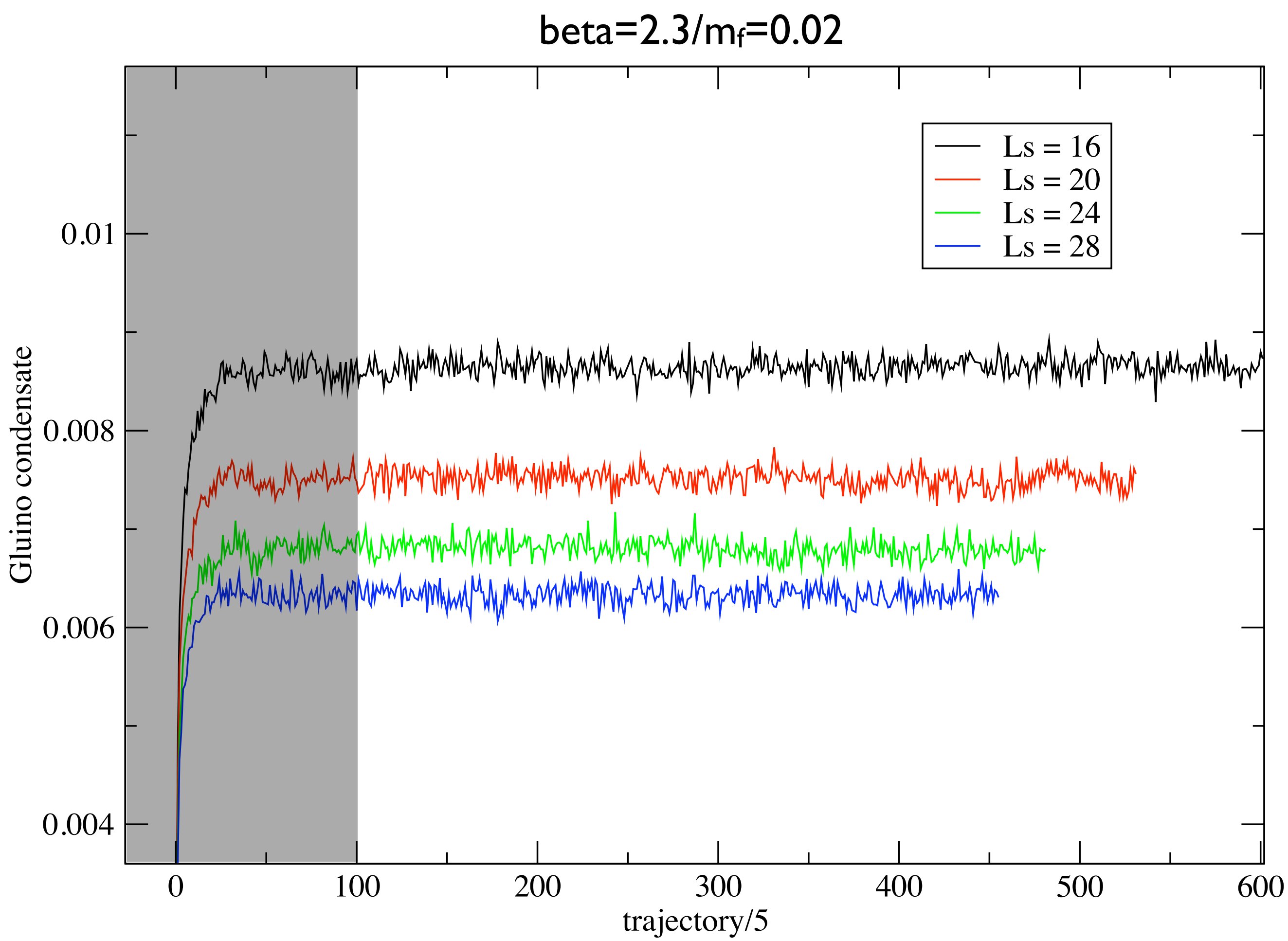
- Wilson gauge action with domain wall fermions
- SU(2) gauge group with adjoint Majorana fermions
- Simulations performed on an appropriately modified version of the Columbia Physics System (CPS)
- $8^3 \times 8 \times L_s$ ensembles were generated and measurements made on QCDOC at Columbia University
- $16^3 \times 32 \times L_s$ ensembles were generated and measurements made on New York Blue (BlueGene/L) at Brookhaven National Laboratory

Simulation parameters for 16^3x32 lattices

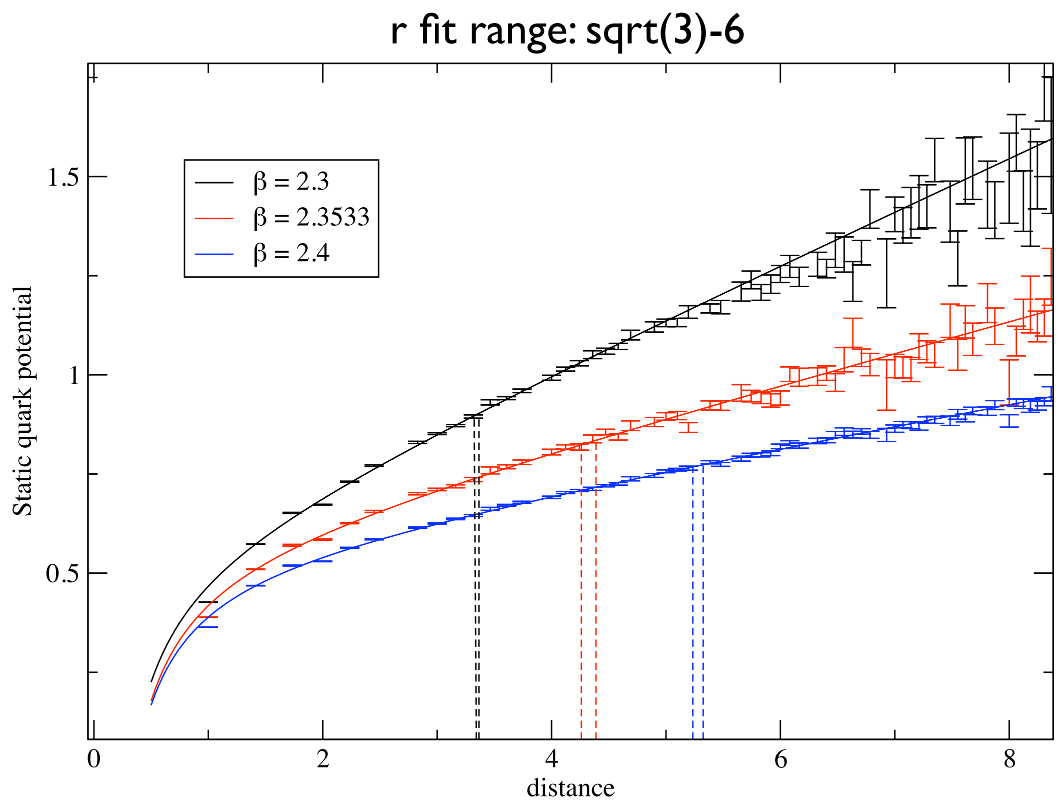
L_s	β	m_f	$\delta\tau$	N_{step}	acceptance	N_{traj}
16	2.3	0.01	0.16	5	0.75	500-1855
16	2.3	0.02	0.16	5	0.75	500-3200
16	2.3	0.04	0.16	5	0.77	500-2795
20	2.3	0.01	0.155	5	0.71	500-2660
20	2.3	0.02	0.155	5	0.71	500-2660
20	2.3	0.04	0.16	5	0.75	500-2765
24	2.3	0.01	0.145	5	0.78	500-1900
24	2.3	0.02	0.145	5	0.78	500-2625
24	2.3	0.04	0.155	5	0.76	500-2615
28	2.3	0.01	0.145	5	0.80	500-2130
28	2.3	0.02	0.145	5	0.81	500-2080
28	2.3	0.04	0.155	5	0.77	500-2320
16	2.3533	0.02	0.163	5	0.80	500-1950
16	2.4	0.02	0.16	5	0.82	500-2710

- Currently generating additional ensembles at smaller coupling

Thermalization



Coupling dependence of static quark potential



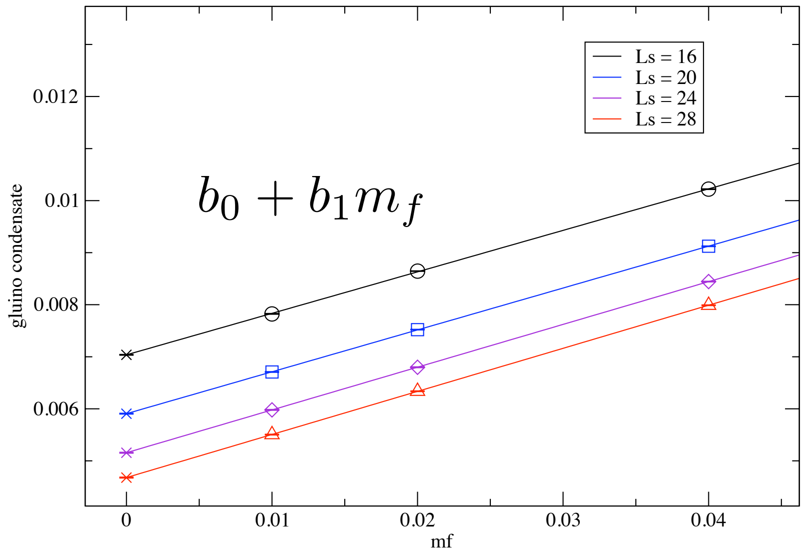
$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$

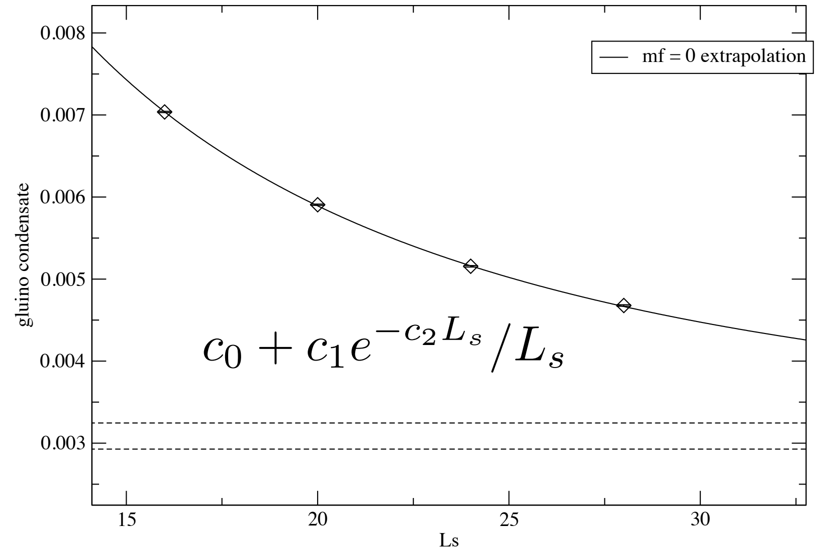
$$r_0^2 V'(r_0) = 1.65$$

Jackknife fits by I. Mihailescu

- Assuming $r_0 = 0.5$ fm, lattice scale is $a^{-1} \approx 1.3$ GeV, 1.7 GeV and 2.1GeV for beta = 2.3, 2.3533 and 2.4, respectively

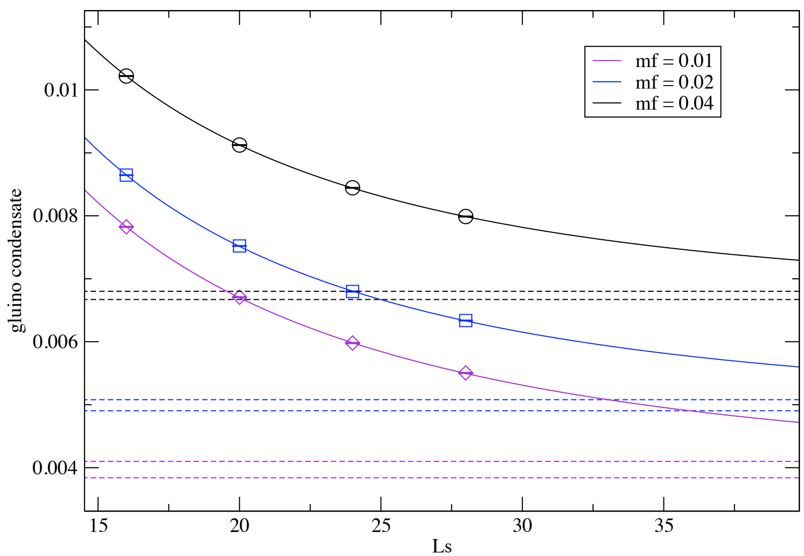
Gluino condensate in the chiral limit

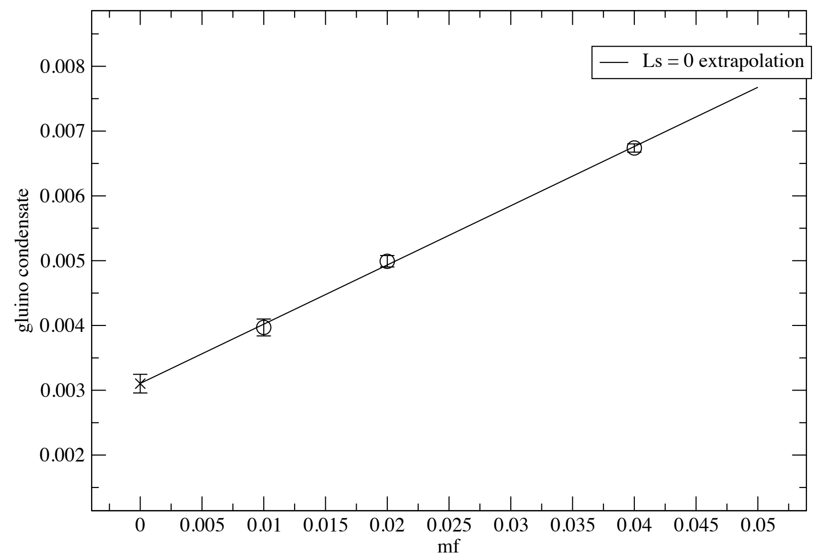




- Two parameter extrapolation: m_f and L_s
- L_s-dependence due to residual mass
- Extrapolation performed by first taking $m_f=0$ then $L_s=\infty$

Gluino condensate in the chiral limit





	gluino condensate	
extrapolation order	m_f then L_s	L_s then m_f
16^3x32	0.003087(159)	0.003102(144)
8^3x8 (Vranas, et. al.)	0.00376(59)	

Spectrum

N=1 SYM	QCD
$Tr\,\bar{\psi}\psi$	0
$Tr\,\bar{\psi}\gamma_5\psi$	η'
$Tr\,F_{\mu\nu}\sigma_{\mu\nu}\psi$	no analogue

Spectrum

N=1 SYM

$$Tr\,\bar{\psi}\psi$$

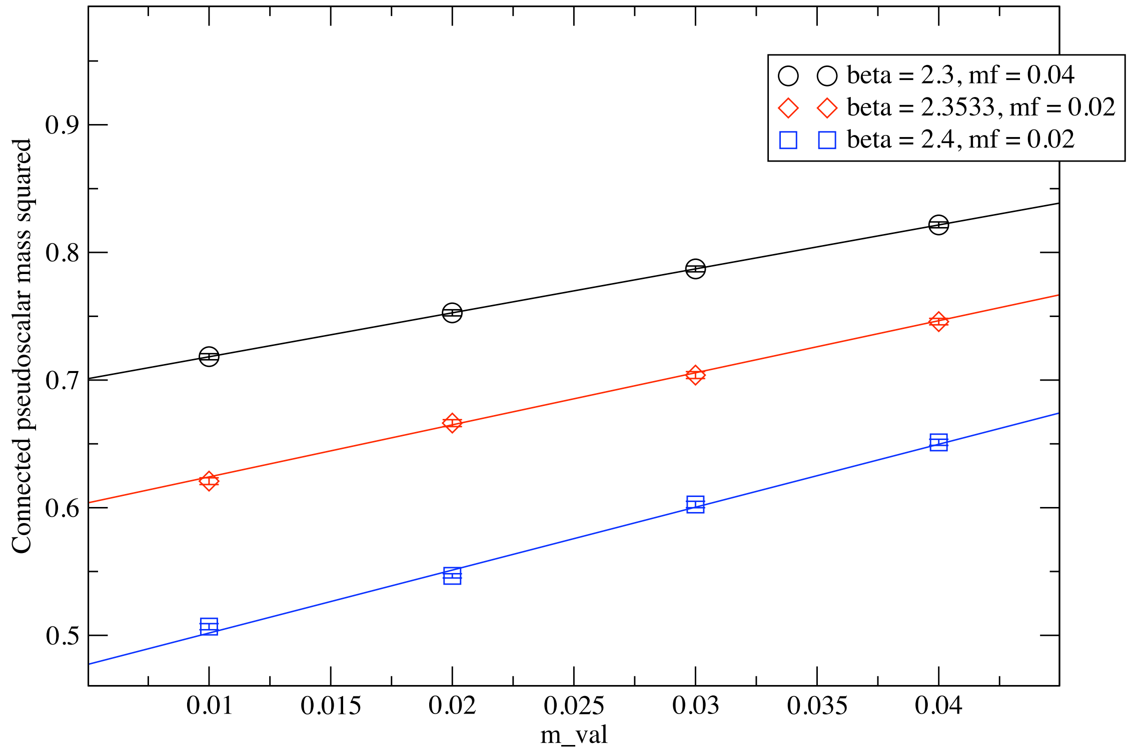
$$Tr\,\bar{\psi}\gamma_5\psi$$

$$Tr\,F_{\mu\nu}\sigma_{\mu\nu}\psi$$

⇒ supermultiplets

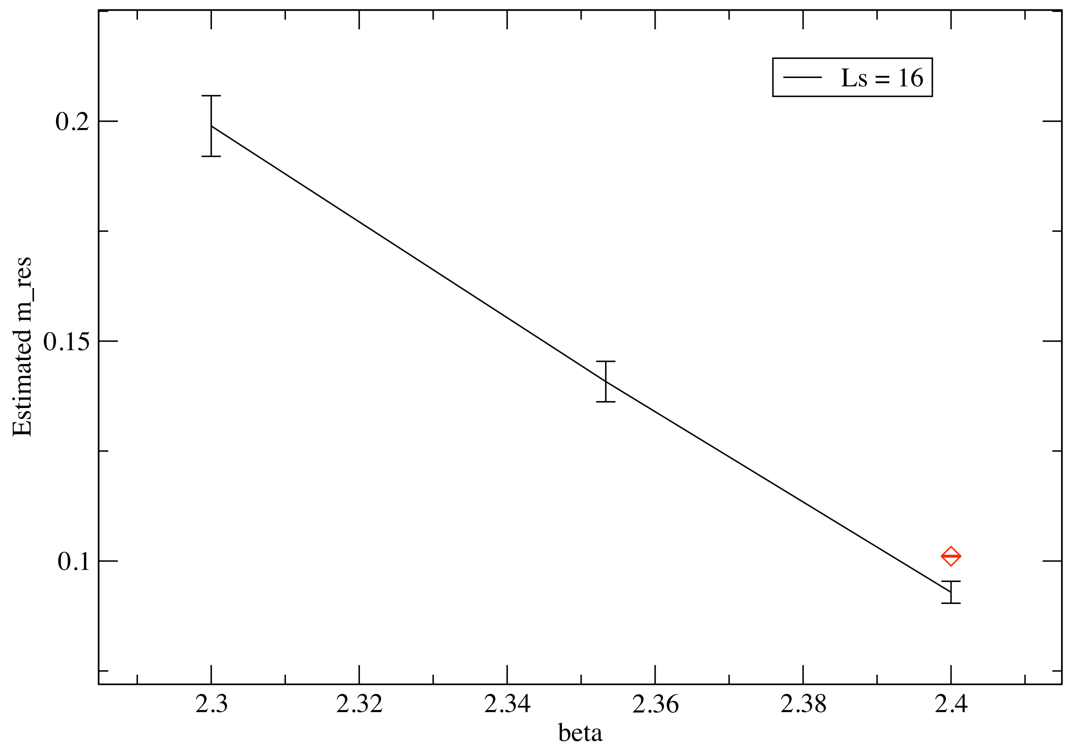
- Low energy described by supermultiplets
  - mass splittings away from SUSY point
  - how big is the residual mass?

Residual mass



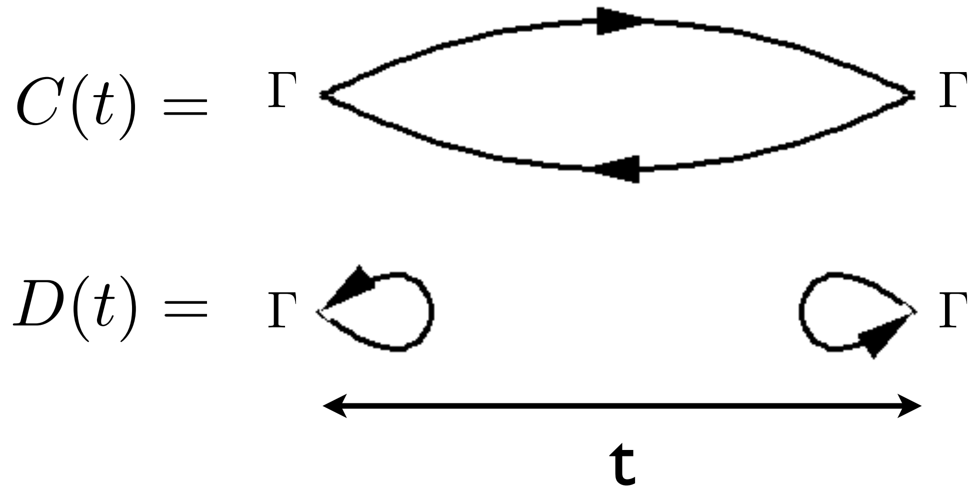
- Estimated m_{res} using extrapolated value of the valence mass (m_{val}) to m_{val}=0 in the partially quenched theory

Coupling dependence of residual mass



- Residual mass is 5-10 times larger than m_f
- Residual mass depends strongly on the coupling

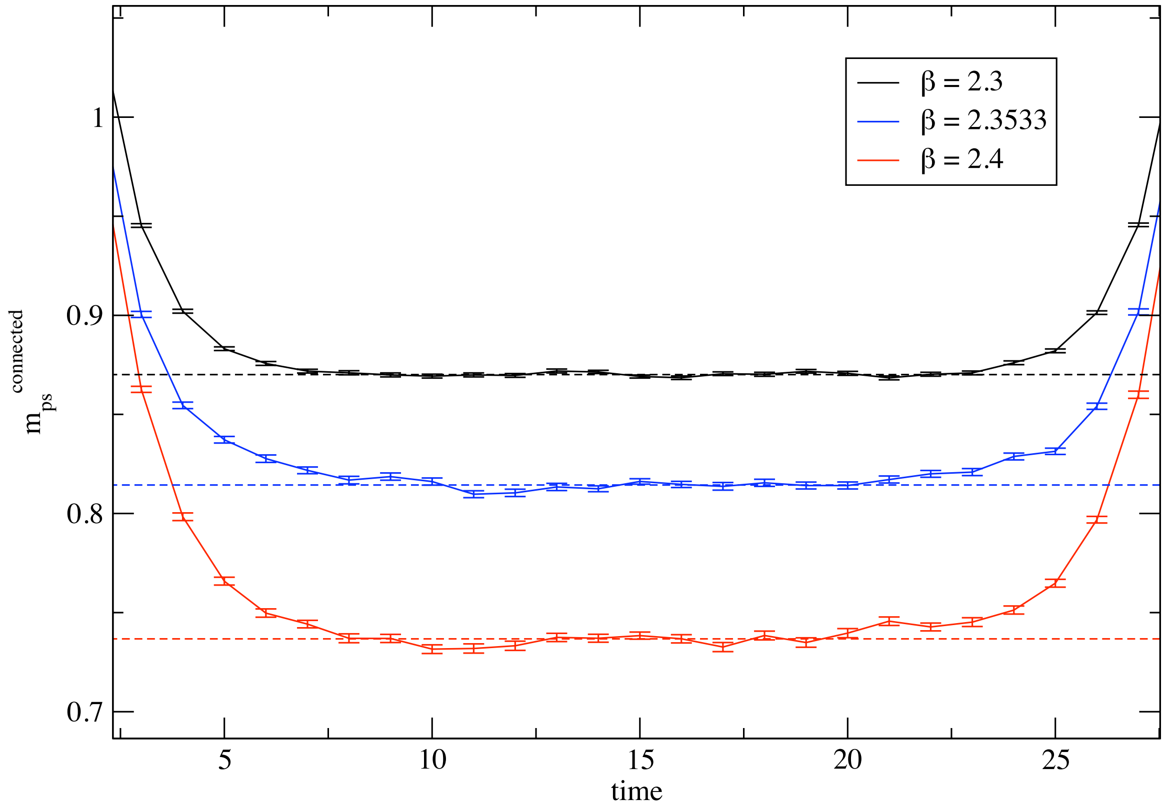
Spectrum: pseudo-scalar and scalar



$$\Gamma = \{1, \gamma_5\}$$

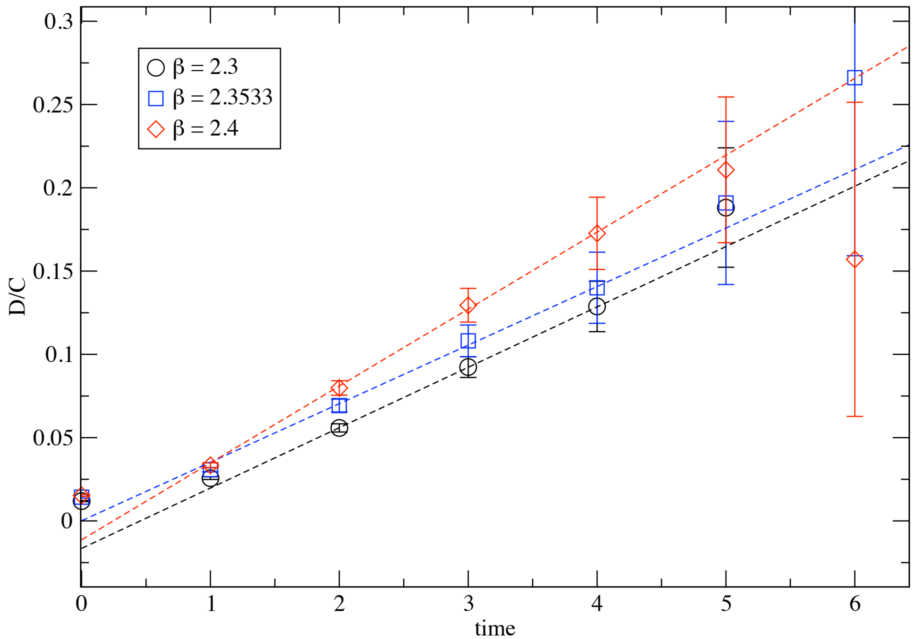
- Connected (C) and disconnected (D) quark contractions
- Stochastic estimator to measure disconnected diagram
  - 1 hit for connected
  - 5 hits for disconnected

Spectrum: connected pseudo-scalar mass C(t)



- Effective mass extracted from the connected part of the pseudo-scaler correlator C(t)
- $\Delta m_{ps}^{connected} \sim 0.7\text{-}0.9$

Ratio D(t)/C(t) for pseudo-scalar



$$\frac{D_{ps}(t)}{C_{ps}(t)} = 2 - d\, e^{-\Delta m_{ps} t}$$

$$\Delta m_{ps} = m_{ps} - m_{ps}^{connected}$$

- Linearity suggests Δm_{ps} t <<1
  - linear fit to data (fit range: 2-4) yields Δm_{ps}~0.02 which is consistent with this assumption

Future tasks and directions

- m_{res} appears to be large
  - need larger L_s, perhaps smaller coupling
- Continue spectrum measurements
  - requires increased statistics and smaller gluino masses
  - measure fermion super-partner
- Continuum extrapolation of gluino condensate
  - requires several couplings

Acknowledgements

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- Simulations performed using a modified version of the Columbia Physics System (CPS v4.9.16).
- Numerical simulations were performed on QCDOC at Columbia University and New York Blue (BlueGene/L) at Brookhaven National Laboratory.