Multi-jet merging at tree-level and at one-loop level

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What is our picture of perturbative QCD ?

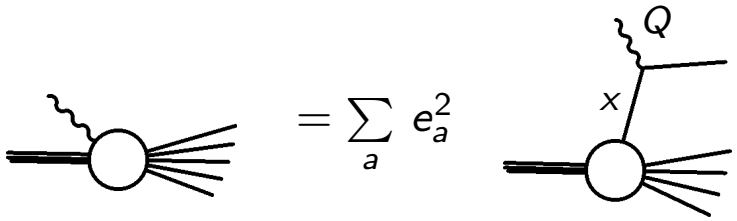
Assume: hadronization is universal and occurs at $Q^2 \approx \mathcal{O}(\Lambda^2_{QCD})$
described by parton distibutions (PDF) and fragmentation functions (FF)

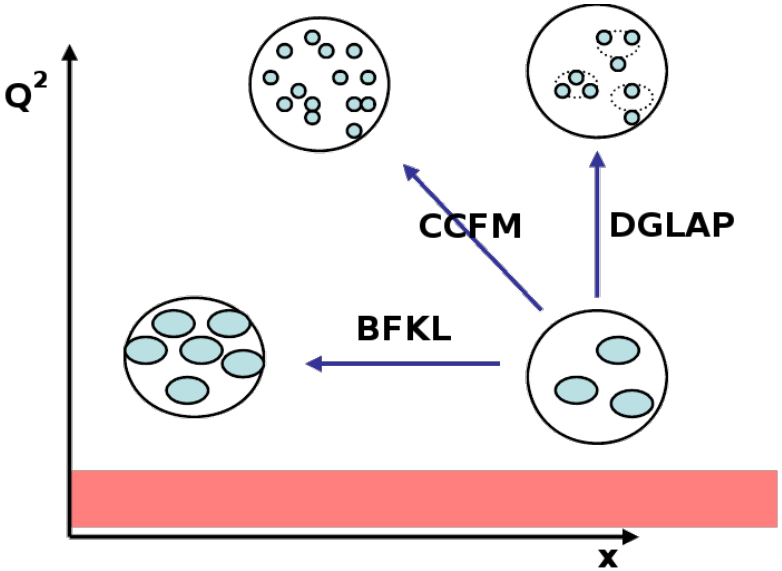
$\Rightarrow$ factorization formula for hadronic cross section

$$\sigma = \sum_a \int \mathrm{d}x\, f_a(x, Q^2)\, \mathrm{d}\hat{\sigma}_a(x, Q^2)$$

$f_a(x, Q^2)$ - PDF probability to extract parton a with energy fraction x from initial hadron at scale Q^2

$\hat{\sigma}_a(x, Q^2)$ - partonic cross section





Energy increases $\Rightarrow$ extra partons
Emission phase space separates into

- DGLAP regime Q^2-evolution
- BFKL regime $1/x$-evolution

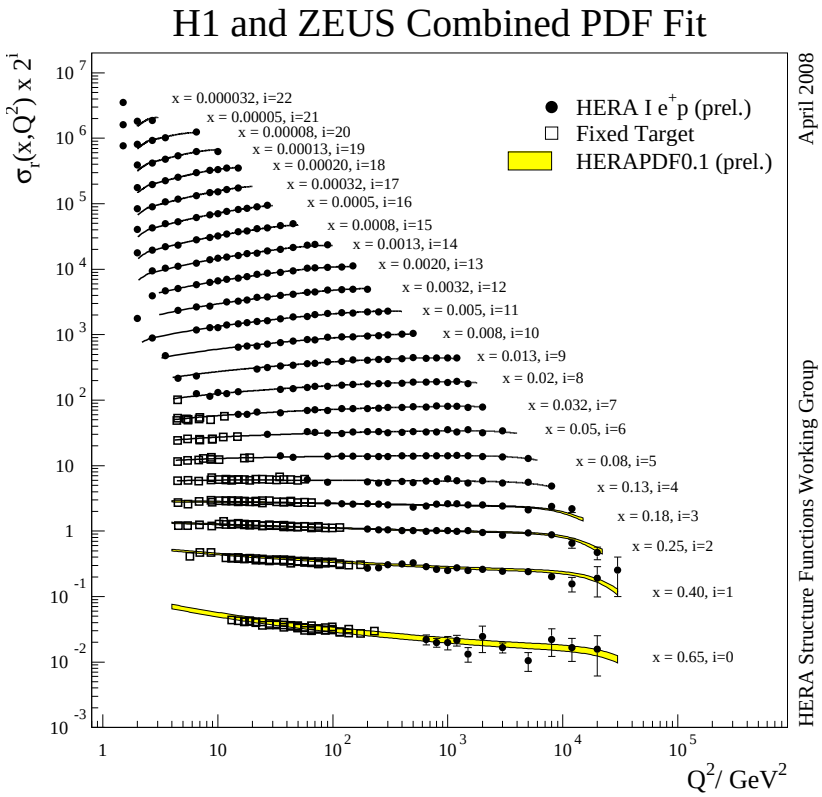
CCFM "interpolates" between DGLAP and BFKL

HERA experiments attempt to test this picture of perturbative QCD

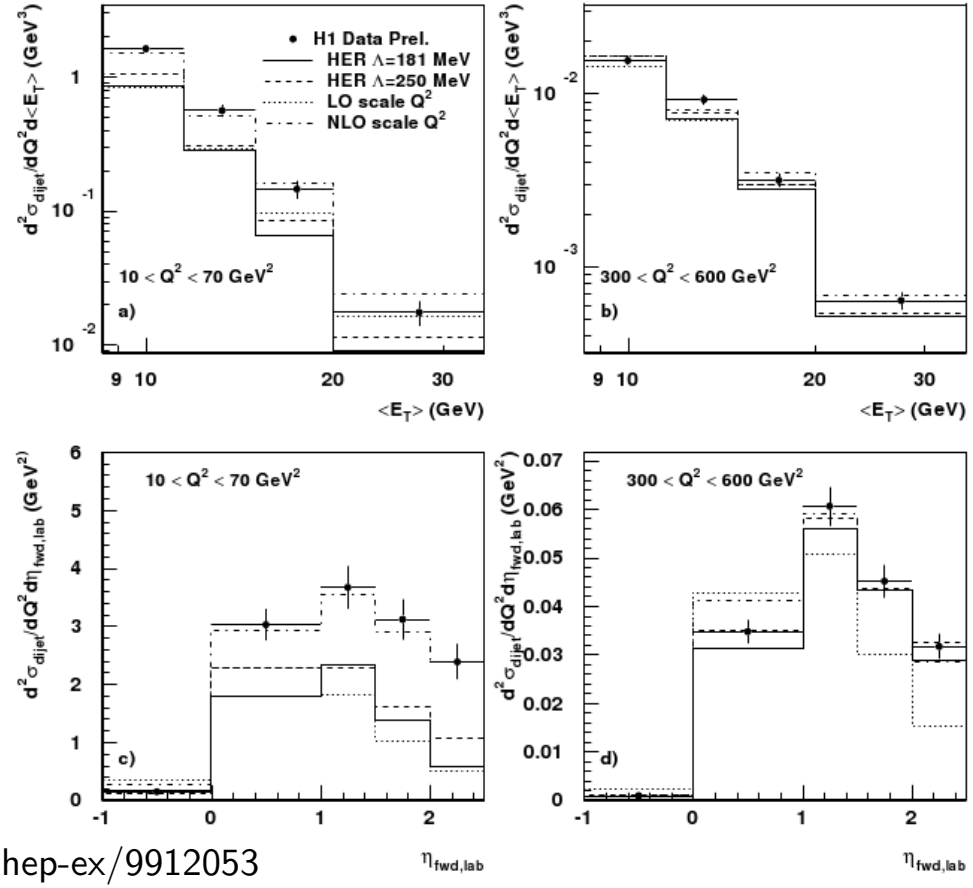
DIS at HERA: Is DGLAP sufficient?

many correct predictions

- Inclusive measurements e.g. F_2
- Jet spectra if computed at NLO



arXiv:0906.1108 [hep-ex]



hep-ex/9912053

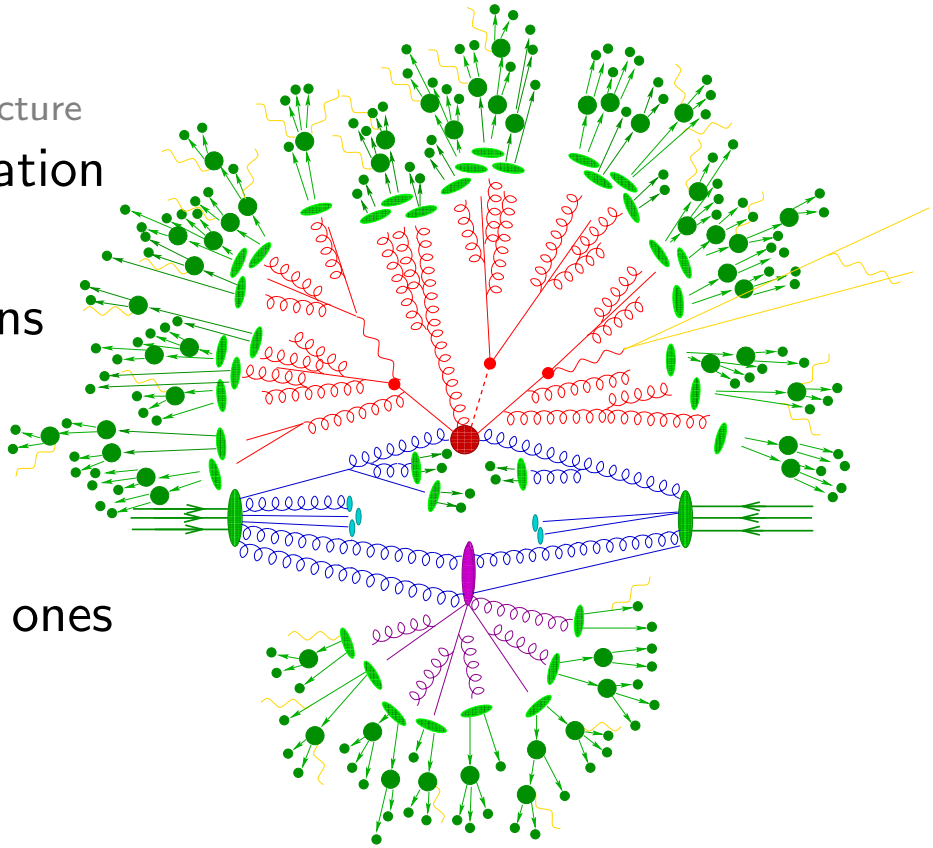
However !

- PS MCs often failed to describe jets
- Low-Q^2 region especially problematic

Introducing ME⊗PS

Event generation in parton shower Monte Carlo

1. Matrix Element (ME) generators red blobs
   simulate "central" part of the event
2. Parton Showers (PS) red & blue tree structure
   produce additional "hard" QCD radiation
3. Multiple interaction models purple blob
   simulate "secondary hard" interactions
4. Fragmentation models light green blobs
   hadronize QCD partons
5. Hadron decay modules dark green blobs
   decay primary hadrons into observed ones
6. Photon emission generators yellow stuff
   simulate additional QED radiation

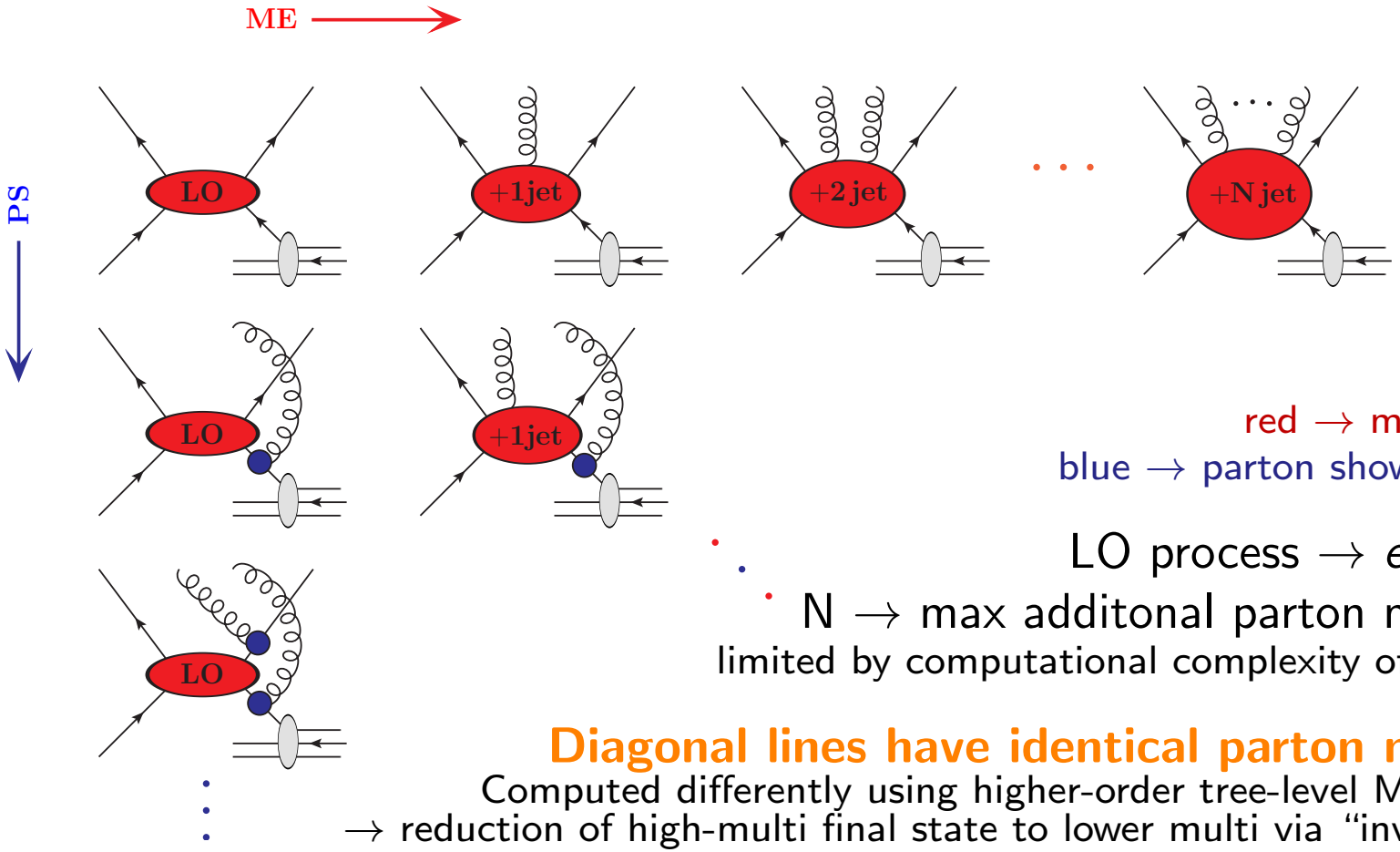


Here we focus on the "hard" QCD part

i.e. ME / PS generation and their interplay

ME⊗PS in a nutshell

Matrix elements and parton showers can describe the same final state !



red → matrix element
blue → parton shower branching

LO process → $e^{\pm} q \to e^{\pm} q$

N → max additonal parton multi in ME
limited by computational complexity of the problem

Diagonal lines have identical parton multiplicity
Computed differently using higher-order tree-level ME or/and PS
→ reduction of high-multi final state to lower multi via "inverse" shower

ME⊗PS idea: Use ME/PS in regime of their respective strengths

ME → hard emissions / PS → soft/collinear regime JHEP11(2001)063, JHEP05(2009)053

Inclusive ME⊗PS

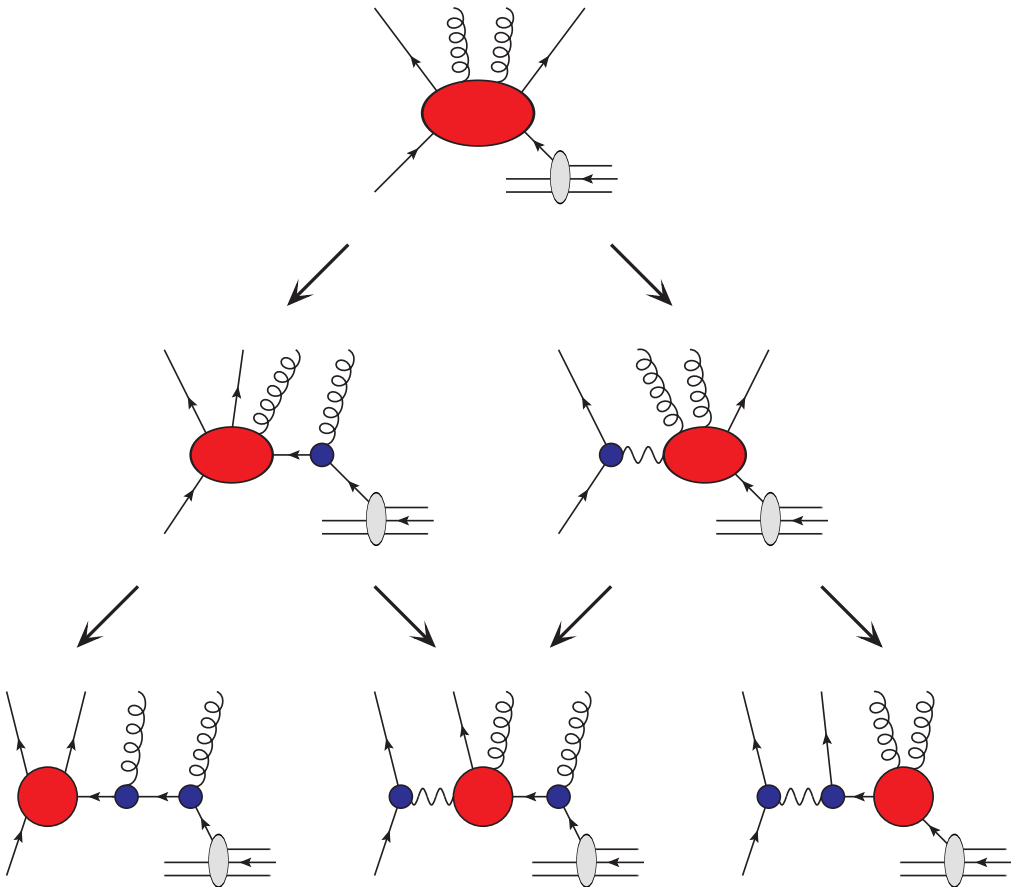
Matrix elements can have very different PS equivalents depending on kinematics

Must reduce full high-multi ME to either of these configurations in order to start PS

Radiation effects off intermediate legs must be modelled to account for Sudakov suppression → truncated PS

Method: PRD57(1998)5767, JHEP05(2009)053

- Probability to identify splitting given by PS's branching eqns
- Reduced ME configuration defined by "inverted" PS splitting kinematics
- Continue until $2 \to 2$ "core"



Core processes set the hardness scale of events $\to \mu_F$

i.e. no scale should be larger than this PRD70(2004)114009, JHEP05(2009)053

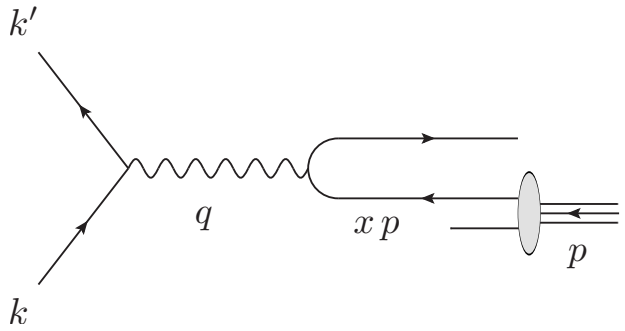
What have we learned at HERA?

Leading order $e^{\pm}p$ - scattering in collinear factorization Breit frame

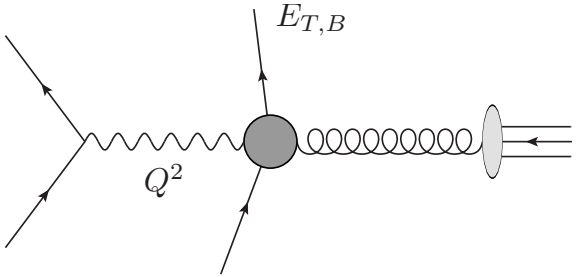
- There are no jets !
- Kinematical variables

$$Q^2 = -q^2 = (k' - k)^2 \text{ and } x = \frac{Q^2}{2\,q \cdot p}$$

- Hadronic cm energy $W = Q\sqrt{(1-x)/x}$



What happens at higher orders ?



- Multiple QCD scales, e.g. $E^2_{T,B}$
- $e^{\pm}q \to e^{\pm}q$ if $E^2_{T,B} \lesssim Q^2$
  $\gamma^* g \to jets$ if $Q^2 \lesssim E^2_{T,B}$

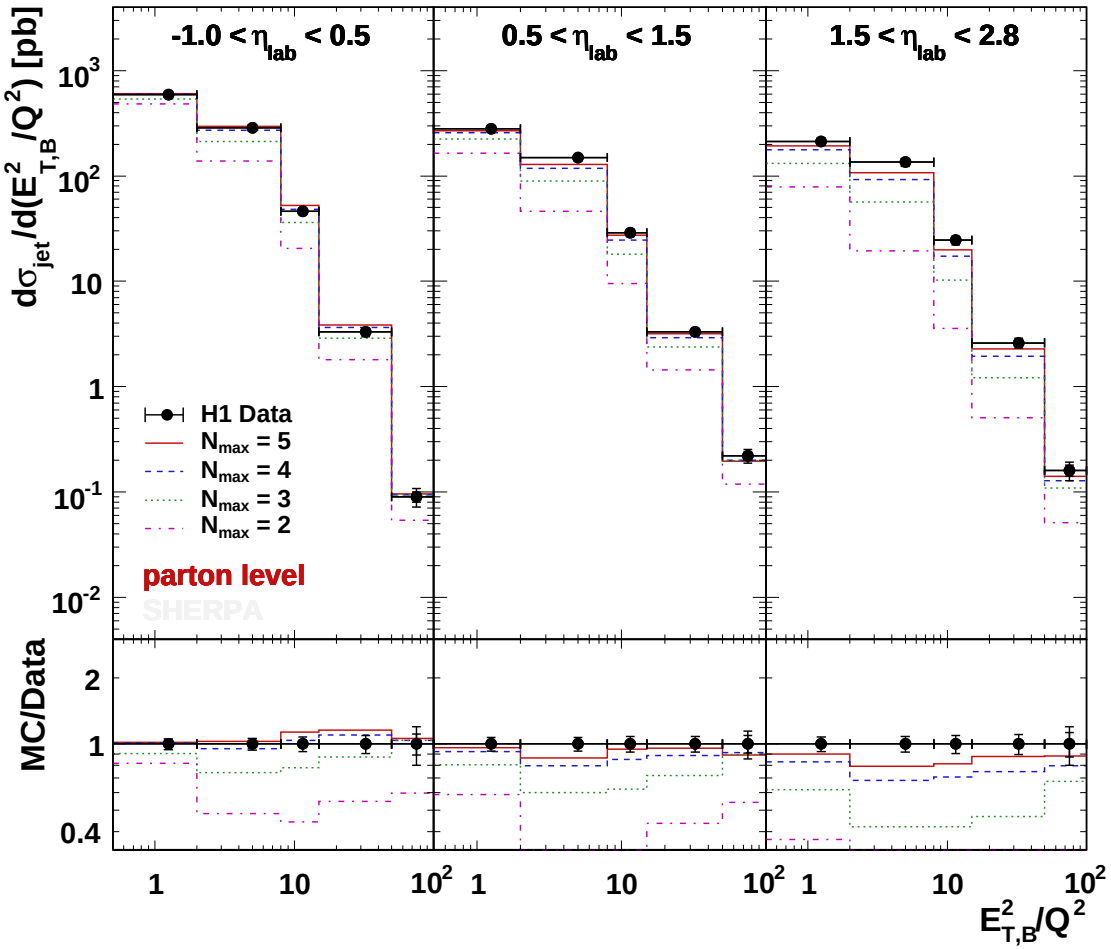
What makes DIS any different from $e^+e^- \to jets$ and $pp \to e^+e^-$?

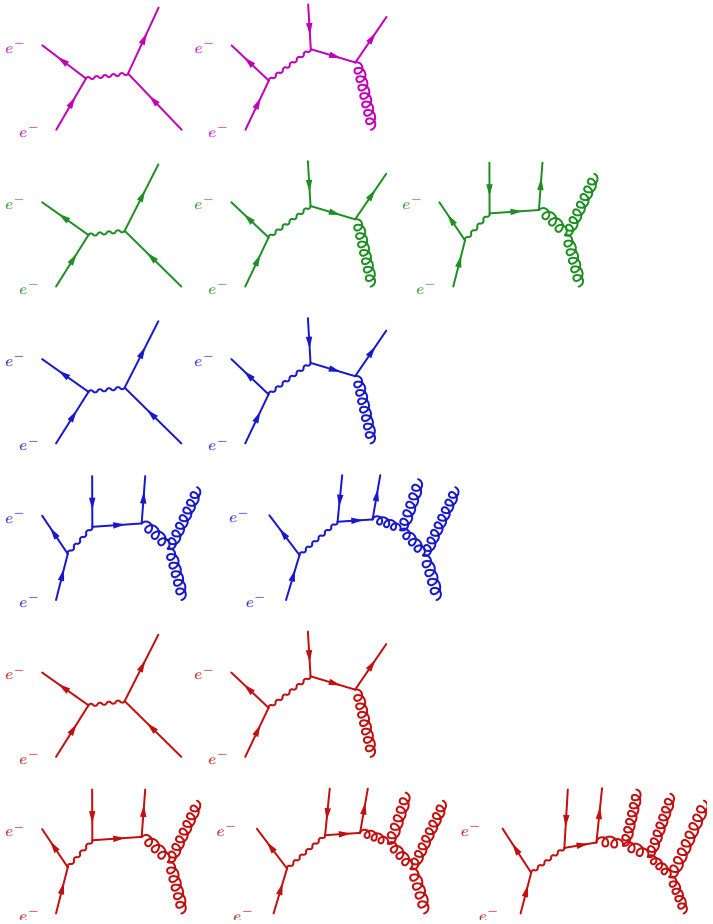
The virtuality of the exchanged photon tends to be close to zero !
PS starting scale is Q^2 $\to$ no phase space for emissions in parton-shower approach

Possible solution: Use higher-order tree-level ME and define core dynamically
$\to$ "inclusive" ME$\otimes$PS merging incorporating electroweak effects see previous slide

Inclusive jets in DIS PLB542(2002)193

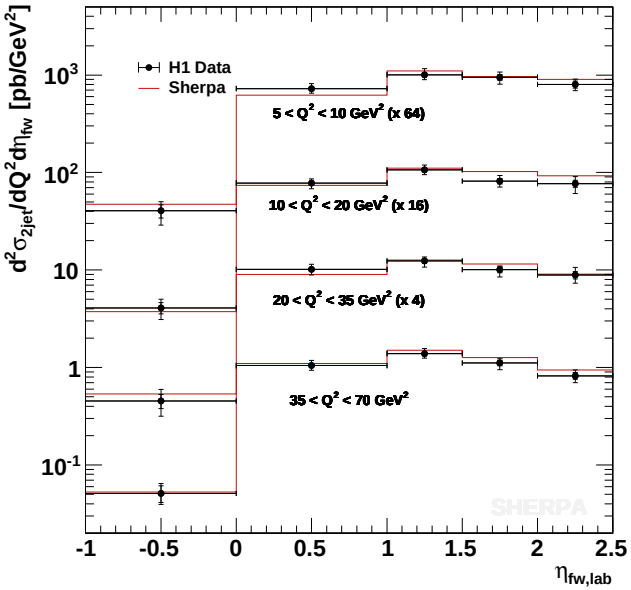
Variation of maximum matrix-element multiplicity, $N_{\max}$ [2]

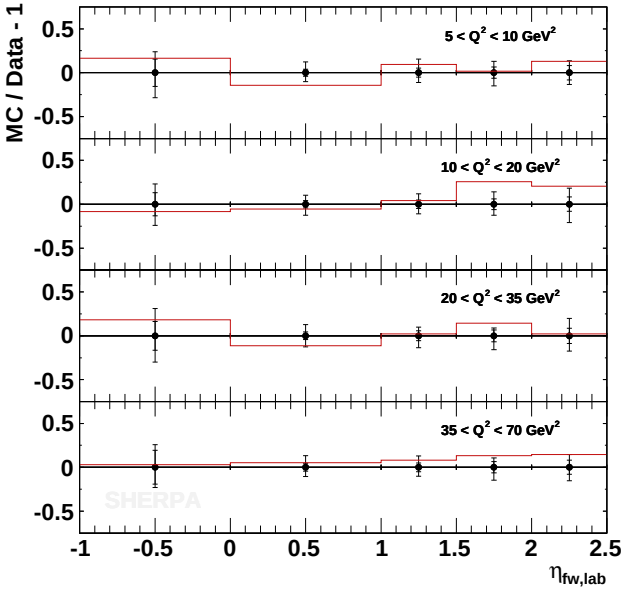




[2] SHERPA ⊕ HZTool from EPJC67(2010)73

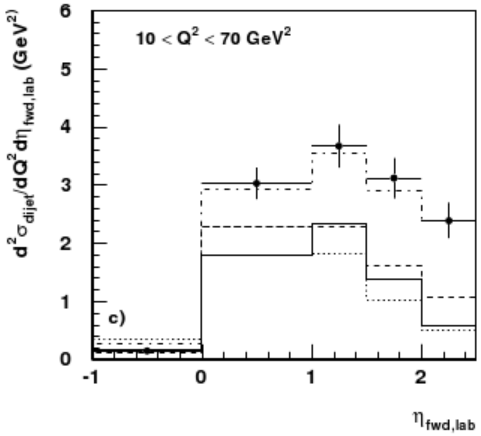
Inclusive dijets in DIS EPJC19(2001)289

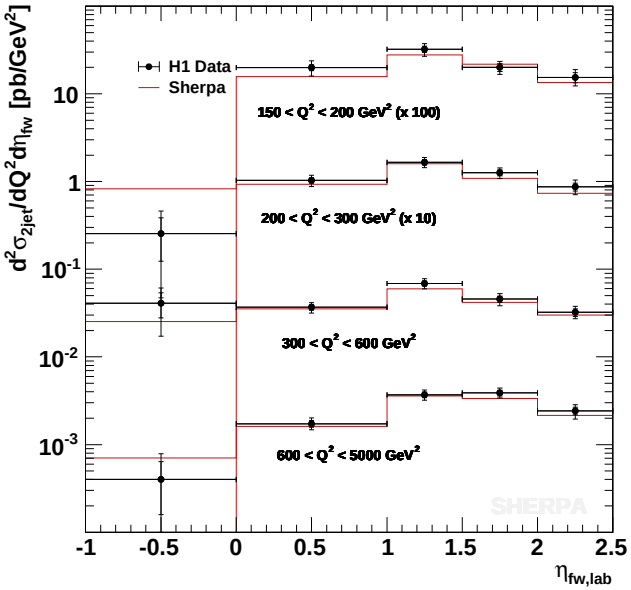


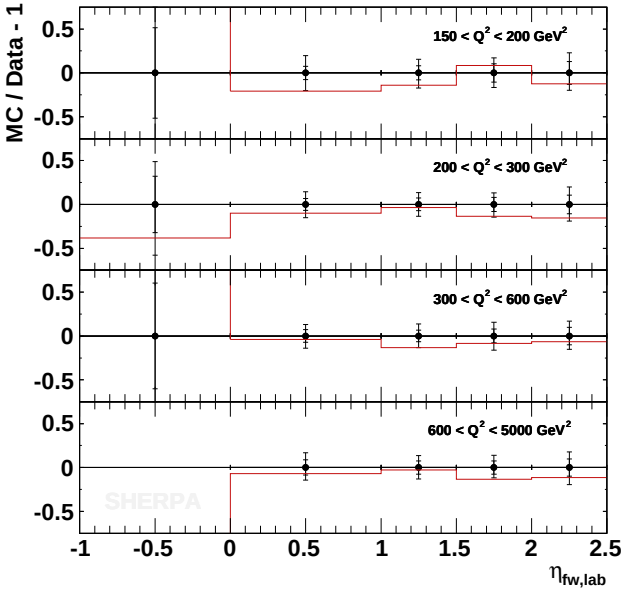


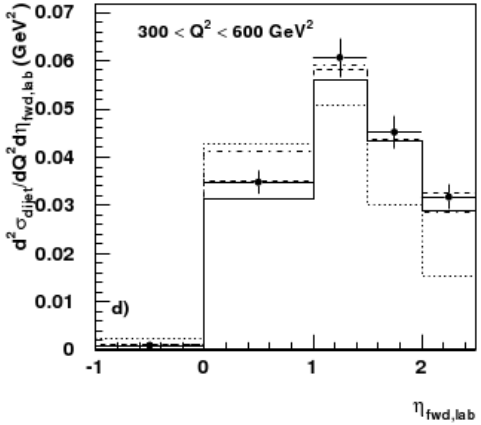
Compare to introduction

→ MC status 1999 vs. 2010

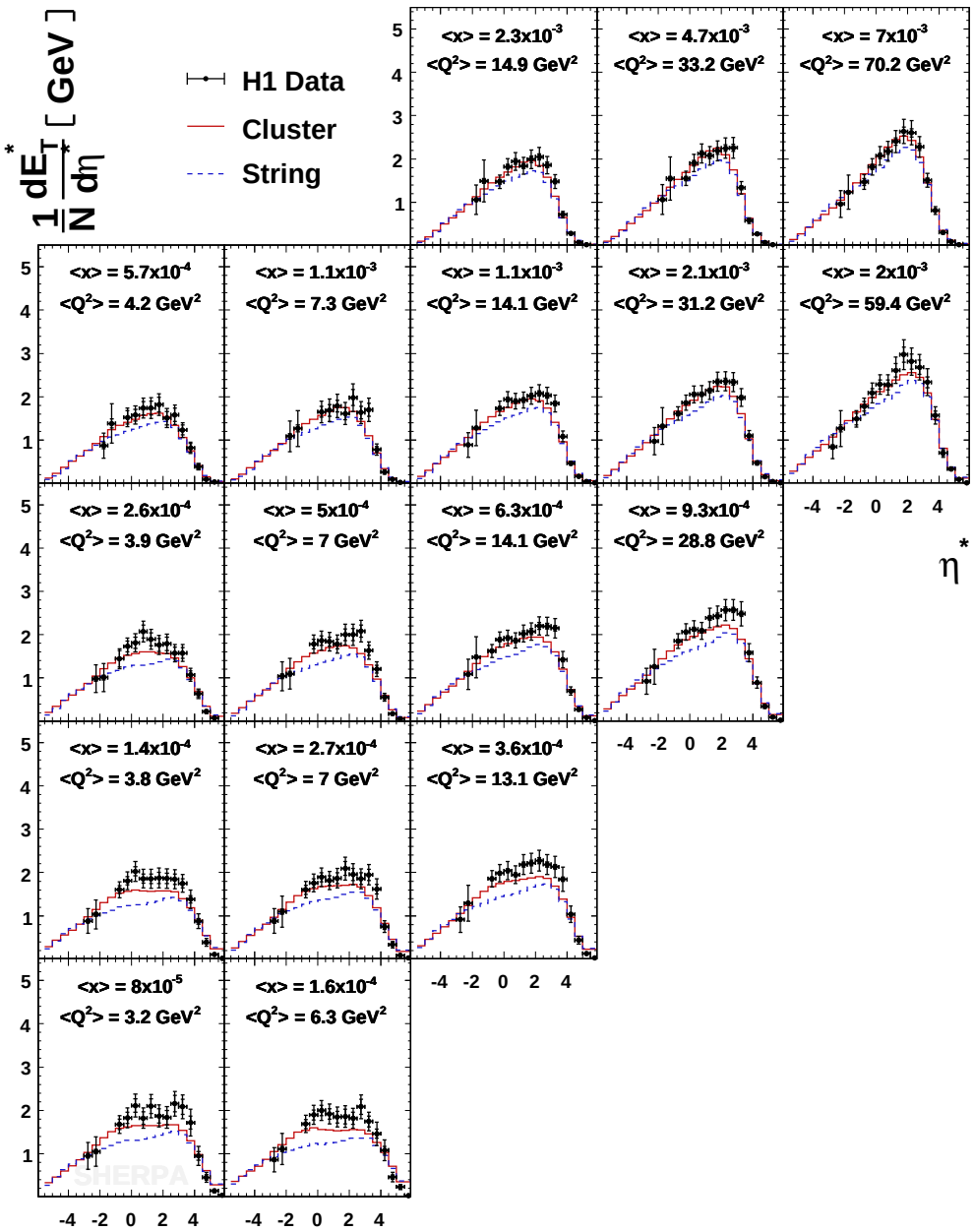






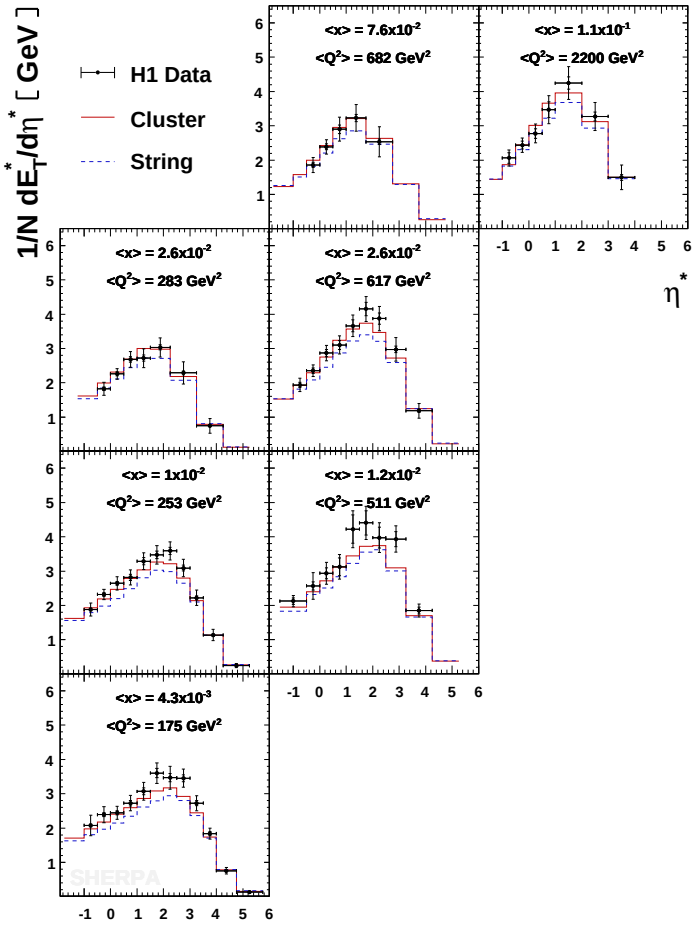


Energy flows in DIS EPJC12(2000)595



Transverse energy flow

SHERPA cluster fragmentation vs. Lund string fragmentation



Why investigate photon production?

Jet definition at low E_T

- "Clean" $\gamma + jet$ signature defines jet characteristics via p_T balance
- Detector response to γ well known

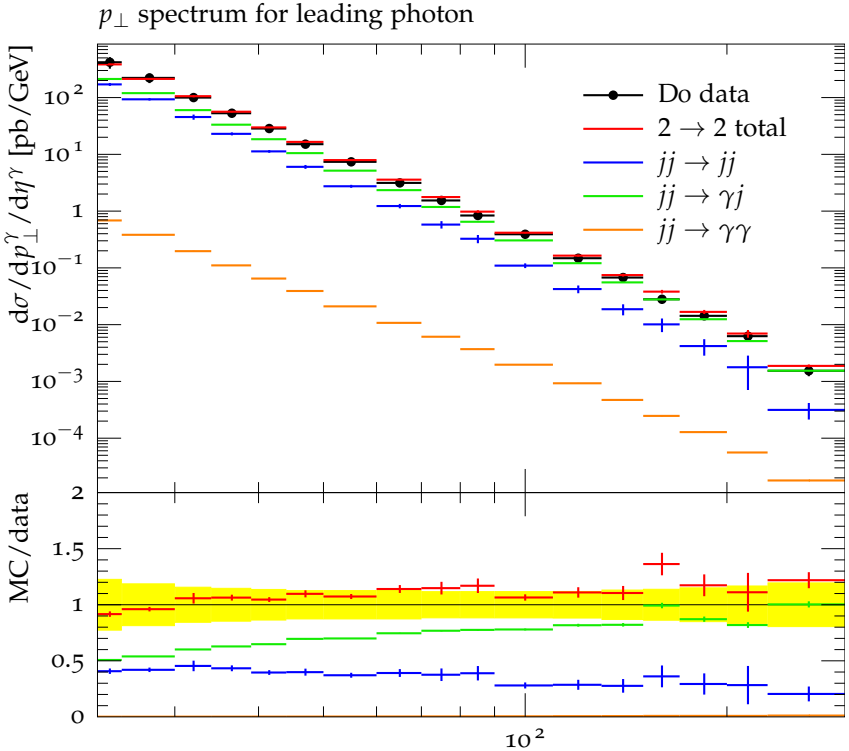
New & old physics background

- $h \to \gamma\gamma$ (+ jets)
- BSM signatures often include γ's
- High-p_T γ's interesting for AGC's

Similarity to QCD

- Resummation possible YFS
- Often need to improve with NLO
- High-p_T tails often interesting

MC simulation often treacherous

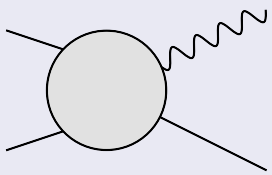


Example: DØ analysis PLB639(2006)151

despite strict cuts on photon p_T $p_T > 23\,\mathrm{GeV}$
and isolation requirements $E_{\mathrm{EM}}/E_{\mathrm{tot}} > 0.95$
Large fraction of events from $jj \to jj$!

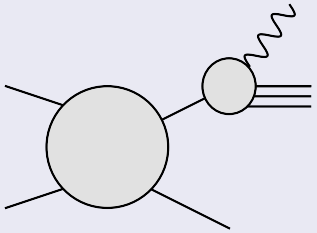
The perturbative QCD approach

"Direct" component
fixed-order calculation with isolated γ



- γ+jet (JetPhox) PRD73(2006)094007
- $\gamma\gamma$ (DiPhox) EPJC16(2000)311
- $\gamma\gamma$+jet JHEP04(2003)059
- $gg \to \gamma\gamma g$ PLB460(1999)184

"Fragmentation" component
factorization of $q\gamma$ collinear singularity



- Gives rise to fragmentation function $D^{\gamma}_{q,g}$ PLB79(1978)83
- Contribution depends on photon isolation criterion

Prerequisites

- Photon isolation criterion for IR-safe cross-section definition
  cone PRD42(1990)61, smooth isolation PLB429(1998)369, democratic approach ZPC62(1994)311
- Nonperturbative input for $D^{\gamma}_{q,g}$ from measurements or model

Non-prompt component not included in this approach, but experimentally separable
$\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, ...

The Monte Carlo approach

"Democratic" Monte Carlo model idea based on ZPC62(1994)311

- Treat partons and photons fully democratically
- Combine matrix elements of various parton/photon multiplicity with
- Interleaved QCD$\oplus$QED parton shower evolution $\Rightarrow$ evolution of $D^{\gamma}_{q,g}$

Modifying the parton shower

Add splitting functions for $qq\gamma$ similarly implemented in Ariadne, Herwig & Pythia

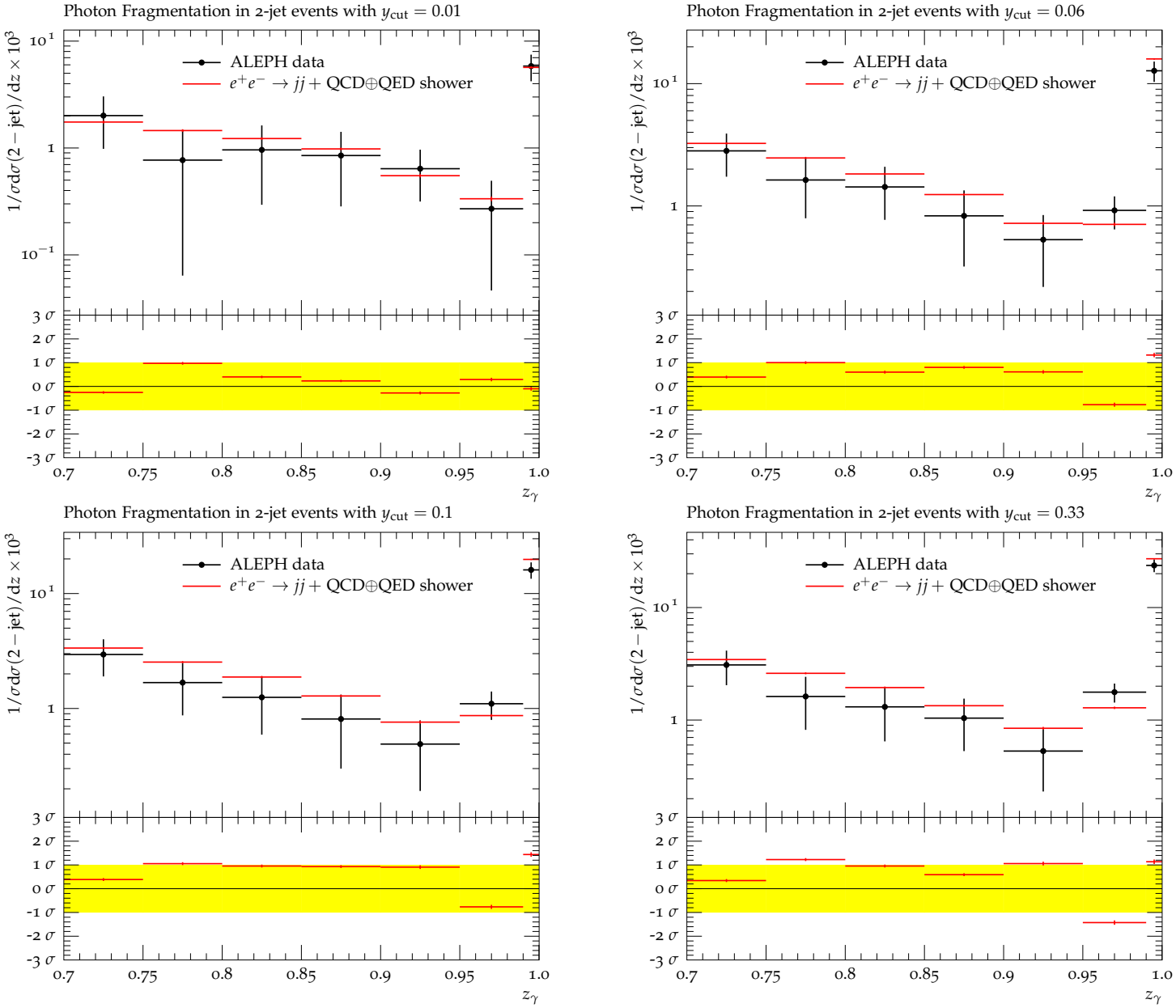
$$\Delta_a(Q_0^2, Q^2) = \Delta_a^{(\mathrm{QCD})}(Q_0^2, Q^2)\Delta_a^{(\mathrm{QED})}(Q_0^2, Q^2)$$

Here: Splitting kernels based on spin-averaged CS dipole functions
$\Rightarrow$ Recoil partners are all oppositely charged particles in the event
Spin dependence restored by real emission ME used to correct the PS

How to treat the nonperturbative regime ?

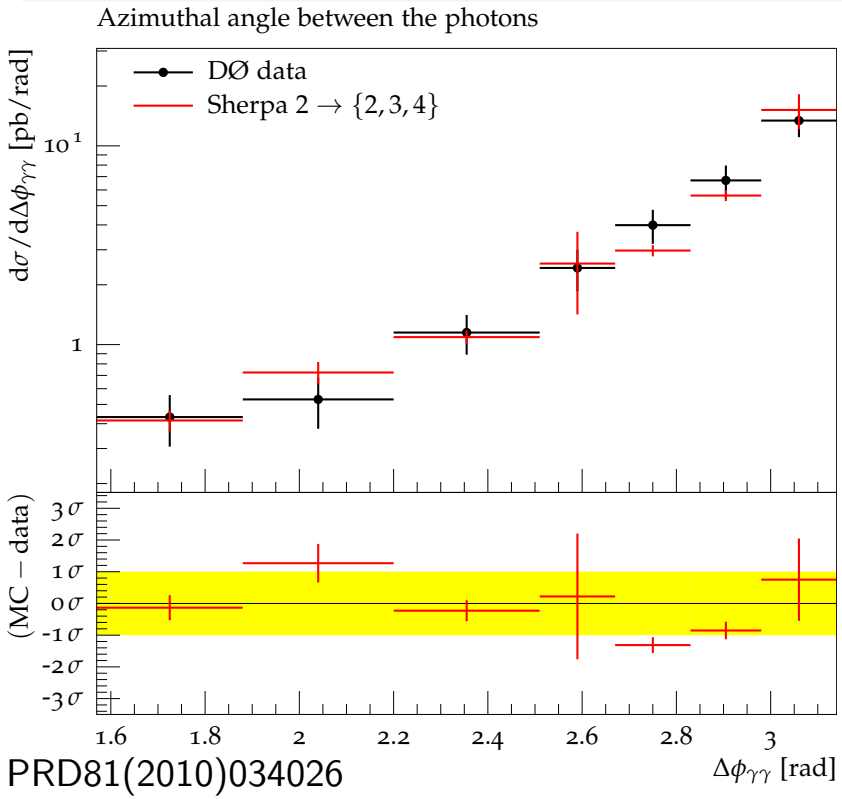
- Parton-to-hadron transition via fragmentation models cluster/string model
- Non-prompt γ-production for free via hadron decay packages
- No new parameters !

Photon fragmentation function at LEP ZPC69(1996)365

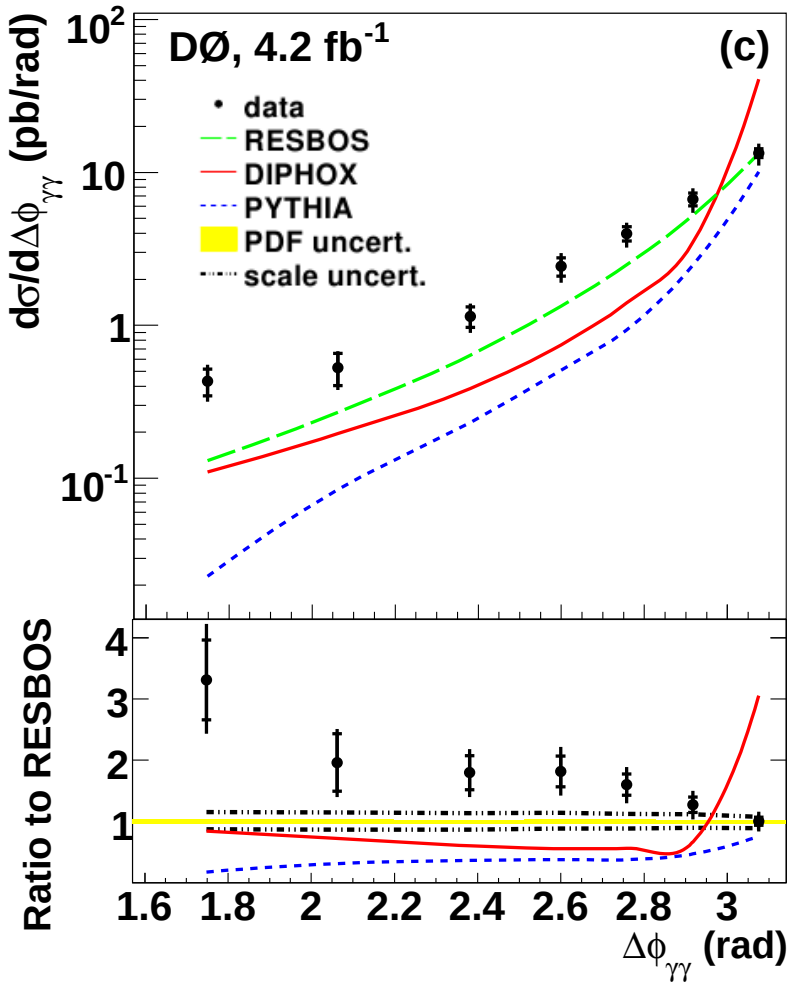


Diphoton production at the Tevatron PLB690(2010)108

$E_T^{\gamma 1} > 21$ GeV, $E_T^{\gamma 1} > 20$ GeV,
$|\eta^\gamma| < 0.9$, $E_T^{R=0.4} - E_T^\gamma < 2.5$ GeV



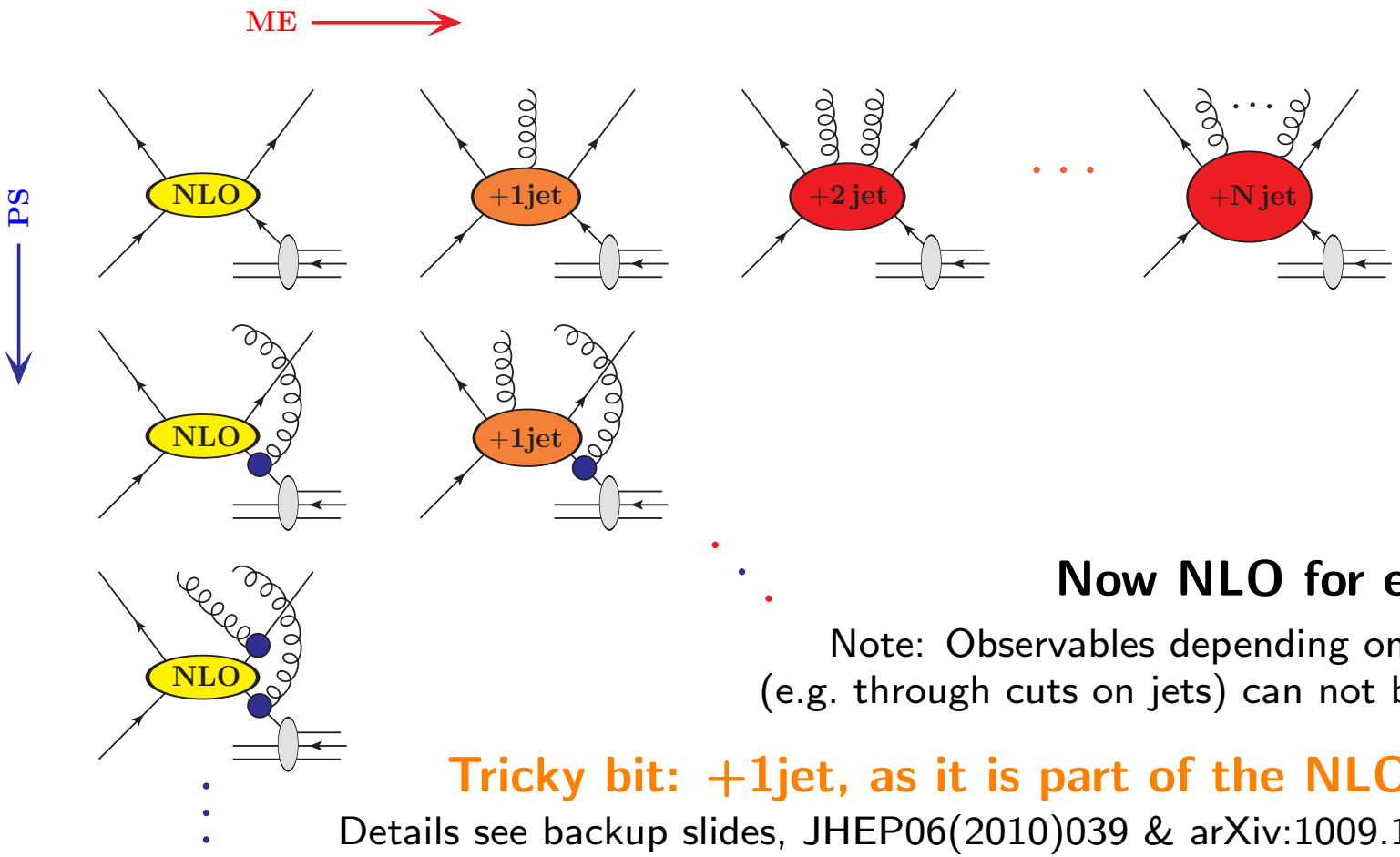
PRD81(2010)034026



SHERPA prediction: Merged 2→{2+3+4}-jet/γ plus $gg \to \gamma\gamma box$

Going NLO: The MENLOPS idea

Adding in full NLO accuracy for the core is easy, just replace LO → NLO



Now NLO for $e^{\pm}q \to e^{\pm}q$

Note: Observables depending on higher multi (e.g. through cuts on jets) can not be improved !

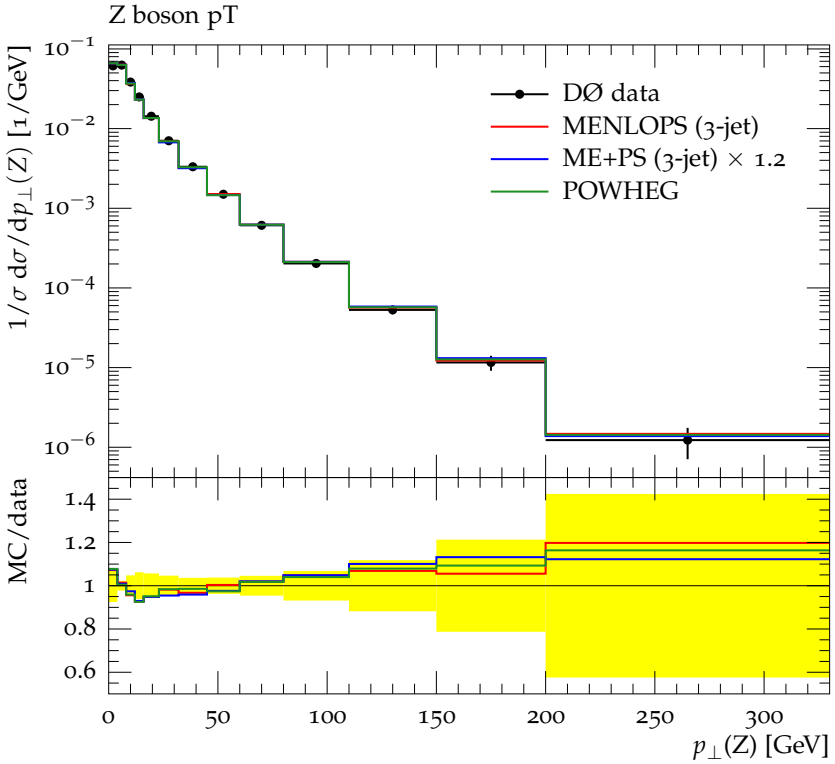
Tricky bit: +1jet, as it is part of the NLO core, too

Details see backup slides, JHEP06(2010)039 & arXiv:1009.1127 [hep-ph]

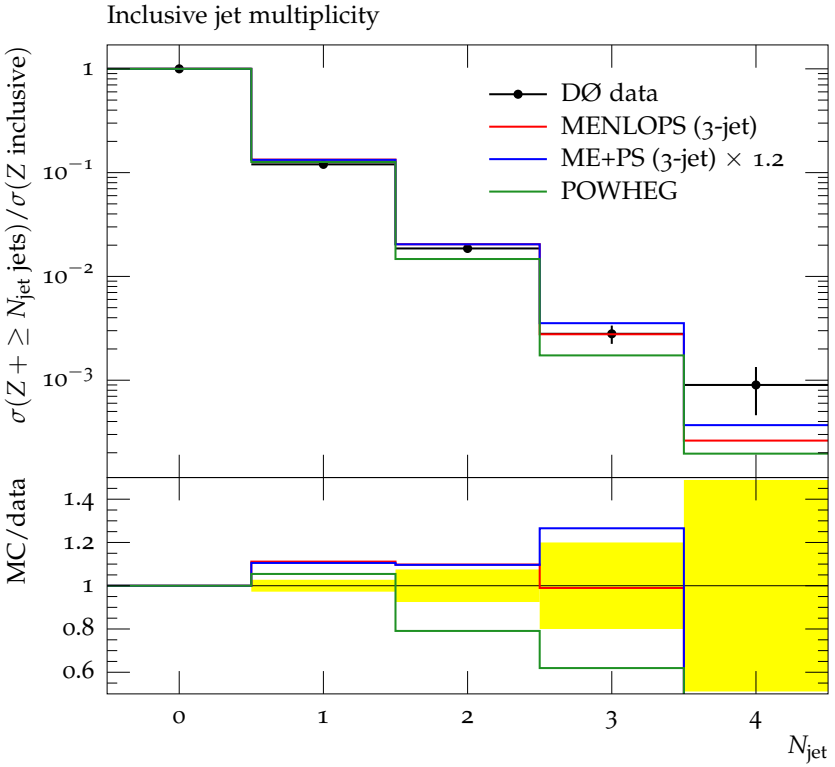
Note that we don't merge NLO with higher-multi NLO yet !

Results: Z+jets arXiv:1009.1127 [hep-ph]

Z-boson p_T arXiv:1006.0618 [hep-ex]



Inclusive jet multiplicity PLB658(2008)112

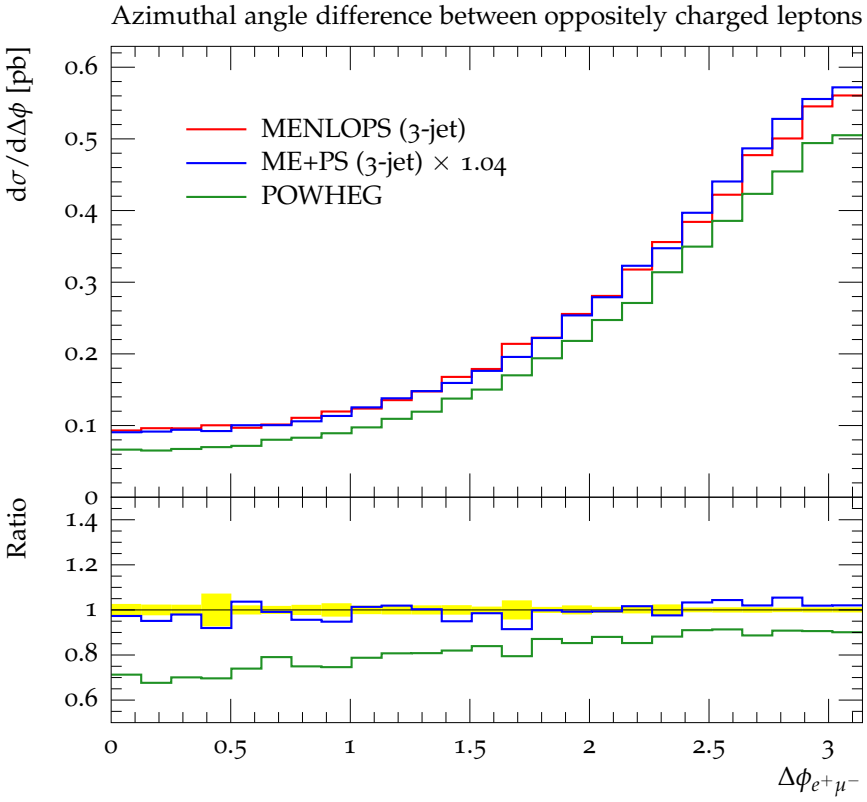


First Run II measurement using μ's
Data corrected to the particle level

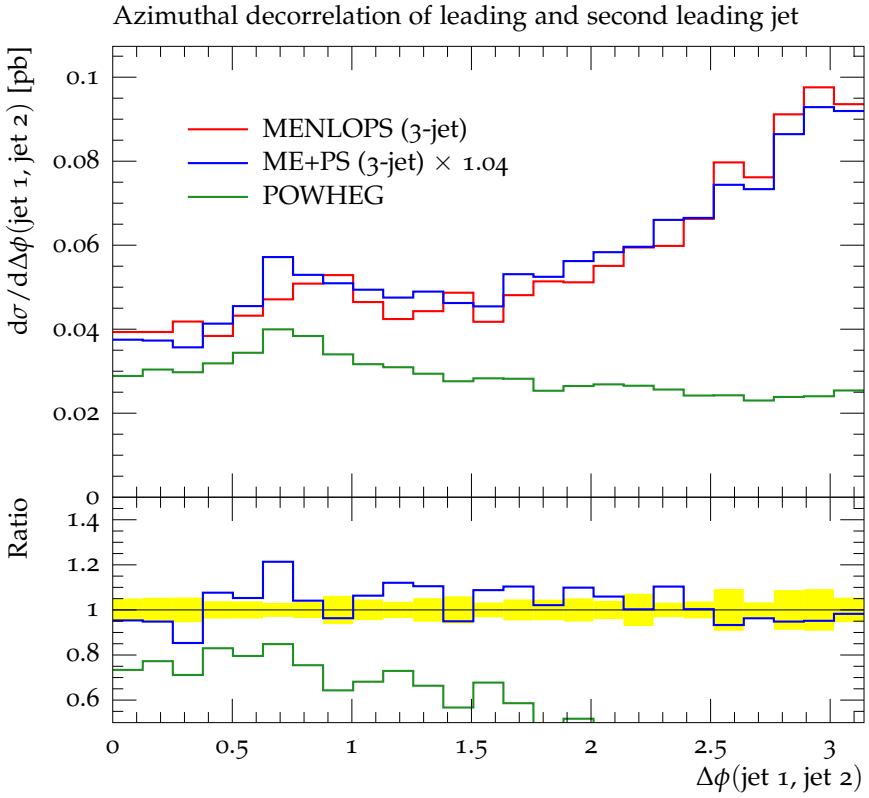
SHERPA prediction: $p\bar{p} \to \ell\ell$@NLO
$\oplus$ $p\bar{p} \to \ell\ell$+{1,2,3}-jets@LO

Results: W^+W^-+jets arXiv:1009.1127 [hep-ph]

Lepton-$\Delta\phi$ (LHC @ 14 TeV)



Jet-$\Delta\phi$ (LHC @ 14 TeV)



Large unitarity violations in both ME⊗PS and MENLOPS compared to POWHEG

Follows from large W^+W^-+2-jet contribution, cf. PRD72(2005)034028, PLB683(2010)154

Note: **This is by no means a shortcoming of the method!**

Large phase space for 2 additional jets. $\Delta\phi_{jj}$ only LO_{1j}⊗LL in POWHEG

Summary

What has recently improved in ME⊗PS...

- Largely reduced systematics with truncated PS JHEP05(2009)053
- Successful first analysis of hadronic final state data from HERA
- Simultaneous description of jet and photon production
- Combination with POWHEG into MENLOPS

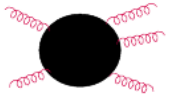
To do ...

- Even more tests and validation
- Extension to NLO ⊗ higher-multi NLO

This is important preparatory work for LHC data analyses !

Don't expect your MC to model backgrounds at 7-14 TeV,
if you can't describe HERA and Tevatron data ...

NLO calculations

Parts of NLO calculations $\longrightarrow$ $\mathcal{A}_{\mathrm{LO}}$:  $\mathcal{A}_{\mathrm{NLO,Virtual}}$: $\mathcal{A}_{\mathrm{NLO,Real}}$:

$$\sigma^{NLO} = \int \mathrm{d}\Phi_B \left(\mathrm{B} + \tilde{\mathrm{V}}\right) + \int \mathrm{d}\Phi_R \, \mathrm{R} = \int \mathrm{d}\Phi_B \left[\left(\mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}\right) + \int \mathrm{d}\Phi_{R|B} \left(\mathrm{R} - \mathrm{S}\right)\right]$$

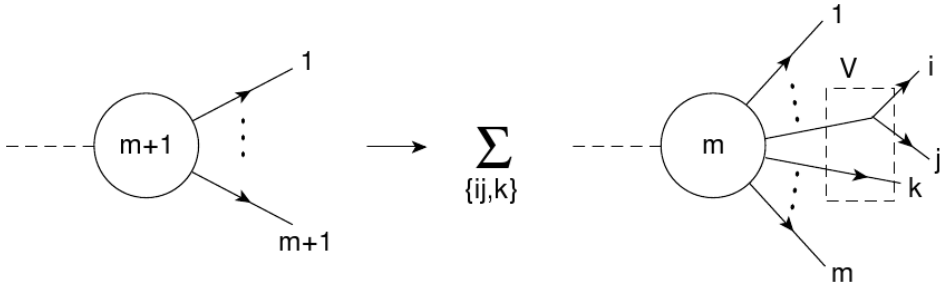
S - subtraction term constructed such that IR singularities in R are removed

I - integrated subtraction term locally (in Φ_B) compensates $\mathrm{S} \to 0 \stackrel{!}{=} \mathrm{I} - \int \mathrm{d}\Phi_{R|B}\, \mathrm{S}$

Consider for example Catani-Seymour dipole subtraction

NPB485(1997)291, NPB627(2002)189

Schematically: $\mathrm{S} \to \sum_{\{i,j\}} \sum_{k \neq i,j} D_{ij,k}$



$D_{ij,k} = \mathrm{B}_{\widetilde{ij},k} \otimes V_{ij,k}$ - dipole terms

$V_{ij,k}$ - process-independent dipole functions

Φ_R-kinematics different from Φ_B $\rightarrow$ impossible to combine into one MC event

Approximate solutions by MC@NLO JHEP06(2002)029 and POWHEG JHEP11(2004)040

Routemap for this section:
- **Soft/collinear limits of S $\rightarrow$ parton showers**
- **Adding NLO back in $\rightarrow$ POWHEG and ME$\otimes$PS**

NLO calculations in the POWHEG approach

Need to be specific now: $\{\vec{a}\}$ means set of partons flavours $\{\vec{f}\}$, momenta $\{\vec{p}\}$

$\rightarrow$ **Real-emission contribution to NLO cross section**

$$\mathrm{d}\sigma_R(\{\vec{p}\}) = \sum_{\{\vec{f}\}} \mathrm{d}\sigma_R(\{\vec{a}\}) \qquad \mathrm{d}\sigma_R(\{\vec{a}\}) = \mathrm{d}\Phi_R(\{\vec{p}\})\,\mathrm{R}(\{\vec{a}\})$$

where $\mathrm{R}(\{\vec{a}\}) = \mathcal{L}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})$ and $\mathcal{L}(\{\vec{a}\};\mu^2) = x_1 f_{f_1}(x_1,\mu^2)\; x_2 f_{f_2}(x_2,\mu^2)$
$\mathrm{d}\Phi_R$ contains initial-state phase space $\mathrm{d}\log x_1\,\mathrm{d}\log x_2$
$\mathcal{R}(\{\vec{a}\}) = |\mathcal{M}_R|^2(\{\vec{a}\})/[F(\{\vec{a}\})S(\{\vec{f}\})]$ with symmetry factor S, flux F

Similar formulas for Born-level term $\mathrm{B}(\{\vec{a}\})$ one parton less, of course

Assume generalised "dipole terms", such that think of $D_{ij,k}$ on previous slide

$$\mathcal{R}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \sum_{\{i,j\}} \sum_{k\neq i,j} \mathcal{D}_{ij,k}(\{\vec{a}\})$$

Define partition of real-emission term $\mathcal{R}(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k\neq i,j} \mathcal{R}_{ij,k}(\{\vec{a}\})$

$$\mathcal{R}_{ij,k}(\{\vec{a}\}) := \rho_{ij,k}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})\,, \quad \text{where} \quad \rho_{ij,k}(\{\vec{a}\}) = \frac{\mathcal{D}_{ij,k}(\{\vec{a}\})}{\sum_{\{m,n\}} \sum_{l\neq m,n} \mathcal{D}_{mn,l}(\{\vec{a}\})}$$

Note: Holds throughout the phase space ! most important formula in the talk

NLO calculations in the POWHEG approach

$\mathcal{D}_{ij,k}(\{\vec{a}\})$ defines parton maps think of Catani-Seymour dipoles

$$b_{ij,k}(\{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_i, f_j\} \cup \{f_{\tilde{ij}}\} \\ \{\vec{p}\} \to \{\vec{\tilde{p}}\} \end{cases} \quad \leftrightarrow \quad r_{\tilde{ij},\tilde{k}}(f_i, \Phi_{R|B}\,; \{\vec{a}\}) = \begin{cases} \{\vec{f}\} \setminus \{f_{\tilde{ij}}\} \cup \{f_i, f_j\} \\ \{\vec{\tilde{p}}\} \to \{\vec{p}\} \end{cases}$$

- $b_{ij,k}$ converts real-emission configuration to Born-level
- $r_{\tilde{ij},\tilde{k}}$ converts Born-level to real-emission needs extra flavour & phase space

Trivially factorise real-emission term into Born and radiative contribution

$$\mathrm{d}\sigma_R(\{\vec{a}\}) = \sum_{\{i,j\}} \sum_{k \neq i,j} \mathrm{d}\sigma_B(b_{ij,k}(\{\vec{a}\}))\, \mathrm{dP}_{ij,k}(\{\vec{a}\})$$

differential emission probability is $\mathrm{dP}_{ij,k}(\{\vec{a}\}) = \mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\}) \dfrac{\mathrm{R}_{ij,k}(\{\vec{a}\})}{\mathrm{B}(b_{ij,k}(\{\vec{a}\}))}$

Subtraction algorithms predict $\mathrm{dP}_{ij,k}$ in the soft/collinear limits via

$$\mathcal{D}_{ij,k}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{1}{2\,p_i p_j}\, 8\pi\,\alpha_s\, \mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k)\ ,$$

Note the symmetry factors $\leftrightarrow$ factorization of invariant ME, not of specific process

$\otimes \to$ spin & colour-correlations between $\mathcal{B}$ and V

NLO calculations in the POWHEG approach

Now make an approximation replace correlated with uncorrelated dipole kernel

$$\mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k) \to \mathcal{B}(b_{ij,k}(\{\vec{a}\}))\, \mathcal{K}_{ij,k}(p_i, p_j, p_k)$$

Parametrize radiative phase space: $\mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\}) = \frac{1}{16\pi^2}\,\mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,J_{ij,k}(t,z,\phi)$

Assume phase space gets filled successively in $t \leftrightarrow$ partons can be distinguished

Must adapt symmetry factors: $\frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \to \frac{1}{S_{ij}} = \begin{cases} 1/2 & \text{if } i,j > 2,\ b_i = b_j \\ 1 & \text{else} \end{cases}$

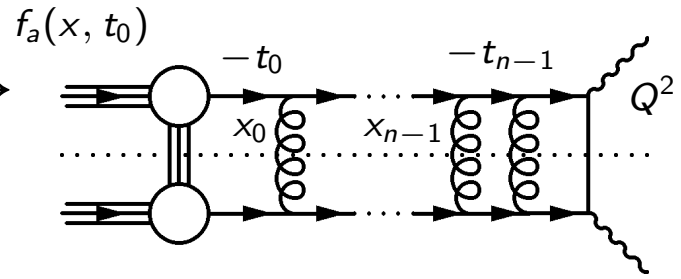
Combining everything gives differential radiation probability

$$\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\}) = \frac{\mathrm{d}t}{t}\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\,\frac{1}{S_{ij}}\,J_{ij,k}(t,z,\phi)\,\mathcal{K}_{ij,k}(t,z,\phi)\,\frac{\mathcal{L}(\{\vec{a}\};t)}{\mathcal{L}(b_{ij,k}(\{\vec{a}\});t)}$$

Iterate this equation for higher-multi ME

$\to$ ladder-like structure of amplitude squared with strong ordering in scales $t_0 < \ldots < t_n$

$\leftrightarrow$



Factorization at any stage above Λ_{QCD}

can split emissions off ME one by one

Corrections induced by $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}$ can be large and must be resummed

In inclusive case $t \in [0, \infty)$ divergences in $\mathcal{K}_{ij,k}$ cancel ε-poles in V $\to$ unitarity !

NLO calculations in the POWHEG approach

→ **No-emission probability** from Poisson statistics implementing unitarity constraint

$$\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t',t'';\{\vec{a}\}) = \exp\left\{-\sum_{f_i=q,g}\int_{t'}^{t''}\int_{z_{\min}}^{z_{\max}}\int_0^{2\pi}\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))\right\}.$$

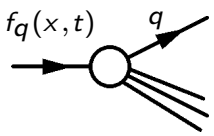
Note: $r_{\widetilde{ij}\tilde{k}}$ implicitly and uniquely defined by subtraction scheme, i.e. $\mathcal{K}_{ij,k}$

Assume IF-splitting → Lumi ratio $\frac{x}{z}f_{f_i}(\frac{x}{z},t)/xf_{f_{\widetilde{ij}}}(x,t)$, symmetry factor 1

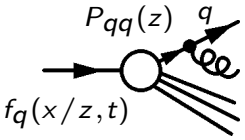
$$\frac{\partial\log\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})}{\partial\log(t/\mu^2)} = \int_x^{z_{\max}}\frac{\mathrm{d}z}{z}\int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\sum_{f_i=q,g}\frac{\alpha_s}{2\pi}J_{ij,k}(t,z,\phi)\,\mathcal{K}_{ij,k}(t,z,\phi)\,\frac{f_{f_i}(\frac{x}{z},t)}{f_{f_{\widetilde{ij}}}(x,t)}$$

Voilà, the DGLAP equation ! imagine $J_{ij,k}(t,z,\phi)\mathcal{K}_{ij,k}(t,z,\phi)\to P_{i\,\widetilde{ij}}(z)$

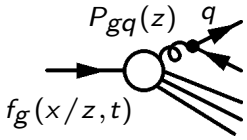
$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$$



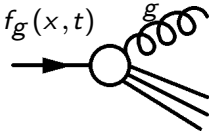
$$= \int_x^1\frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$



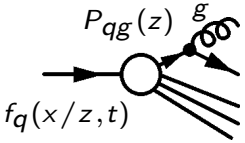
$$+ \int_x^1\frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$



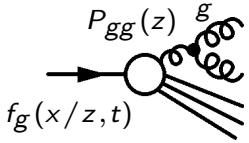
$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$$



$$= \sum_{i=1}^{2\,n_f}\int_x^1\frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$



$$+ \int_x^1\frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}$$



NLO calculations in the POWHEG approach

The Monte Carlo implementation of this is called a parton shower:

- Generate emission scale t from parton a at t' via

$$1 - \mathcal{P}^{(PS)}_{\widetilde{ij},\tilde{k}}(t, t'; \{\vec{a}\}) \stackrel{!}{=} \# \,, \qquad \text{where} \qquad \# \in [0,1] \text{ random}$$

  nonzero probability for unresolved emission ($z > z_{max}$ or $z < 1 - z_{max}$)
  $\rightarrow$ nonzero probability for no splitting at all

- Dice splitting variable z, angle ϕ and flavour f_i according to
  $\alpha_s/(2\pi)\, J_{ij,k}(t,z,\phi)\, \mathcal{K}_{ij,k}(t,z,\phi)\, f_{f_i}(x/z,t)/z\, f_{f_{\widetilde{ij}}}(x,t)$

What does the form of $\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t, t'; \{\vec{a}\})$ imply ?

- Exclusive radiation patterns, i.e. $z_{max} < 1$
- Preservation of hard cross section $\hat{\sigma}$ i.e. **unitarity**
  either splitting or no splitting $\rightarrow$ no additional events & no rejections,
  just "dress" hard ME with additional partons

Various choices for evolution and splitting variable possible, e.g.

- Virtuality t and energy fraction z PYTHIA, ISAJET, APACIC++
- Angle θ and light cone momentum fraction $\tilde{z}$ fHERWIG, HERWIG++

NLO calculations in the POWHEG approach

Recover NLO-accurate radiation pattern from PS through extra weight

$$w_{ij,k}(\{\vec{a}\}) = \mathrm{dP}_{ij,k}(\{\vec{a}\}) \,/\, \mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\})$$

Easy to compute in general-purpose Monte-Carlo, all input is tree-level only

Approximate "seed cross section" using local K-factor $\bar{\mathrm{B}}/\mathrm{B}$

$$\frac{\bar{\mathrm{B}}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} = 1 + \frac{\tilde{\mathrm{V}}(\{\vec{a}\}) + \mathrm{I}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} + \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij,k}_{R|B} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\})) - \mathrm{S}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}$$

Note: Implies wrong dependence of observables on final-state momenta $\{\vec{p}\}$ $\rightarrow$ resolved by PS

Combine reweighting and local K-factor $\rightarrow$ observable O to $\mathcal{O}(\alpha_s)$ from

$$\begin{aligned}\langle O\rangle^{(\mathrm{POWHEG})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\, \bar{\mathrm{B}}(\{\vec{a}\}) \Bigg[& \mathcal{P}^{(\mathrm{ME})}(t_0, \mu^2; \{\vec{a}\})\, O(\{\vec{p}\}) \\ + \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, & J_{ij,k}(t,z,\phi) \\ \times \frac{1}{S_{ij}} \frac{S(r_{\widetilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})} & \mathcal{P}^{(\mathrm{ME})}(t, \mu^2; \{\vec{a}\})\, O(r_{\widetilde{ij},\tilde{k}}(\{\vec{p}\}))\Bigg]\end{aligned}$$

This is the POWHEG master formula JHEP11(2004)040 JHEP11(2007)070

Tree-level calculations in the ME⊗PS approach

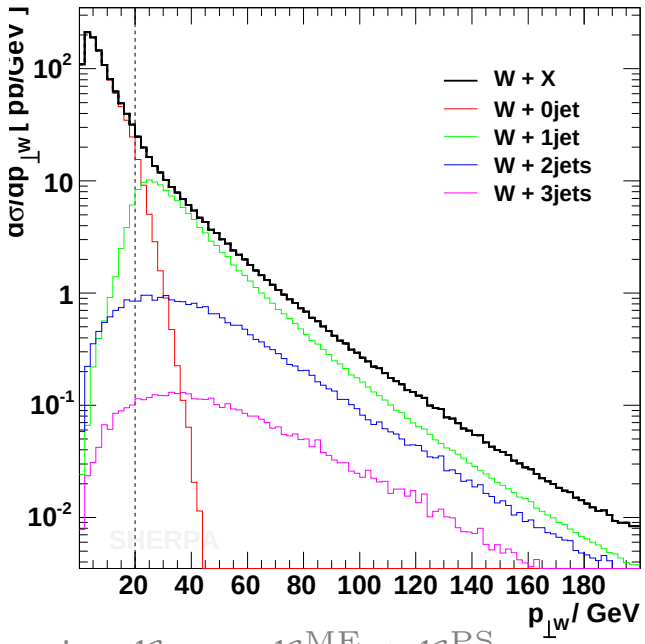
ME⊗PS idea: **Separate phase space into "hard" and "soft" region**

- Matrix elements populate hard domain
- Parton shower populates soft domain

need criterion to define "hard" & "soft" as above and below a certain cut value

→ **Jet criterion** Q e.g. k_T-jet measure

$\longleftrightarrow$



Formally: replace kernels in PS evolution with note that $\mathcal{K}_{ij,k} = \mathcal{K}^{\mathrm{ME}}_{ij,k} + \mathcal{K}^{\mathrm{PS}}_{ij,k}$

$$\mathcal{K}^{\mathrm{ME}}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right] \quad \rightarrow \quad \mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\tilde{ij},\tilde{k}}(t,t')$$

$$\mathcal{K}^{\mathrm{PS}}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right] \quad \rightarrow \quad \mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\tilde{ij},\tilde{k}}(t,t')$$

Reweight PS **emission** by $w_{ij,k}$ in ME domain only

Formally equivalent to POWHEG but **uncorrected no-emission rate** $\mathcal{P}^{(\mathrm{PS})}$

Note: $\mathcal{P}^{(\mathrm{PS})} = \mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}\,\mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}$

Tree-level calculations in the ME⊗PS approach

Starting event generation from tree-level ME is way more efficient !

Interpret as LO ME ⊗ PS branchings

- Identify most likely splitting according to $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}$
- Combine partons into mother according to $b_{ij,k}$
- Continue until $2 \to 2$ core process

$\to$ Cluster algorithm similar to k_T algo

PS starts at core interaction

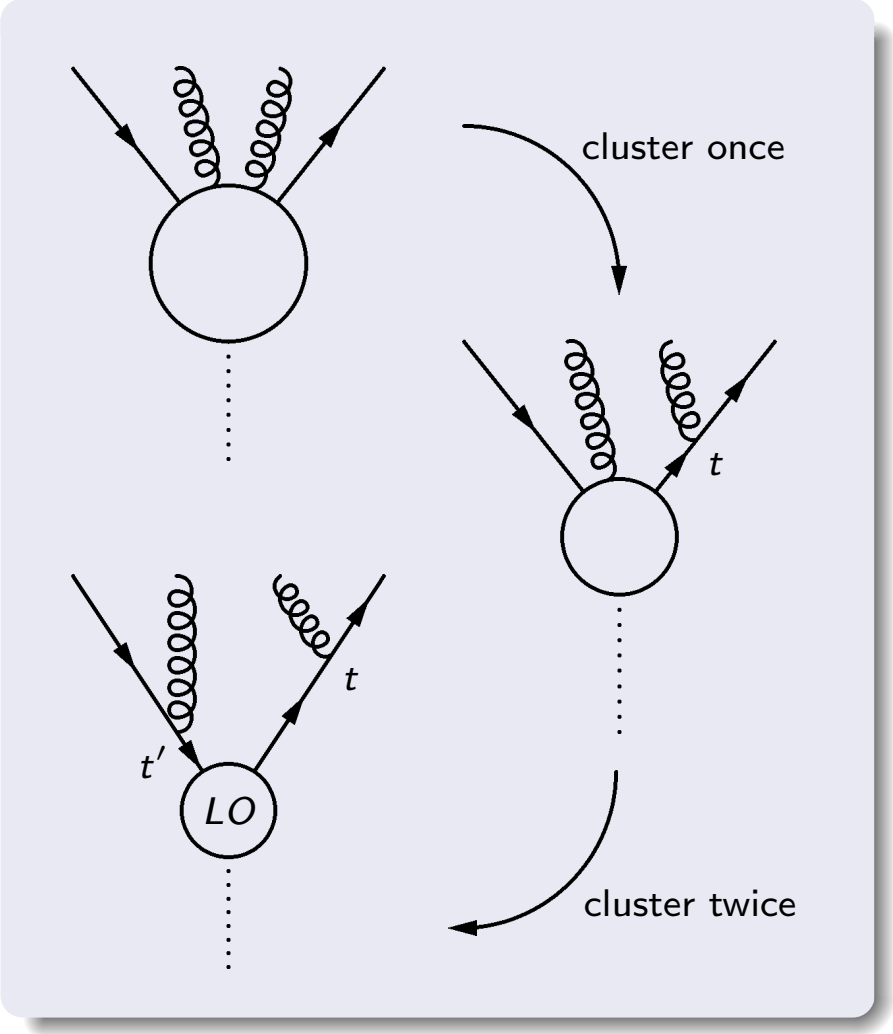
possibly radiates additional partons on intermediate lines "between" ME partons

ME branchings must be respected

i.e. t, z and ϕ must be preserved

$\to$ Truncated shower see next slide

universal concept for ME-PS merging



Tree-level calculations in the ME⊗PS approach

How to interpret $\mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$?

Assume predefined branchings at t and $t' > t$

$$\mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right]$$

means running a vetoed shower

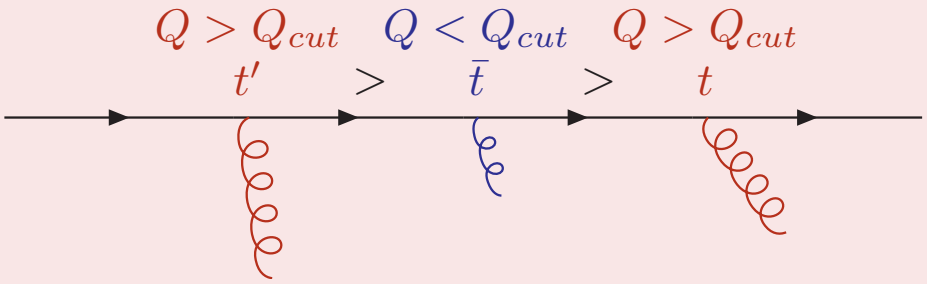
emission phase space is limited from above by Q_{cut}

$$\mathcal{P}^{(\mathrm{PS})\,\mathrm{PS}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$$

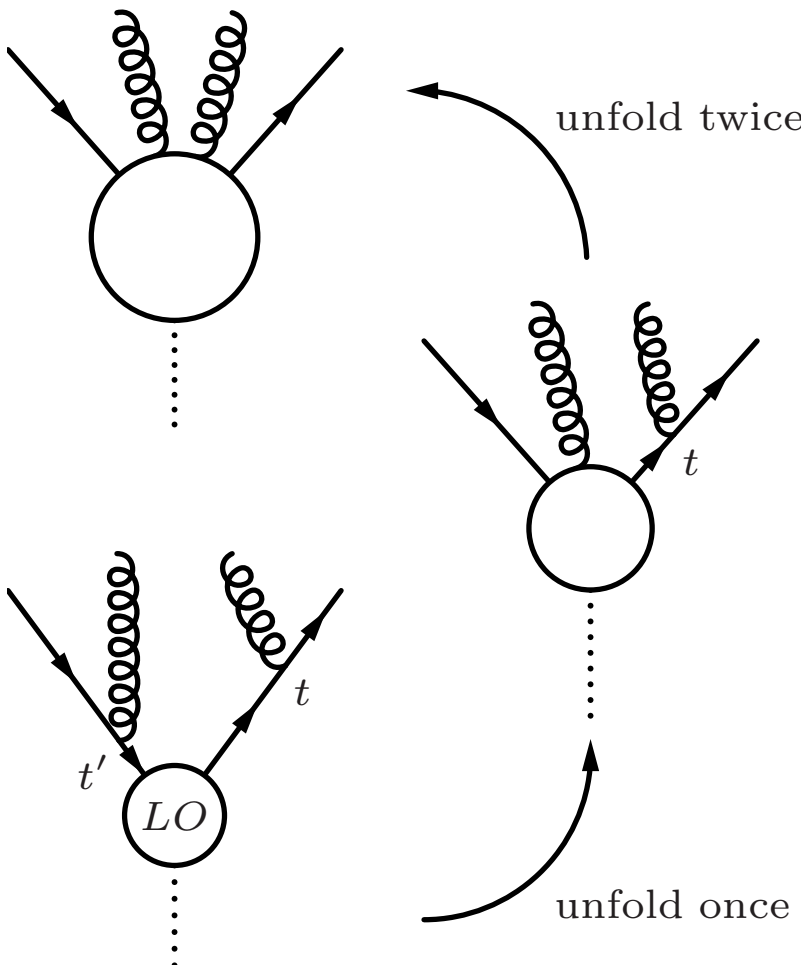
means running a truncated shower

t is larger than global shower cutoff t_0

What is the catch of it ?



Example branching history



Tree-level calculations in the ME⊗PS approach

How to interpret $\mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$?

Assume predefined branchings at t and $t' > t$

$$\mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right]$$

means running a vetoed shower

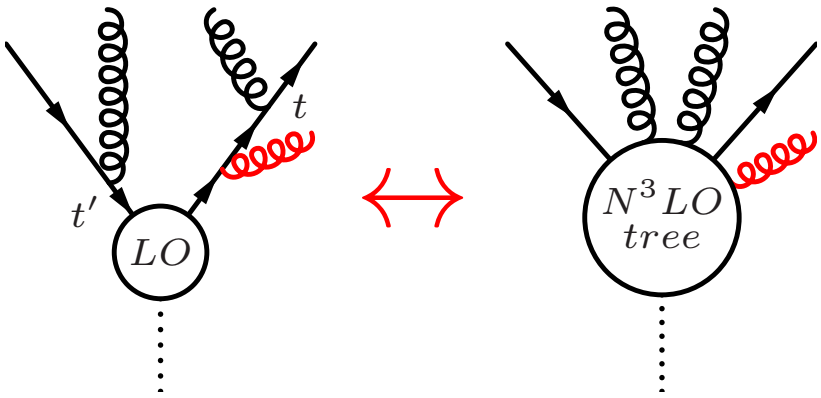
emission phase space is limited from below by Q_{cut}

$$\mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})$$

means running a truncated shower

t is larger than global shower cutoff t_0

Example emission



What happens if we emit something ?

Emission must be implemented to preserve full QCD evolution, i.e. $\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t,t')$

But we want matrix elements to take care of such emissions !

To avoid double-counting, the complete event must be rejected

Event is lost ⇒ rejection reduces initial cross section σ to $\sigma \cdot \mathcal{P}^{(\mathrm{PS})\,\mathrm{ME}}_{\widetilde{ij},\tilde{k}}(t,t')$

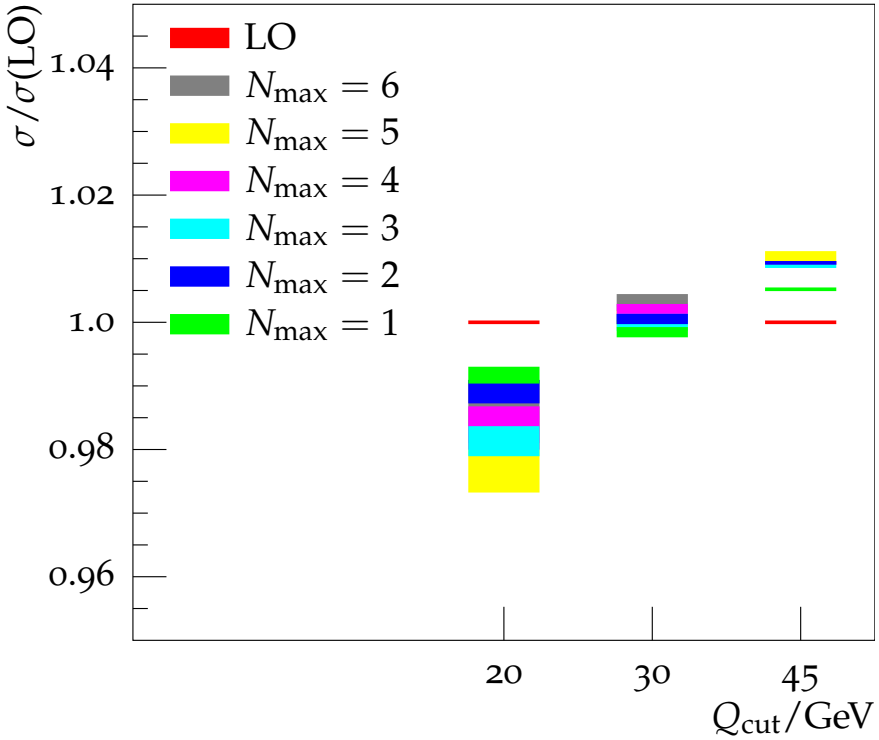
"Gap" is filled by higher order ME⊗PS ⇒ σ preserved up to PS↔ME mismatch

Tree-level calculations in the ME⊗PS approach

ME⊗PS-merging related uncertainties

- Choice of the jet criterion not discussed here
- Value of the phase-space separation cut, Q_{cut}
- Maximum number of jets from hard MEs, N_{max}, within reason
  m-jet observable usually not well described by N-jet ME plus PS if $N < m$

Example: DY-pair production @ Tevatron



pQCD-related uncertainties

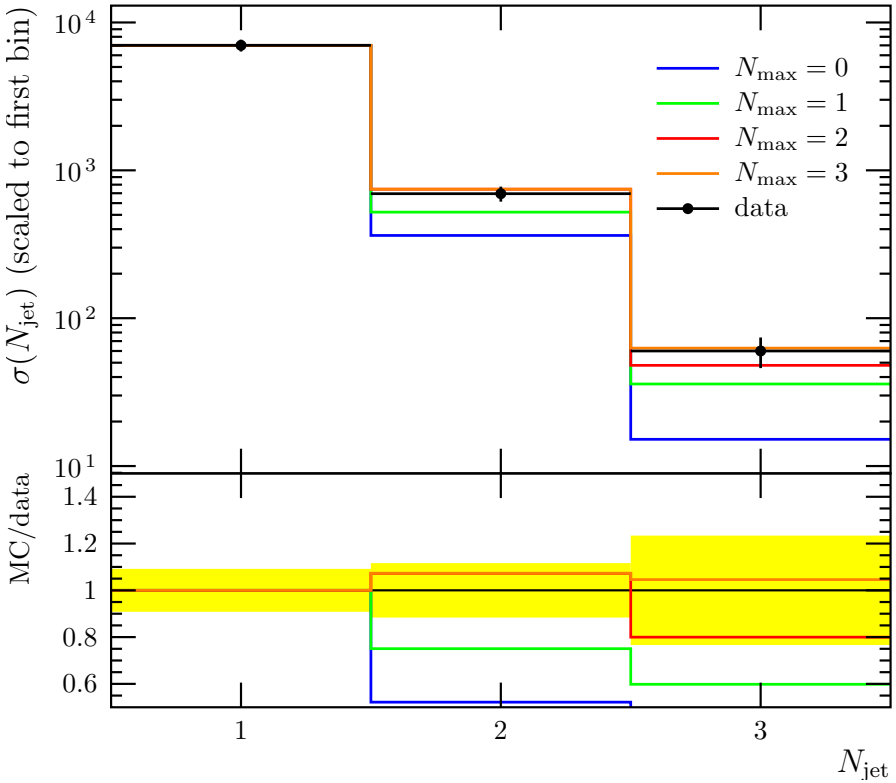
- Scale uncertainties from MEs
  Selection of "core process" and μ_F
- Scale uncertainties from PSs
  Selection of scale for running α_S

pQCD-npQCD transition

- IR-cutoff of the PS
- PDF & α_S uncertainties

Tree-level calculations in the ME⊗PS approach

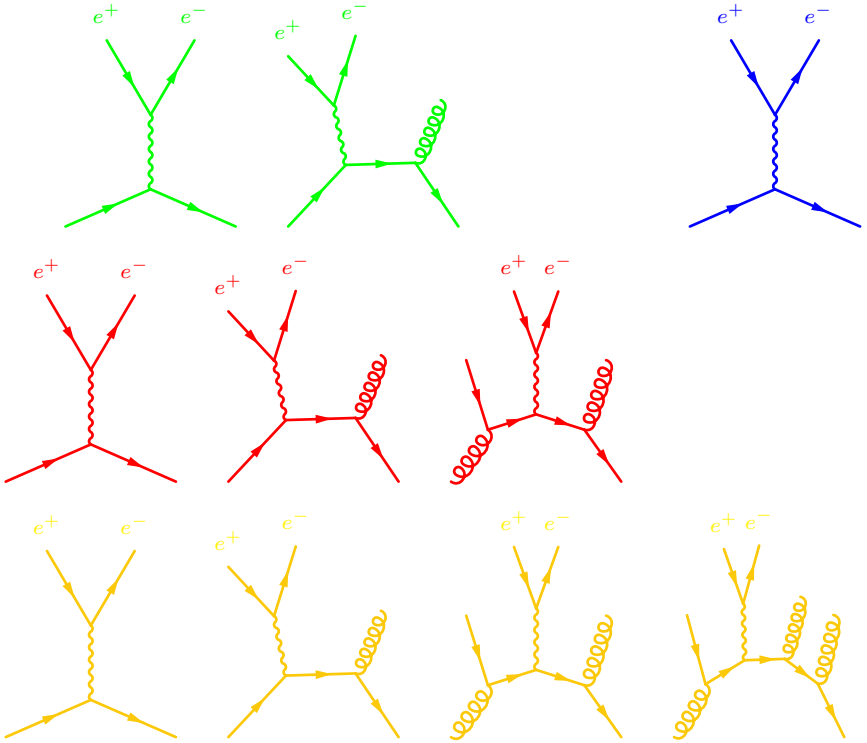
Example: DY-pair production
σ @ Tevatron JHEP05(2009)053



Consequence of the method:

Jet rates and -spectra improved compared to pure PS simulation
due to usage of NLO real ME's

Note: minor corrections to total cross section might still have big effect on rare events !



ME⊗PS ⊗ POWHEG: MENLOPS in a nutshell

Slice the POWHEG phase space in ME⊗PS-style and you have MENLOPS

$$
\begin{aligned}
\langle O\rangle^{(\mathrm{MENLOPS})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\,\bar{\mathrm{B}}(\{\vec{a}\}) \Bigg[& \underbrace{\mathcal{P}^{(\mathrm{ME})}(t_0,\mu^2;\{\vec{a}\})\,O(\{\vec{p}\})}_{\text{unresolved}} \\
& + \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, J_{ij,k}(t,z,\phi)\; O(r_{\widetilde{ij},\tilde{k}}(\{\vec{p}\})) \\
& \times \frac{1}{S_{ij}} \frac{S(r_{\widetilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})} \Bigg(\overbrace{\mathcal{P}^{(\mathrm{ME})}(t,\mu^2;\{\vec{a}\})\,\Theta\left[Q_{\mathrm{cut}} - Q_{ij,k}(t,z,\phi)\right]}^{\text{resolved, PS domain}} \\
& \qquad + \underbrace{\mathcal{P}^{(\mathrm{PS})}(t,\mu^2;\{\vec{a}\})\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\mathrm{cut}}\right]}_{\text{resolved, ME domain}} \Bigg) \Bigg]
\end{aligned}
$$

Note: Local K-factor $\bar{\mathrm{B}}/\mathrm{B}$ must be applied to ME⊗PS before merging

Overall characteristics

- Full NLO accuracy inherited from POWHEG → stable rates for core process
- Higher-order tree-level via ME⊗PS → improved multi-jet predictions
- Unitarity violation inherited from ME⊗PS not necessarily bad, see later !

Proposed independently in JHEP06(2010)039 & arXiv:1009.1127 [hep-ph]