Radiative Physics from Lattice QCD using Distillation

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For The Hadron Spectrum Collaboration

In a nutshell

what do we want?

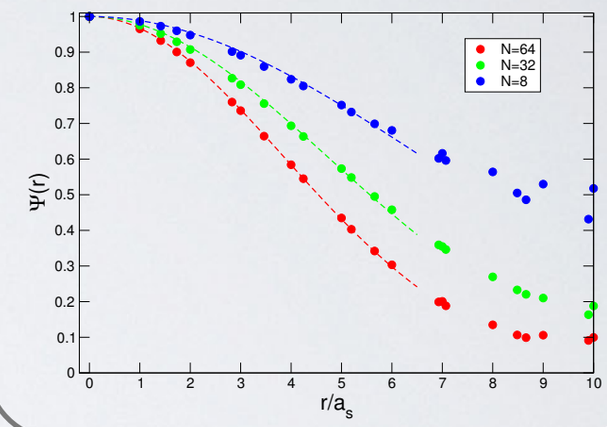
what do we have?

Vector Current Matrix Elements

$$\langle n; p'\lambda' | j^\mu | m; p\lambda \rangle$$

$$j^\mu = \sum_q e_q \bar{\psi}_q \gamma^\mu \psi_q$$

Distillation



HSC, Phys.Rev. D80 (2009) 054506

Optimized Operators

$$\Omega_n^\dagger \sim v_i \mathcal{O}_i^\dagger$$

Variational Method

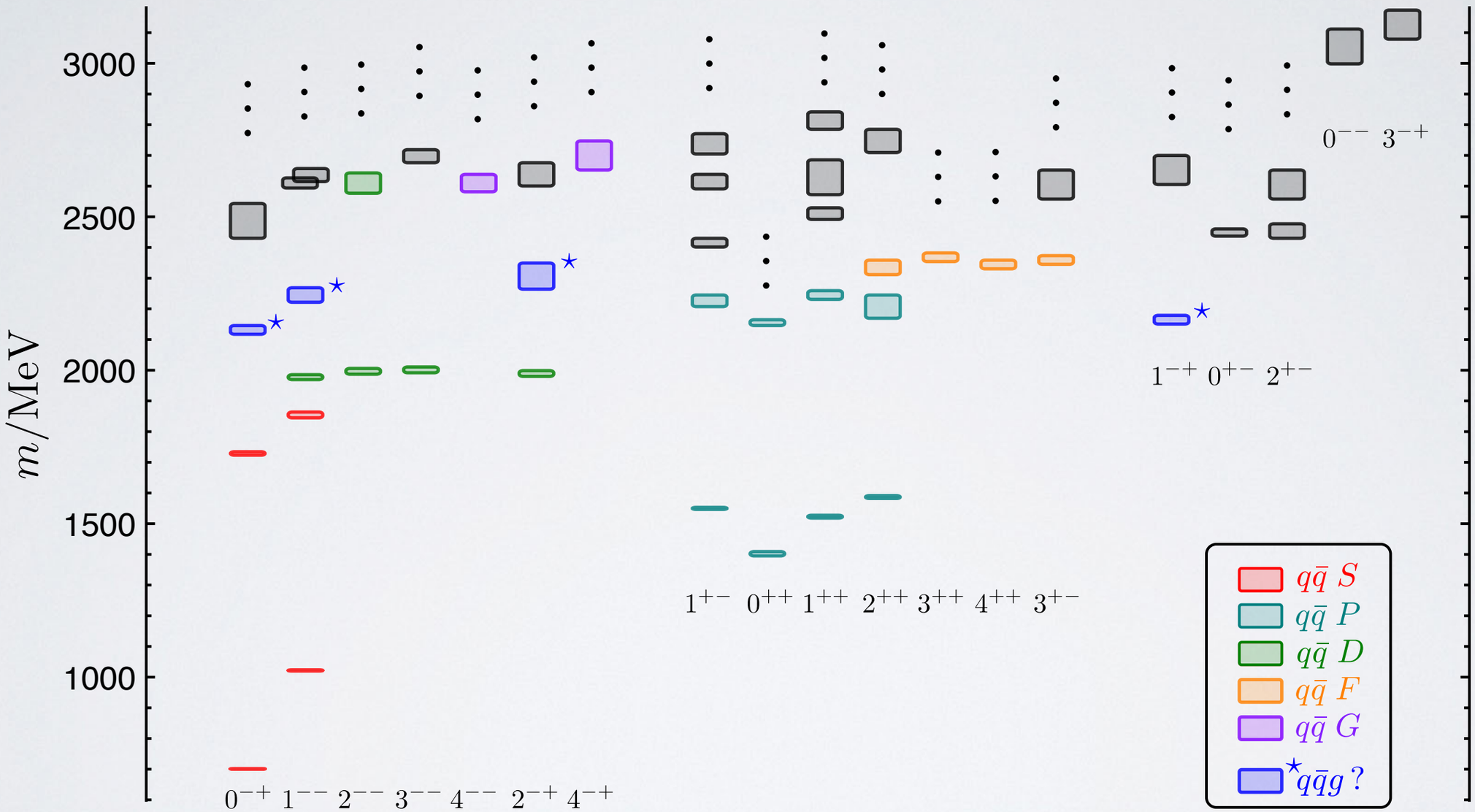
$$C(t)V = C(t_0)V\Lambda$$

Large Basis of Interpolators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a, b, c \to J\lambda)$$

spectroscopy

Spectroscopy

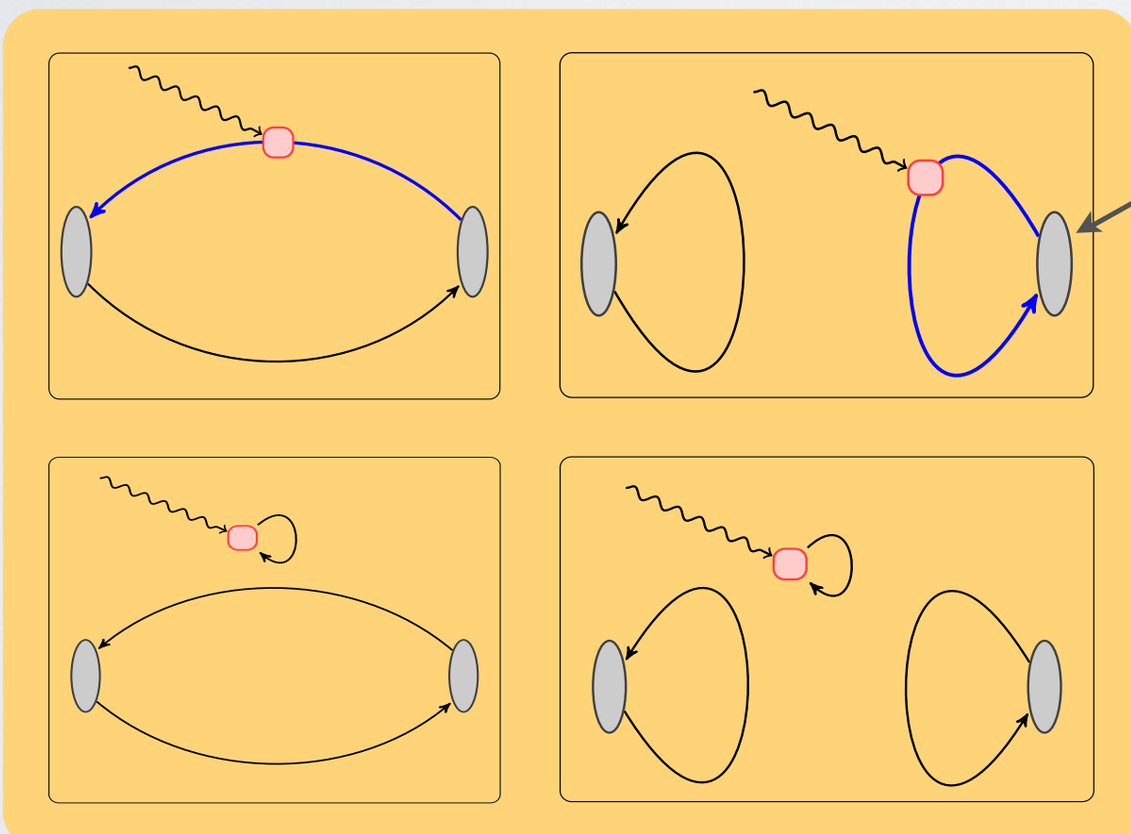


JJDudek, Phys.Rev. D84 (2011) 074023

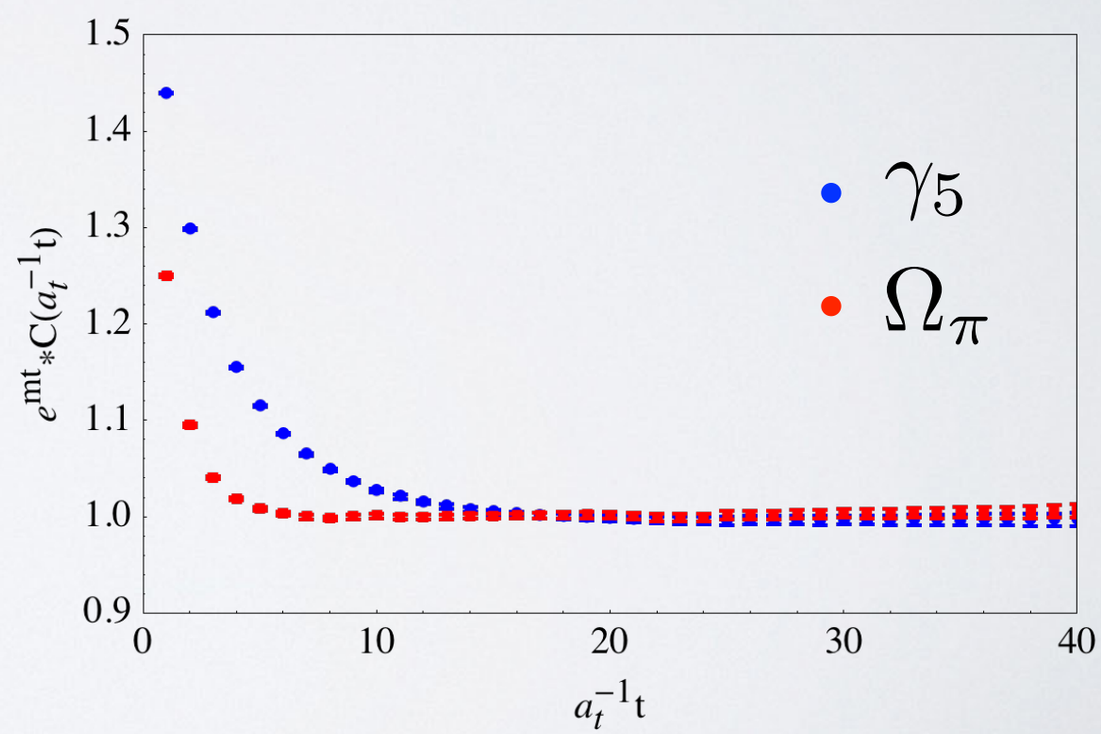
3-pt Functions

Generically

$$C^{\mu}_{n,m}(\delta t, t_\gamma) = \langle 0|\Omega_n(\delta t) j^{\mu}(t_\gamma)\Omega^{\dagger}_m(0)|0\rangle$$



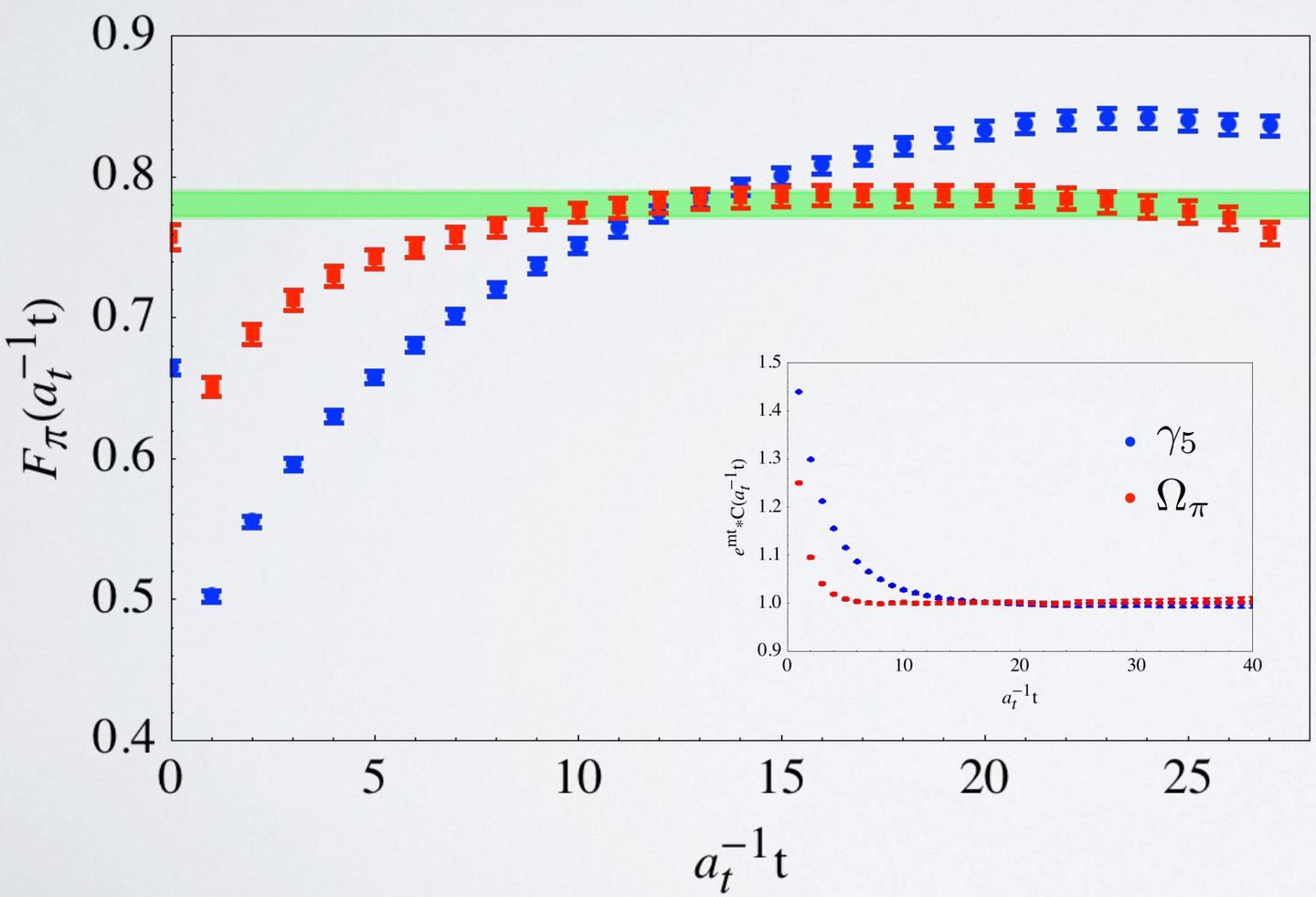
Optimized Operators



3-pt Functions

Decomposed

$$C^{\mu}_{n,m}(\delta t, t_\gamma) = \sum_{\bar{n},\bar{m}} Z^{\bar{n}}_{n} Z^{\bar{m}*}_{m} \frac{e^{-E_{\bar{n}}(\delta t - t_\gamma)}}{2E_{\bar{n}}} \frac{e^{-E_{\bar{m}} t_\gamma}}{2E_{\bar{m}}} \langle \bar{n} | j^{\mu} | \bar{m} \rangle$$



$Q^2 \sim 0.36(\mathrm{GeV}^2)$
$\mathcal{O}^\dagger_\pi \sim \mathrm{A1, p000}$
$\mathcal{O}^\dagger_\gamma \sim \mathrm{D4}_{4\to\mathrm{A1}}, \mathrm{p100}$
$\mathcal{O}_\pi \sim \mathrm{D4}_{4\to\mathrm{A2}}, \mathrm{p100}$

Optimized Operators 'relax' quickly

$$Z^{i}_{\Omega_n} \sim \begin{cases} 2E_n & i = n \\ \epsilon & i \neq n \end{cases}$$

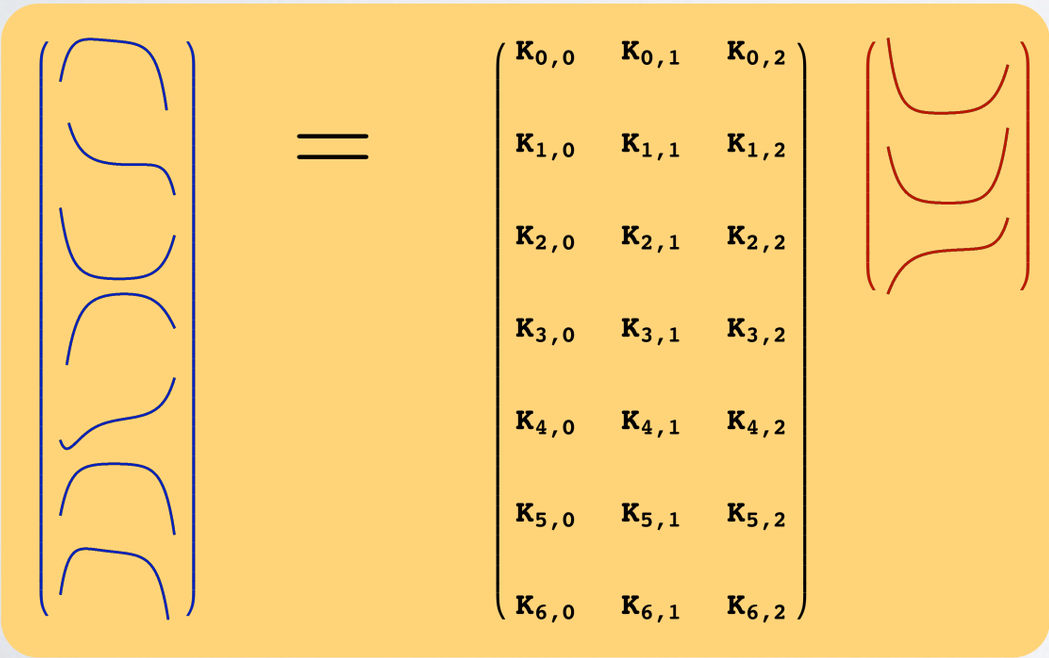
The Linear System

Optimized Operators
'relax' quickly

pollution term

$$C^{\mu}_{n,m}(\delta t, t_\gamma) \sim \frac{e^{-E_n(\delta t - t_\gamma)}}{2E_n}\frac{e^{-E_m t_\gamma}}{2E_m}\langle n|j^\mu|m\rangle + f(\delta t, t_\gamma)$$

Matrix Element Decompositions



Kinematic Factor

Form Factor

$$\langle n|j^\mu|m\rangle = \sum_k K^\mu_k(n,m) F_k(Q^2)$$

A simple example

$$\langle \pi, p'|j^\mu|\pi, p\rangle = (p' + p)^\mu F_\pi(Q^2)$$

Calculation Details

The Lattice

$16^3 \times 128$ Aniso Wilson-Clover SU(3)$_f$

$a_s \sim 0.12\text{fm} \qquad m_\pi \sim 700\text{MeV}$

$\xi \sim 3.4$

Spectroscopy

8 time sources

N=64 distillation smearing

3-pt Functions

1 time source

N=64 distillation smearing

local vector current

The Operators

Rest, up to 3 derivatives

A_1^{++}	13	A_1^{+-}	5	A_1^{-+}	12	A_1^{--}	6
T_1^{++}	22	T_1^{+-}	22	T_1^{-+}	18	T_1^{--}	26
T_2^{++}	22	T_2^{+-}	14	T_2^{-+}	18	T_2^{--}	18
E^{++}	17	E^{+-}	9	E^{-+}	14	E^{--}	12
A_2^{++}	5	A_2^{+-}	5	A_2^{-+}	4	A_2^{--}	6

Phys.Rev. D82 (2010) 034508

Flight, up to 2 derivatives

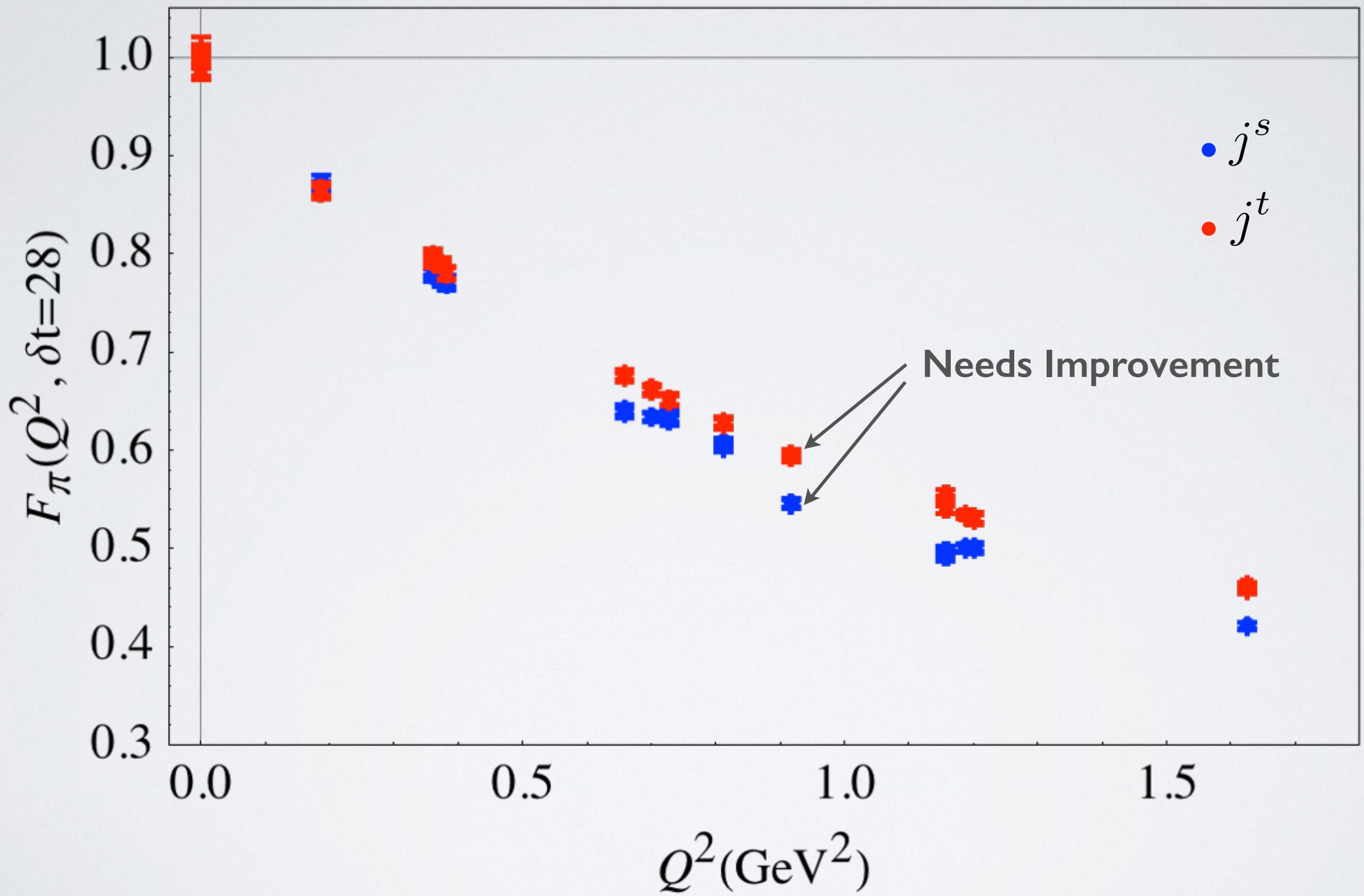
Dic_2 $(n,n,0)$	$A_1^+(1)$	25
	$A_2^+(1)$	31
	$B_1^+(1)$	23
	$B_2^+(1)$	23
	$A_1^-(1)$	27
	$A_2^-(1)$	21
	$B_1^-(1)$	29
	$B_2^-(1)$	29
Dic_3 (n,n,n)	$A_1^+(1)$	15
	$A_2^+(1)$	21
	$E_2^+(2)$	33
	$A_1^-(1)$	21
	$A_2^-(1)$	15
	$E_2^-(2)$	35

Dic_4 $(n,0,0)$	$A_1^+(1)$	14
	$A_2^+(1)$	20
	$E_2^+(2)$	23
	$B_1^+(1)$	11
	$B_2^+(1)$	11
	$A_1^-(1)$	18
	$A_2^-(1)$	12
	$E_2^-(2)$	29
	$B_1^-(1)$	9
	$B_2^-(1)$	9

Phys.Rev. D85 (2012) 014507

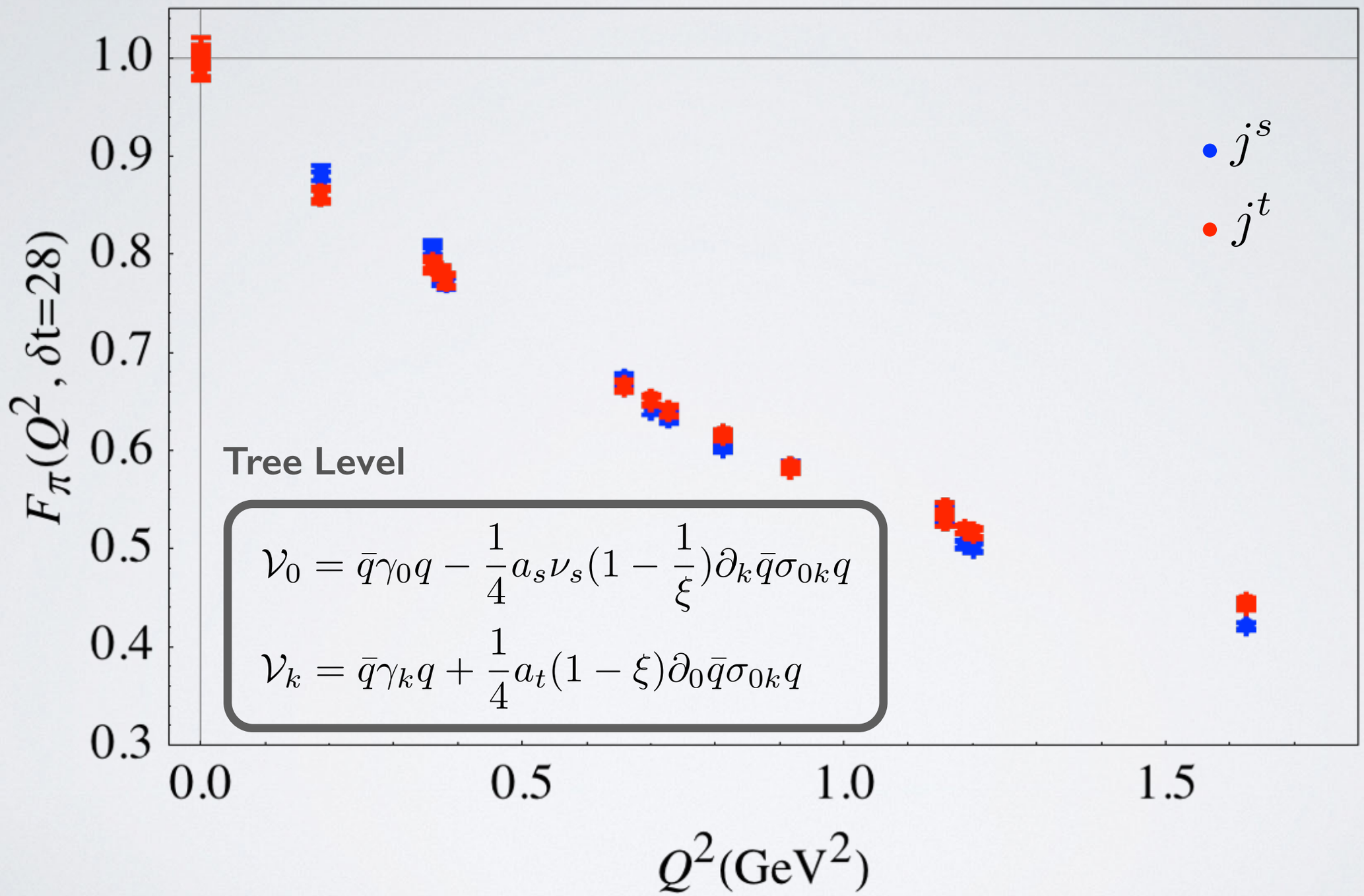
The Pion Form Factor

$\pi \to \pi\gamma$



After Improvement

$\pi \to \pi\gamma$

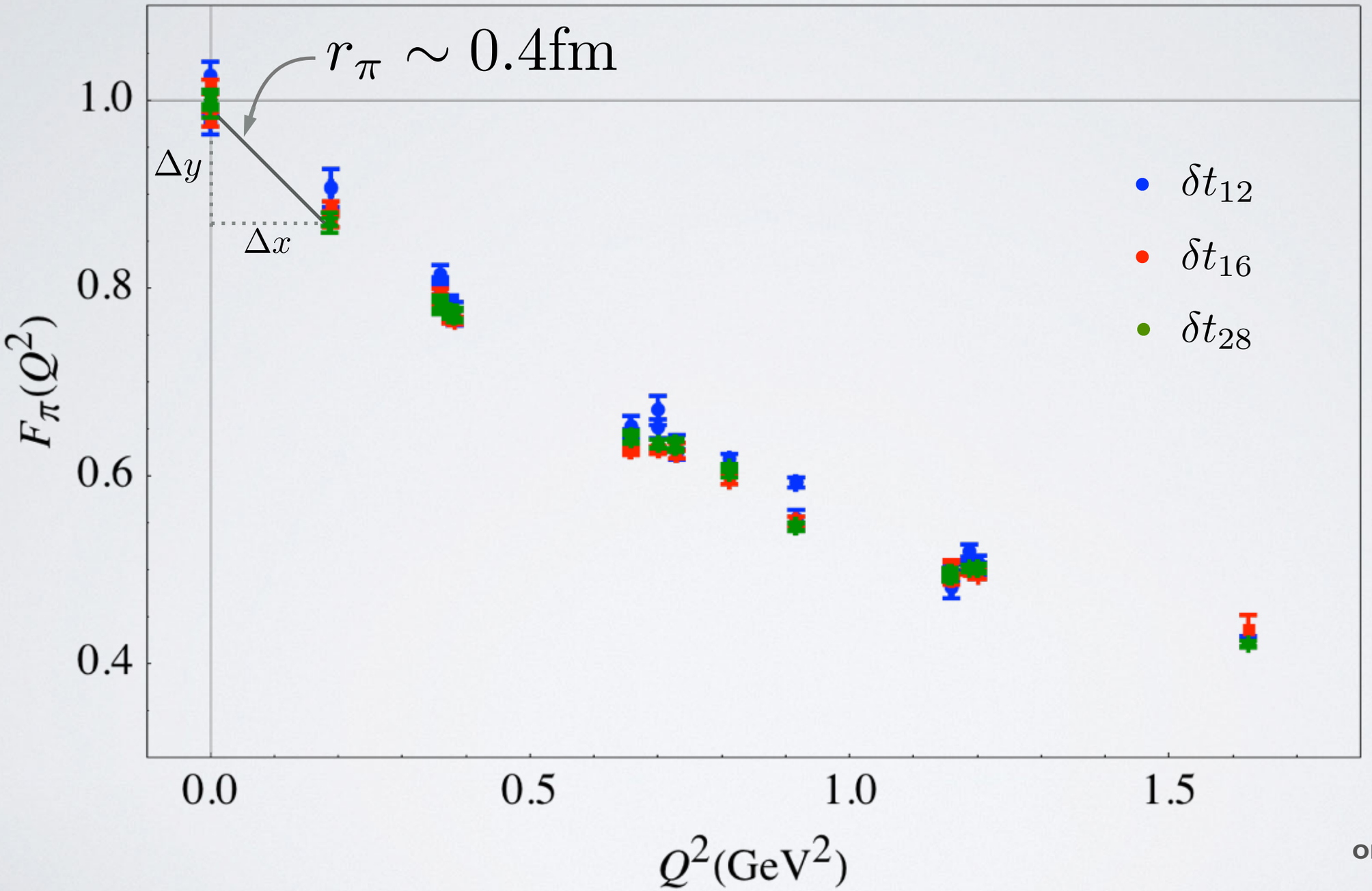


$$\mathcal{V}_0 = \bar{q}\gamma_0 q - \frac{1}{4}a_s\nu_s(1-\frac{1}{\xi})\partial_k\bar{q}\sigma_{0k}q$$

$$\mathcal{V}_k = \bar{q}\gamma_k q + \frac{1}{4}a_t(1-\xi)\partial_0\bar{q}\sigma_{0k}q$$

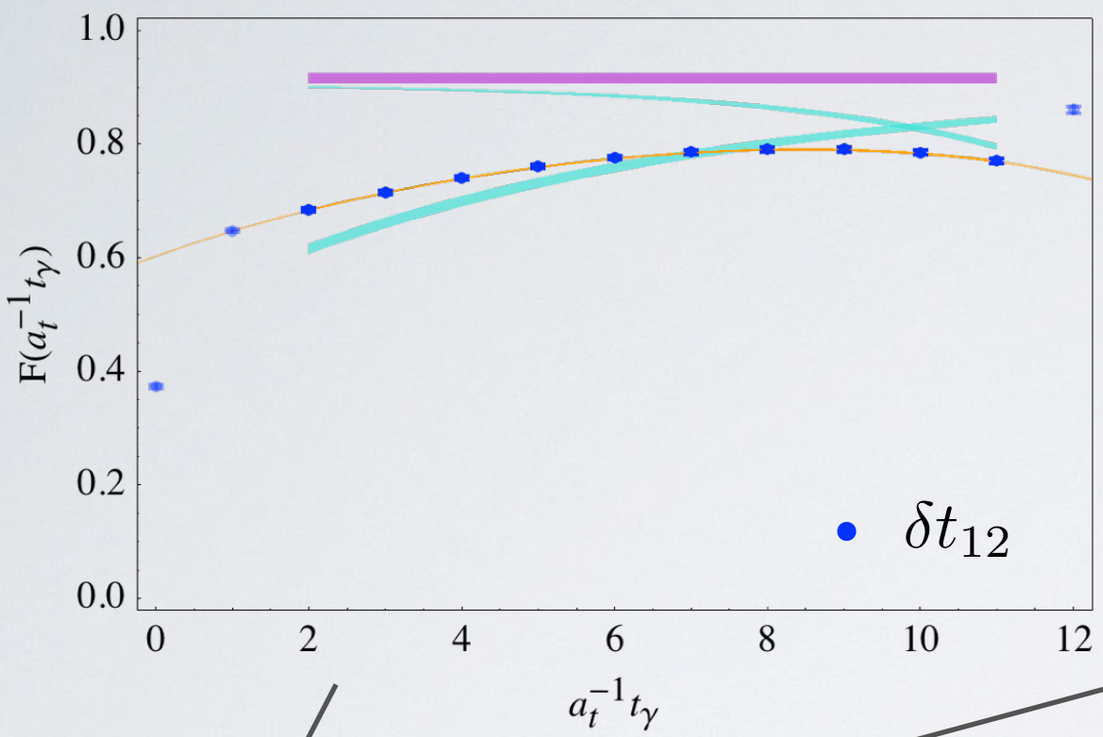
Systematics

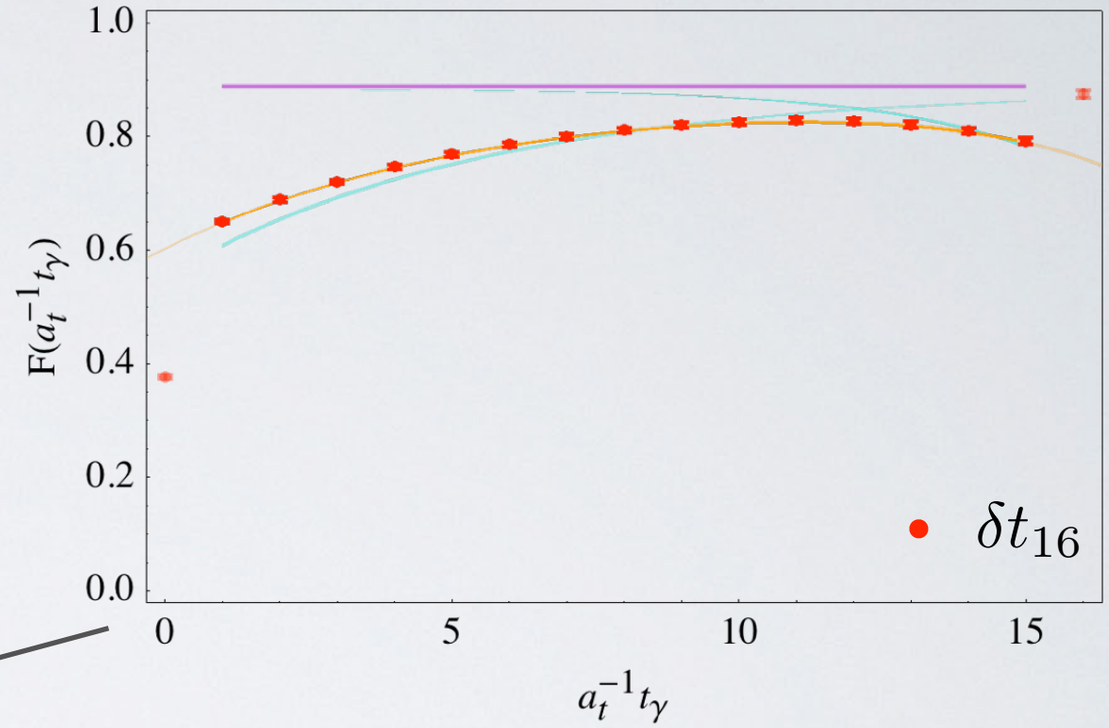
$$C^s_{\pi,\pi}(\delta t, t_\gamma) = \langle \Omega_\pi(\delta t) j^s(t_\gamma) \Omega_\pi(0) \rangle$$

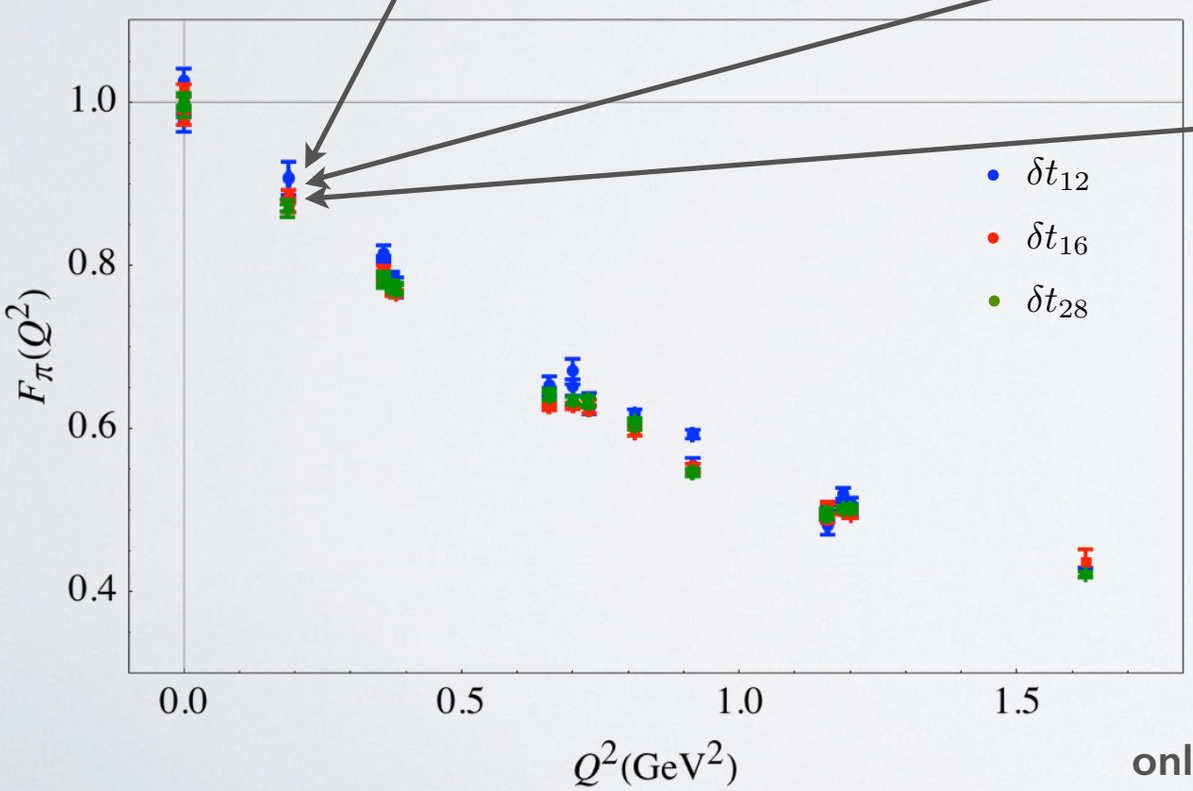


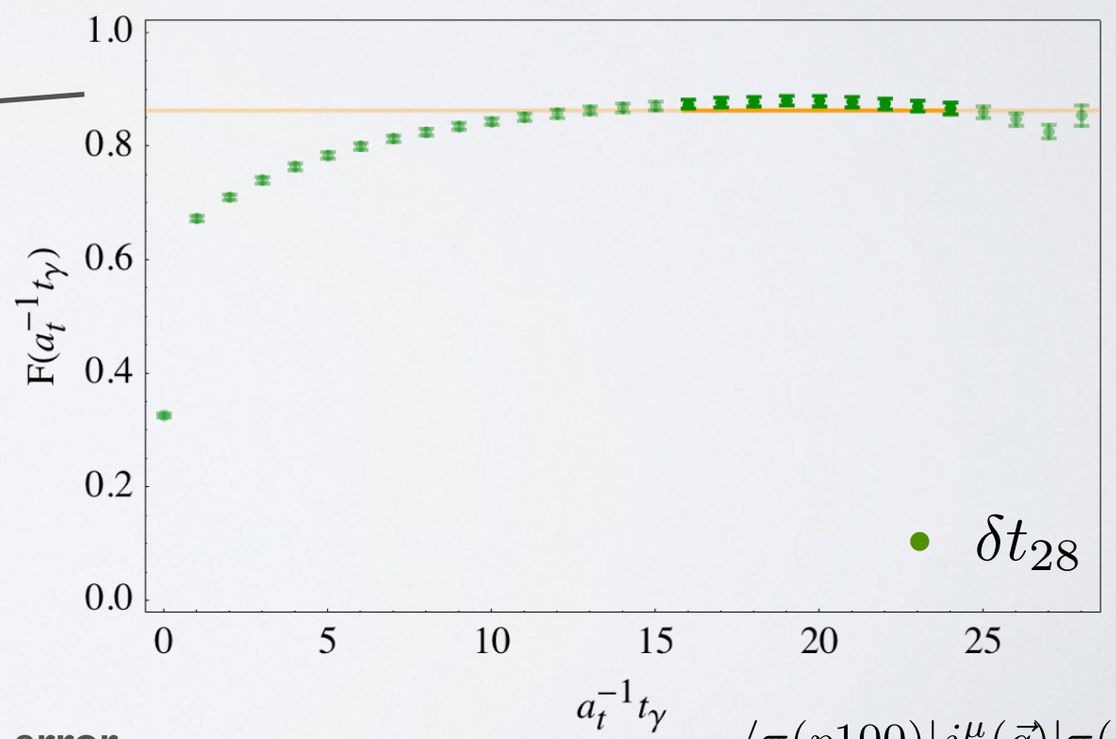
only stat. error

Pollution









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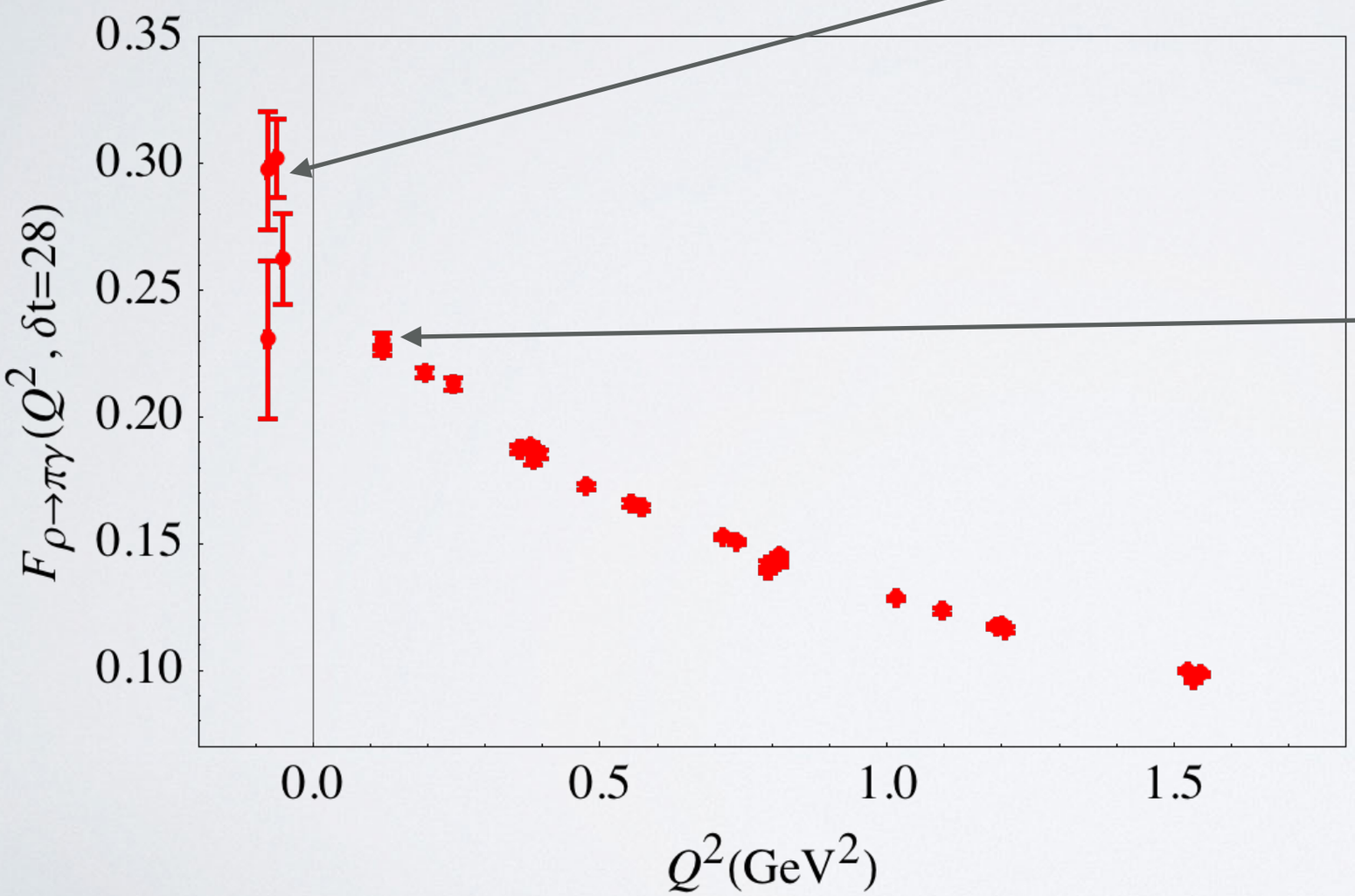
$\langle \pi(p100)|j^\mu(\vec{q})|\pi(p200)\rangle$

A Transition

$\rho \to \pi\gamma$

$$\langle \rho; p'\lambda | j^\mu | \pi; p \rangle = \frac{2F(Q^2)}{m_\rho + m_\pi} \epsilon^{\mu\nu\eta\sigma} p'_\nu p_\eta \varepsilon^*_\sigma(p', \lambda)$$



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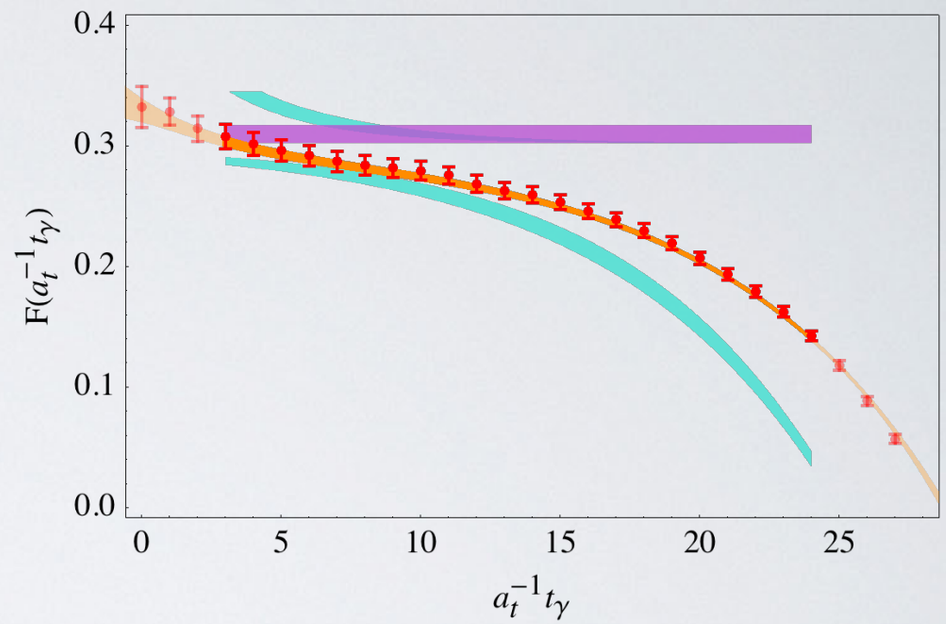
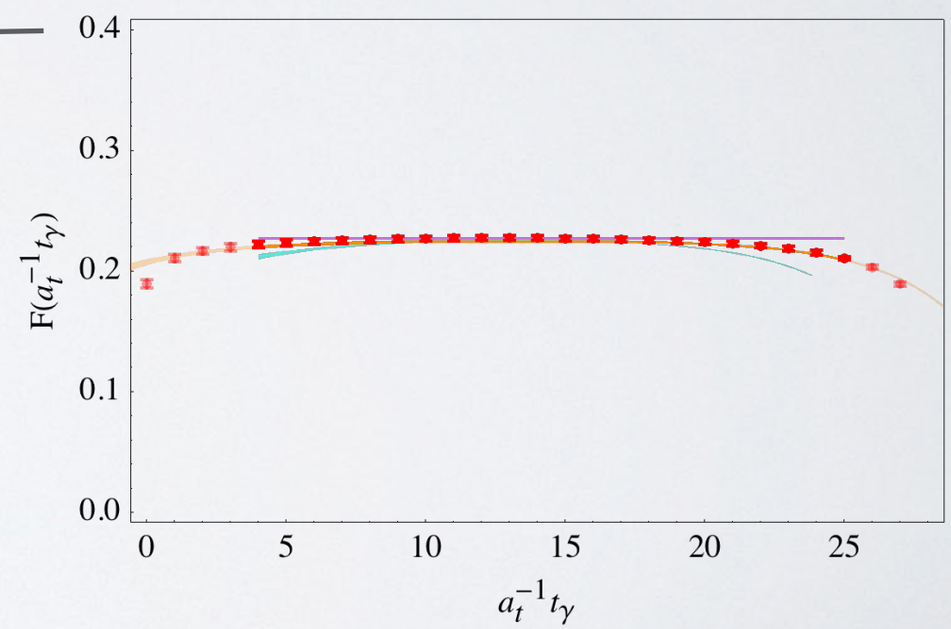
$\langle \rho(p111), E2 | j^\mu(\vec{q}) | \pi(p111) \rangle$





$\langle \rho(p100), E2 | j^\mu(\vec{q}) | \pi(p000) \rangle$

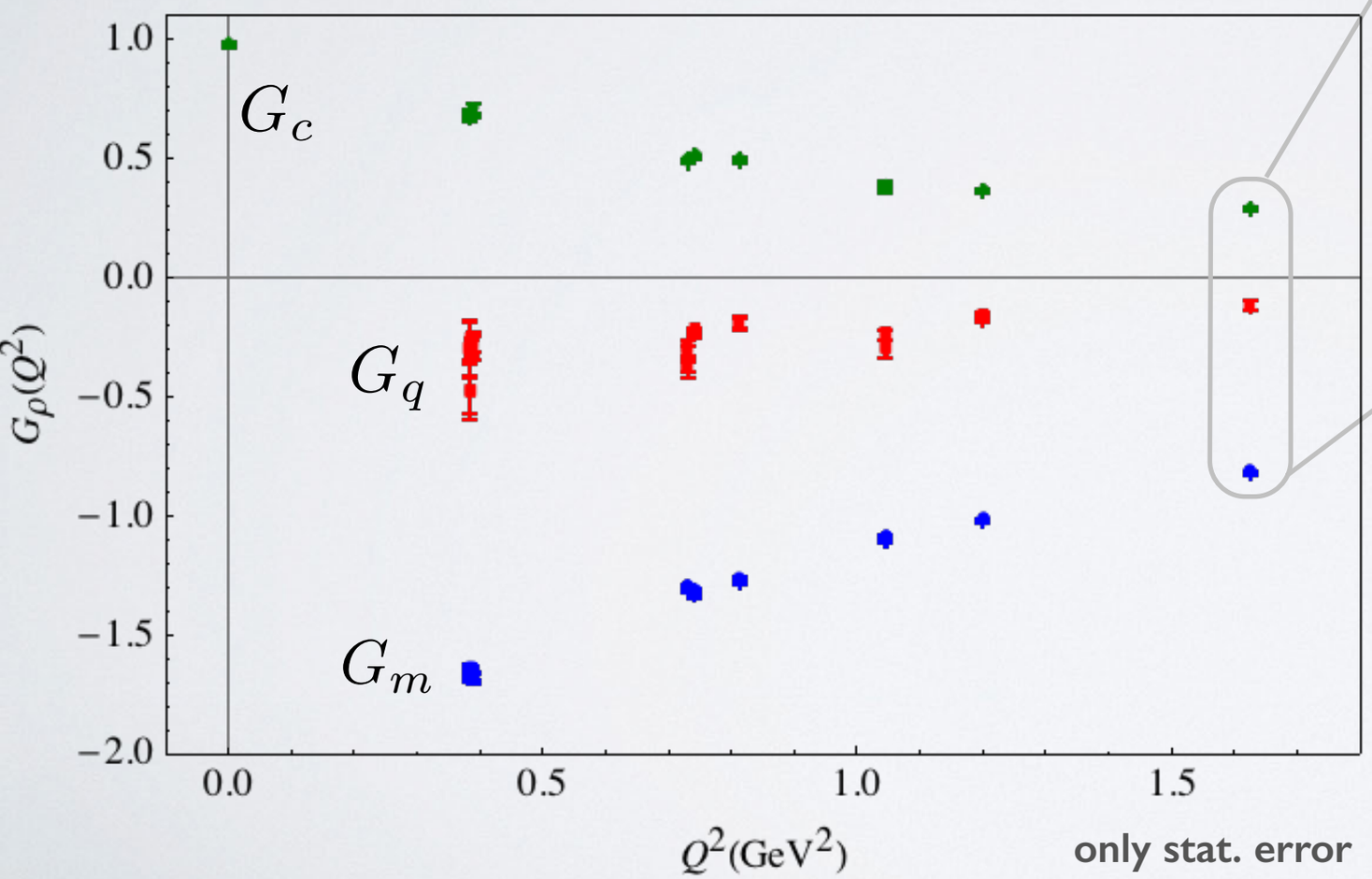
Vector Form Factors

$\rho \to \rho\gamma$

linear combinations of multipoles

$$\langle \rho, p'\lambda' | j^\mu | \rho, p\lambda \rangle = -(p'+p)^\mu \epsilon^*_\alpha(p'\lambda')\epsilon^\alpha(p\lambda) G_1(Q^2)$$
$$+ \left[\epsilon^\mu(p\lambda)\epsilon^*_\alpha(p'\lambda')p^\alpha + \epsilon^{\mu *}(p'\lambda')\epsilon^\alpha(p\lambda)p'_\alpha\right] G_2(Q^2)$$
$$- (p'+p)^\mu \epsilon^*_\alpha(p'\lambda')p^\alpha \epsilon^\beta(p\lambda)p'_\beta \frac{G_3(Q^2)}{2m_\rho^2}$$

solved simultaneously



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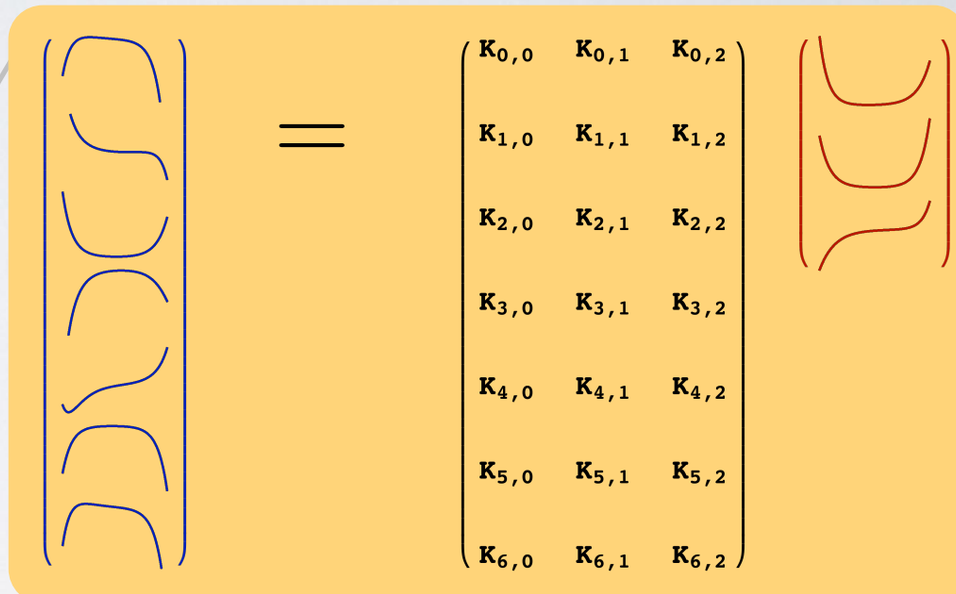


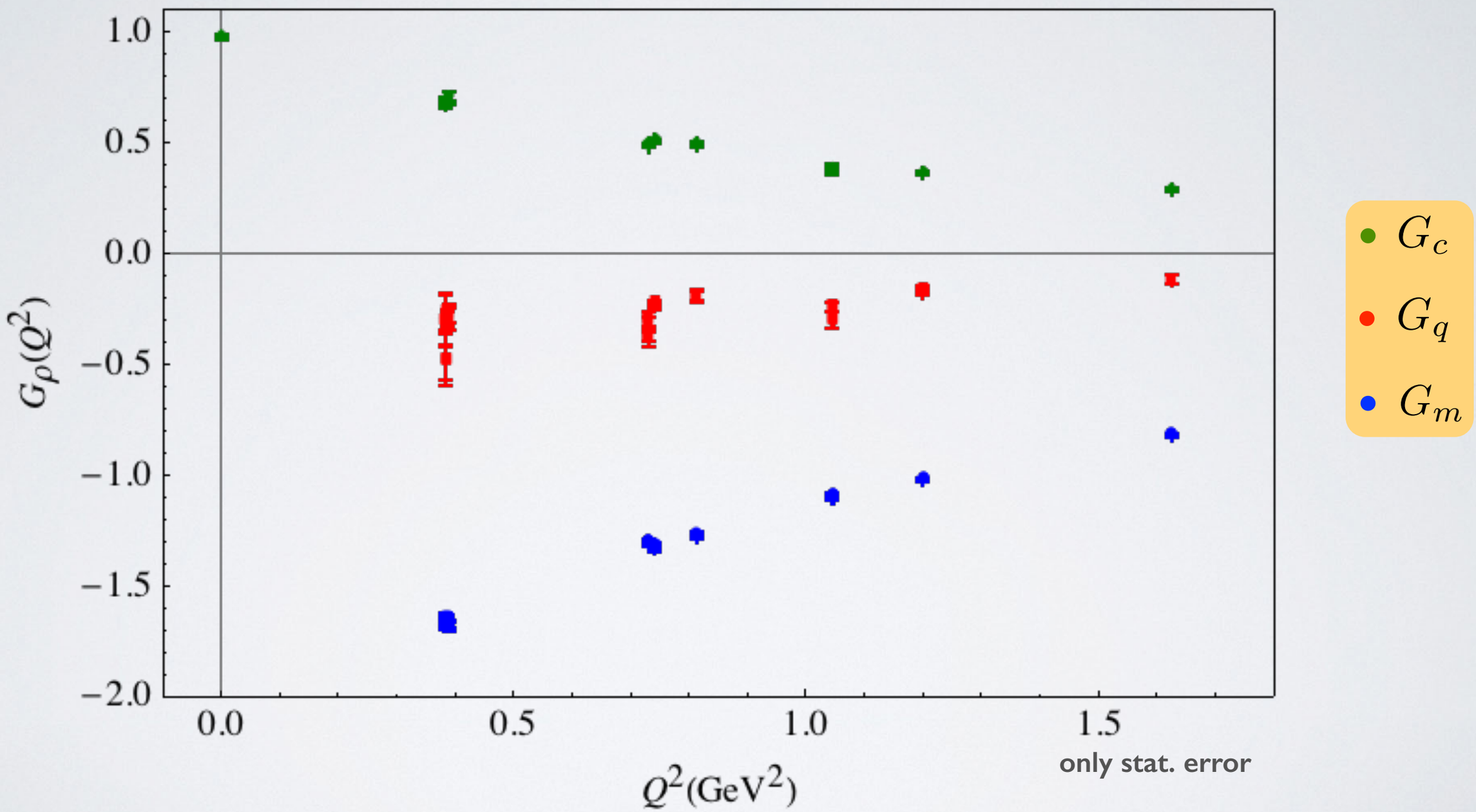
$$\langle n | j^\mu | m \rangle = \sum_k K^\mu_k(n,m) F_k(Q^2)$$

Kinematic Factor

Form Factor

Vector Form Factors

$\rho \to \rho\gamma$



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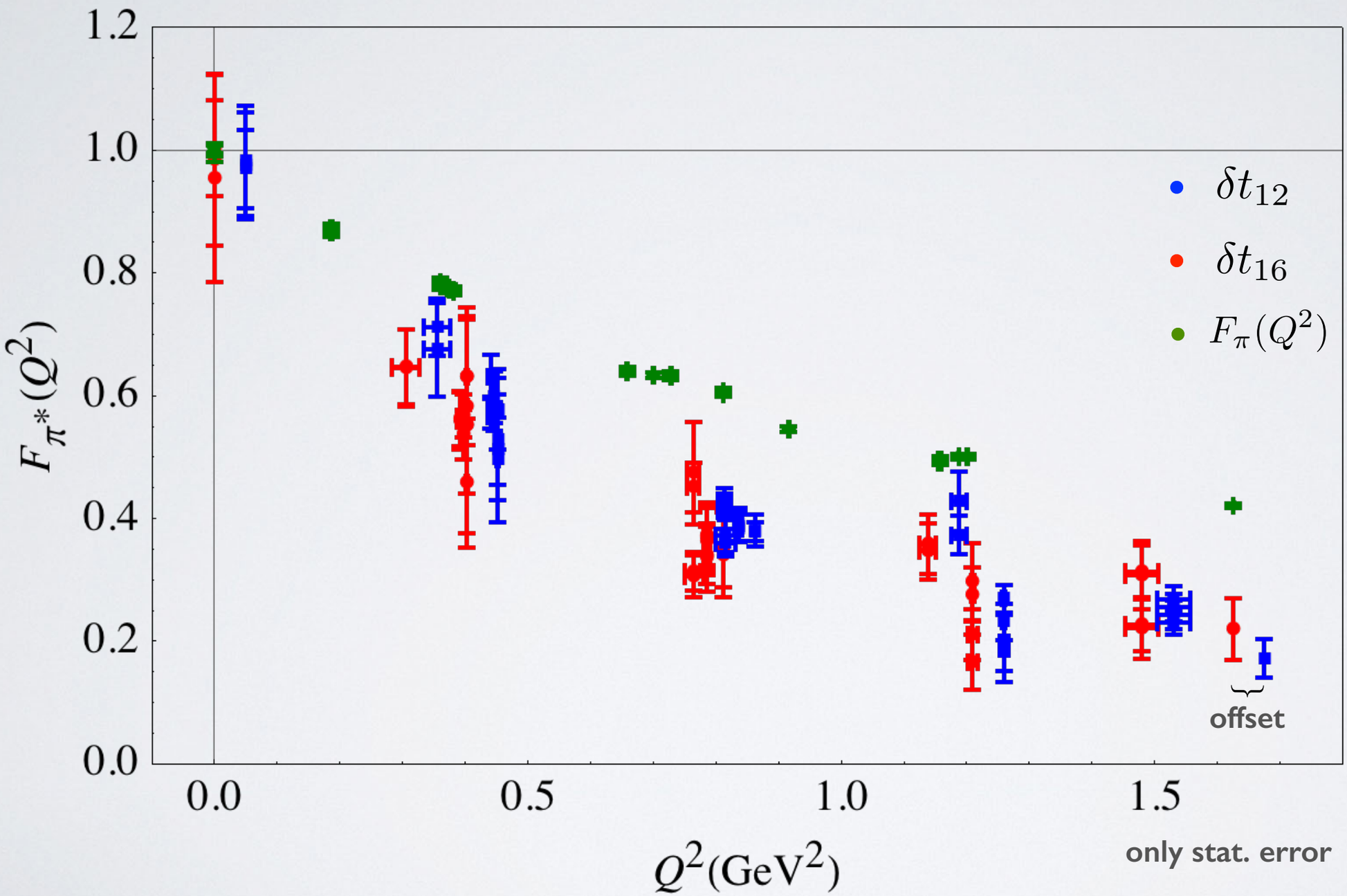
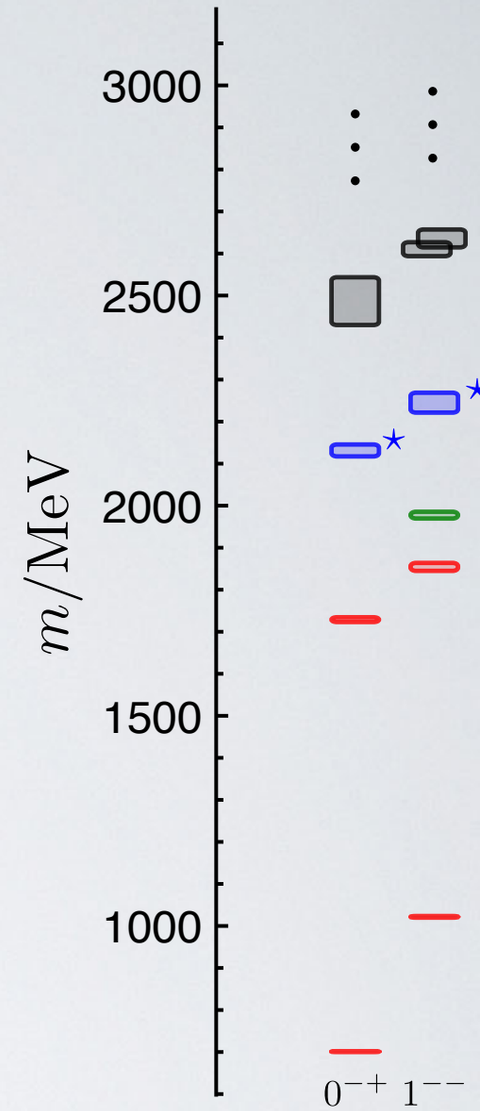
Excited State Form Factor

$\pi^* \to \pi^* \gamma$

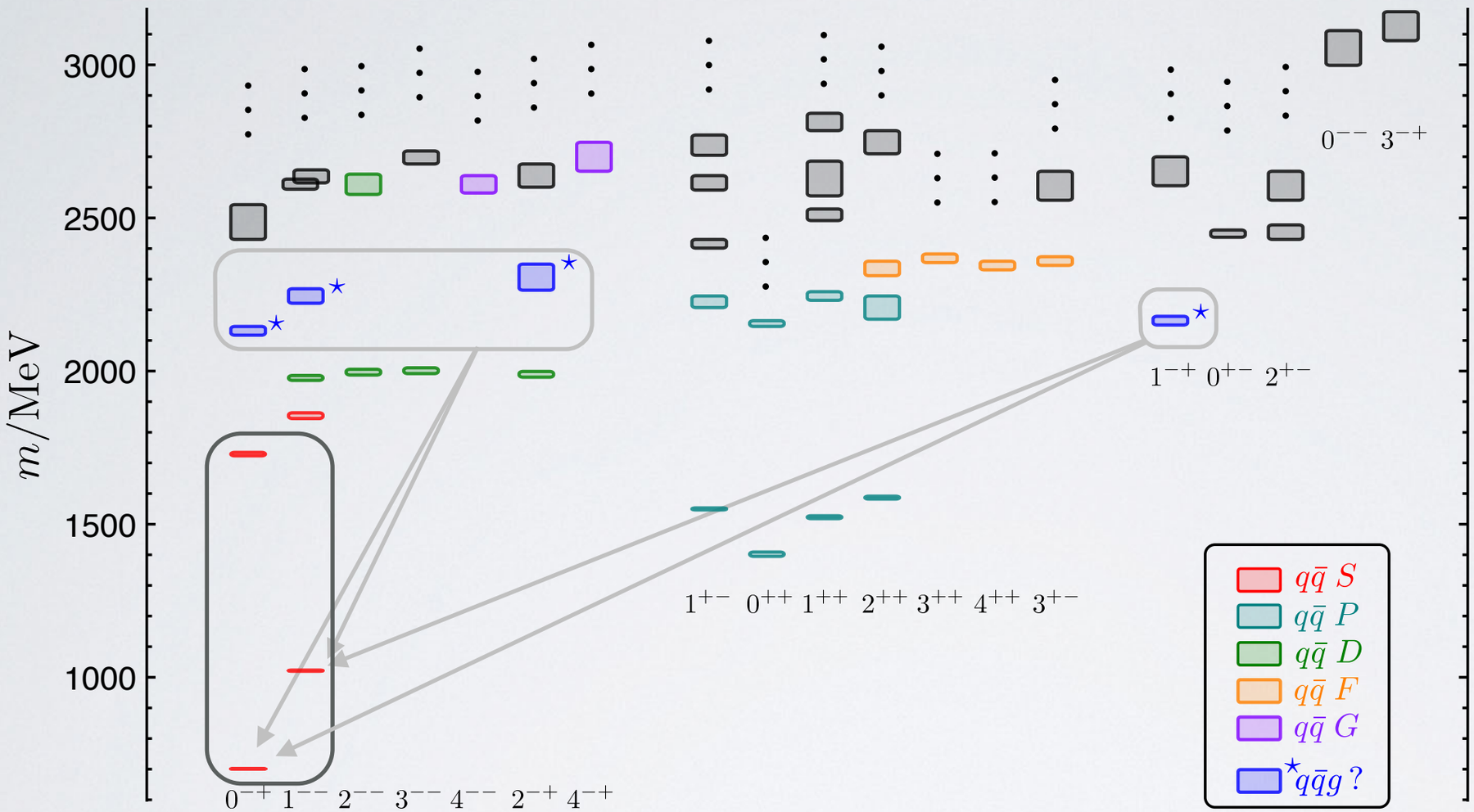




$r_{\pi^*} \sim 0.5\mathrm{fm}$

$r_\pi \sim 0.4\mathrm{fm}$

The Future



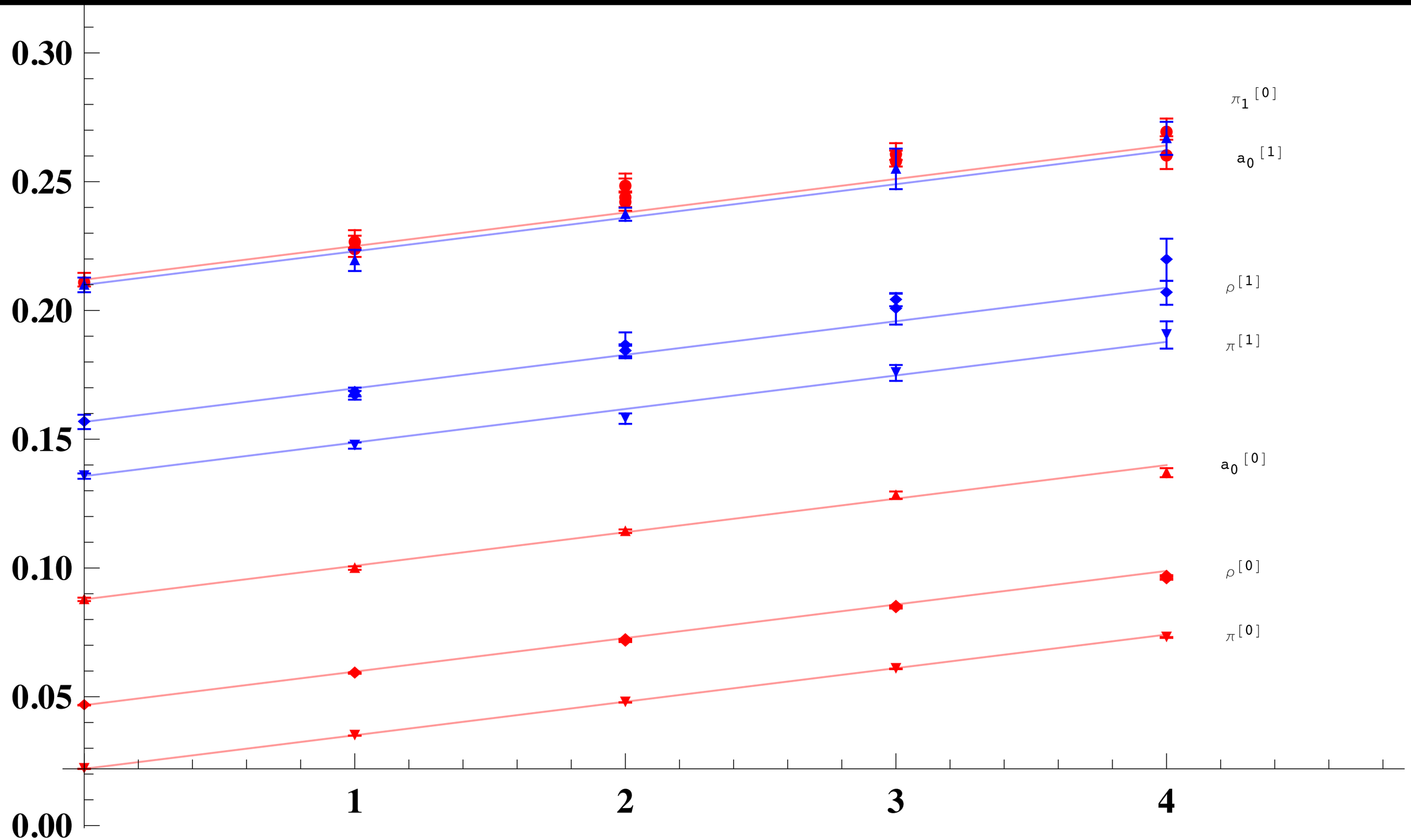
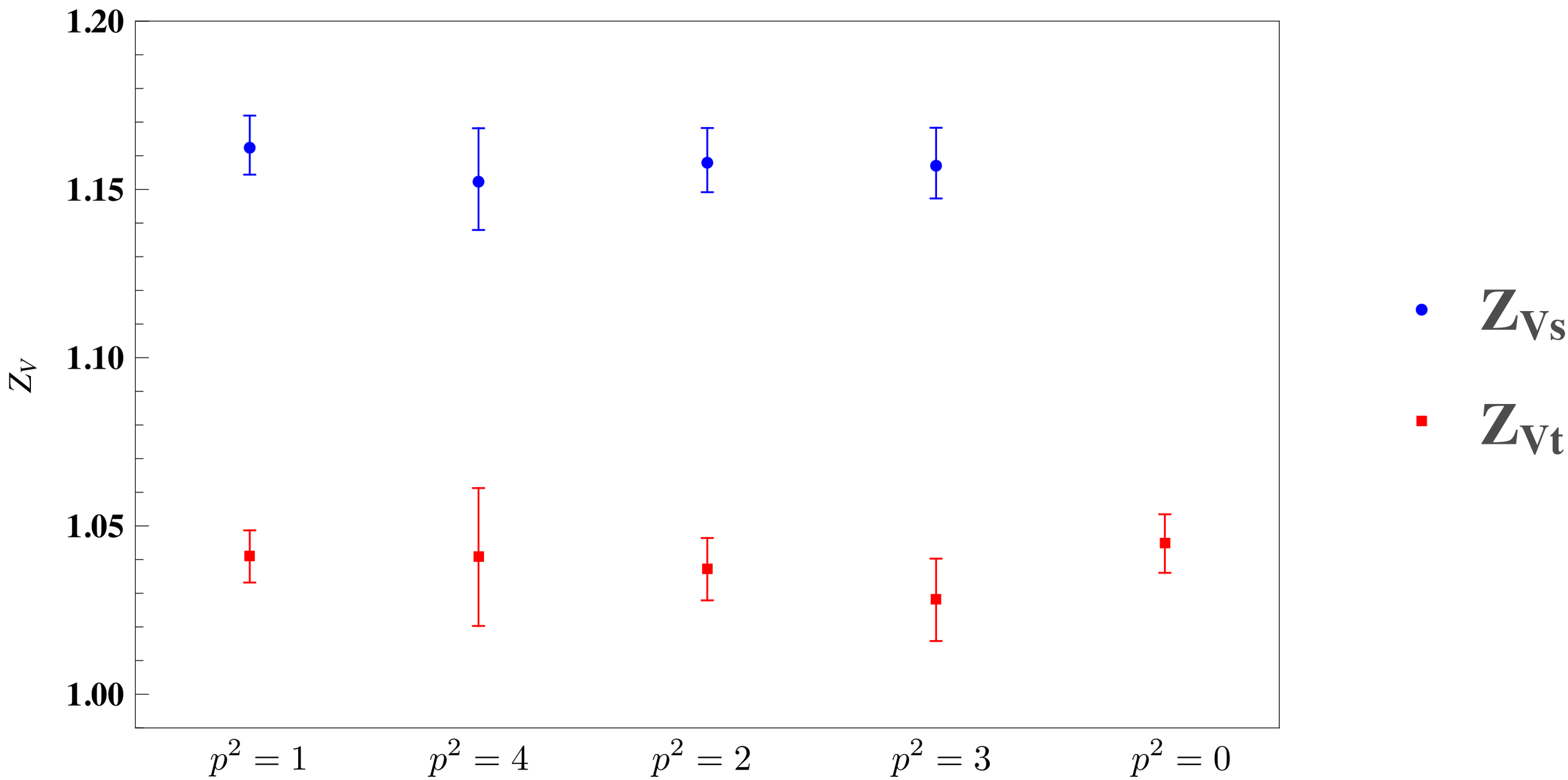
what have we done?

- implemented the formalism
- moved beyond ground states
- explored systematics
- seen real physics

where can we go?

- only stat. error
- explore more of the spectrum
- unstable to stable transition
- charm, tetraquarks?

Dispersion Relation



Renormalization