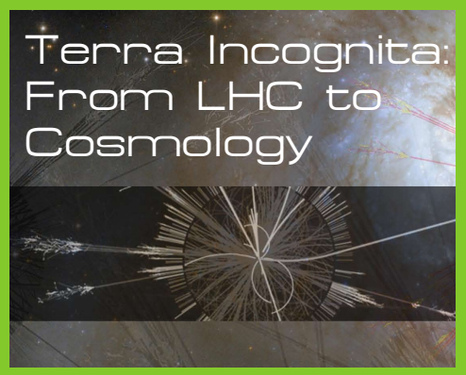
Dirac eigenvalues in nearly conformal gauge theories



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Terra Incognita, BNL, Nov 7 2008

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the bottom line

revived interest in new gauge theories - is a given theory conformal or not?

non-perturbative question: lattice role

eigenvalues of the Dirac operator know if chiral symmetry is spontaneously broken i.e. not conformal

can measure eigenvalues via lattice simulations

outline

- motivation for new gauge theories
- eigenvalues & chiral symmetry
- which theories to study?
- results to date
- outlook

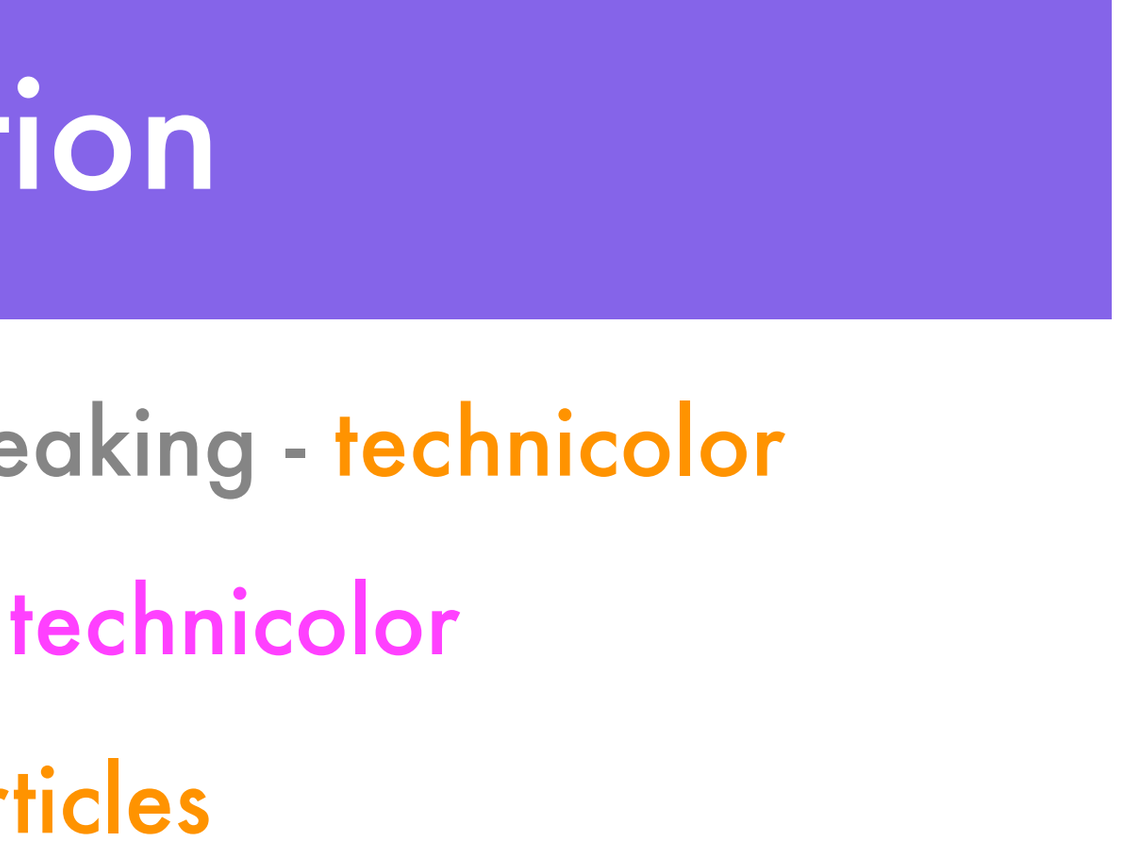
motivation

- origin of EW symmetry breaking - technicolor
- phenomenology - walking technicolor
- conformal theories - unparticles
- field theory challenge

conformal windows

representation, # colors, # flavors

Dietrich, Sannino



eigenvalues & chiral symmetry

Dirac operator D eigenvalue density $\rho(\lambda)$

Banks-Casher relation $\Sigma = -\langle\bar{\Psi}\Psi\rangle = \lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\frac{\pi\rho(\lambda)}{V}$

in finite volume, small eigenvalues closely packed

$$\Delta\lambda = \frac{1}{\rho(0)} = \frac{\pi}{\Sigma V}$$

extract condensate from eigenvalue spacing? can do much better

if chiral SB: tune volume and quark mass $\frac{1}{F_\pi} \ll L \ll \frac{1}{m_\pi}$

lighter than physical pion ϵ-regime (Gasser, Leutwyler)

crazy limit, theory dominated by finite-volume effects

random matrix theory

Chiral perturbation theory dominated by zero-momentum mode

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) + \frac{1}{2}\Sigma\ \mathrm{Tr}\left[M(U+U^\dagger)\right], \qquad U = \exp\left[\frac{i\pi^a T^a}{F_\pi}\right]$$

distributions of lowest eigenvalues identical to those of different theory

$$\mathcal{Z}_\nu(m_1,...,m_{N_f}) = \int dW e^{-\beta\ \mathrm{Tr}(W^\dagger W)} \prod_{i=1}^{N_f} \mathrm{Det}\begin{pmatrix} m_i & W \\ -W^\dagger & m_i \end{pmatrix}$$

$(N+\nu)\times N$ matrix

ν topology

random matrix theory (RMT): complete analytic control

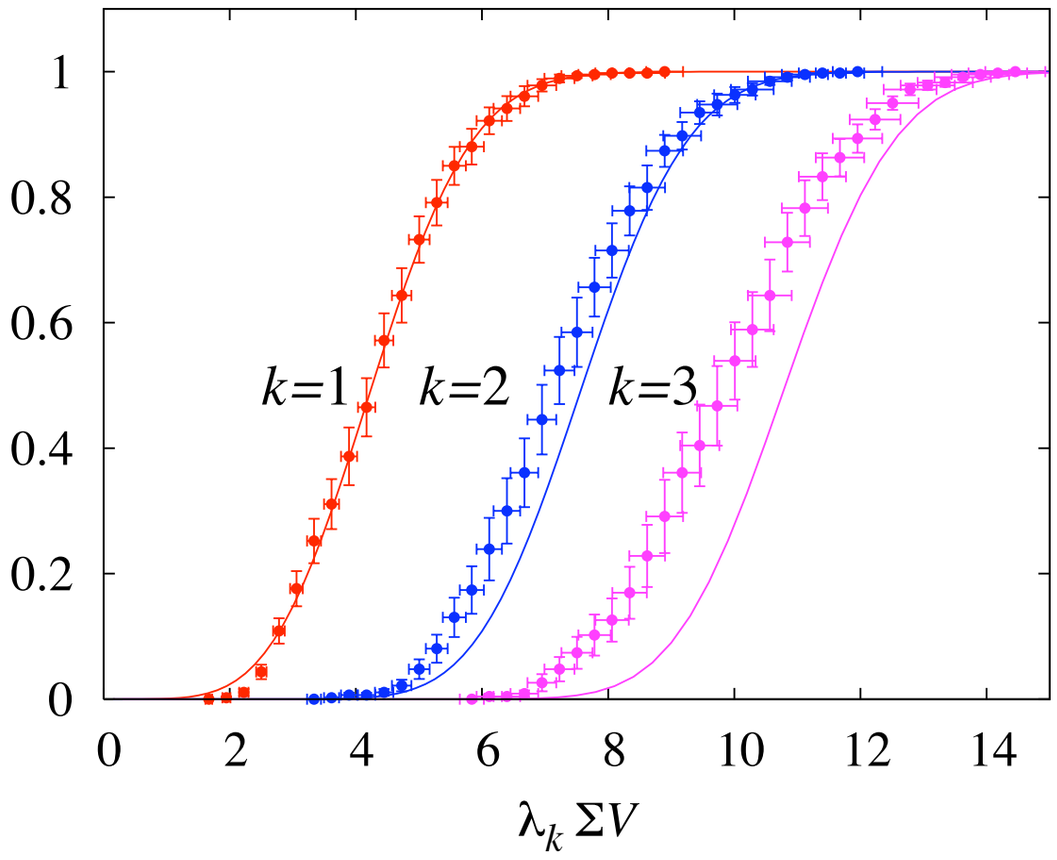
example: $N_f = 1, \nu = 0$

distribution 1st eigenvalue

$$p_1(z) = \frac{1}{2} z e^{-z^2/4}\left(\frac{I_2(\sqrt{z^2+\mu^2})}{I_0(\mu)}\right)$$

rescale mass, eigenvalues: $\mu = m\Sigma V, \quad z = \lambda \Sigma V$ dimensionless

lattice example



JLQCD, TWQCD (2007)

$SU(3)$ color

2 flavors, fundamental

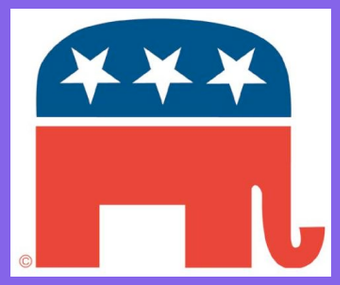
chiral fermion simulations expensive

integrated distributions $\int_0^z p_k(z')dz'$

fit Σ via $\dfrac{\langle \lambda_1 \rangle}{m} = \dfrac{\langle z_1 \rangle}{\mu}$ → predict distributions

lattice RMT

very good agreement with RMT, as expected - theory non-conformal



candidate theories

$SU(3)$ color

8,12 flavors, fundamental

- see crossing into conformal window?
- compare with other lattice studies
- cheap lattice simulations for $N_F = 4, 8, 12, 16, ...$

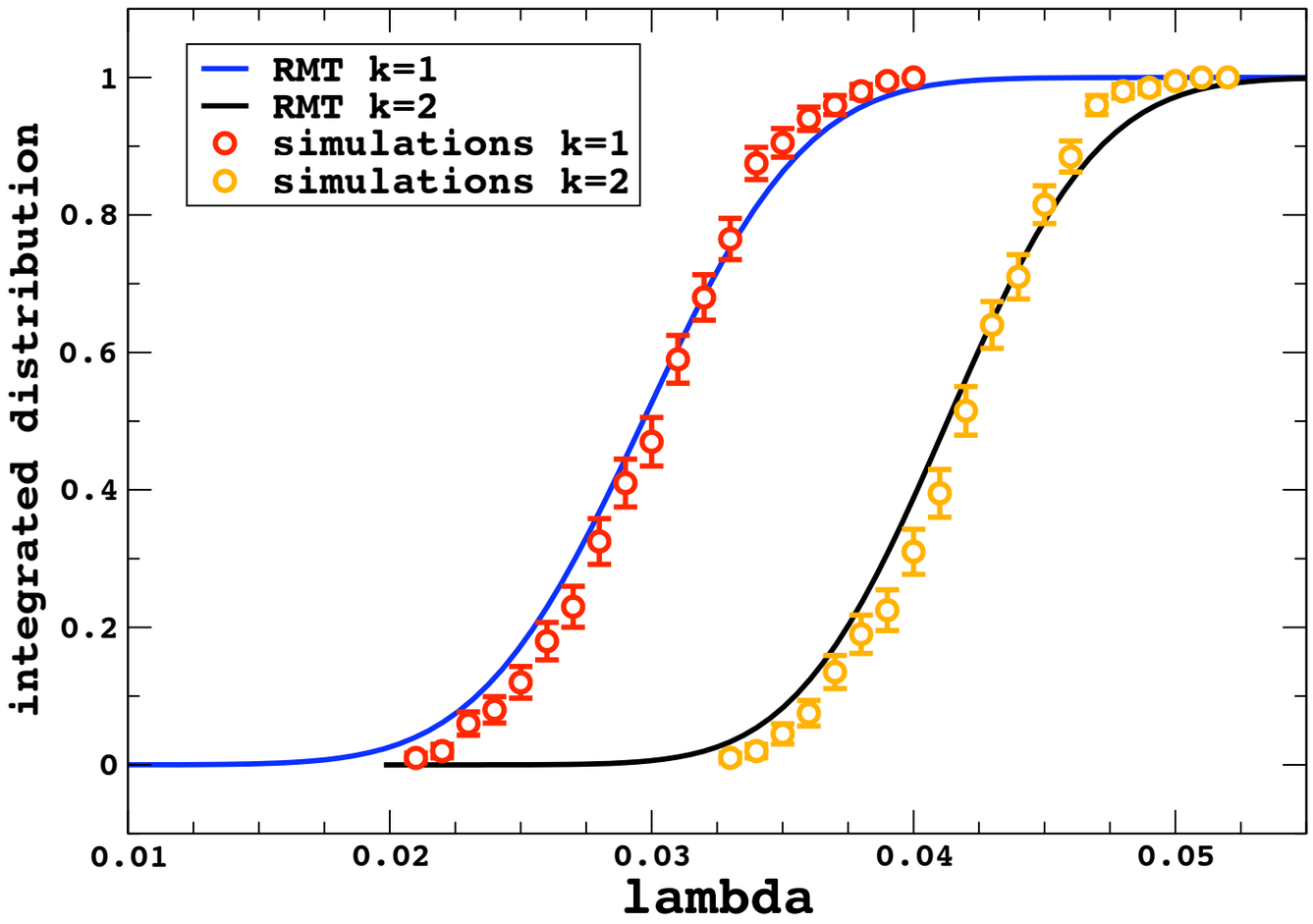
2 flavors, 2-index symmetric

- more realistic for phenomenology?

(walking, S parameter, # Goldstone bosons for W, Z)

- chiral fermion lattice simulation: very expensive

8 flavors, fundamental



improved staggered fermion

(Asqtad)

integrated distributions, k = 1,2

$$\int_0^z p_k(z')dz'$$

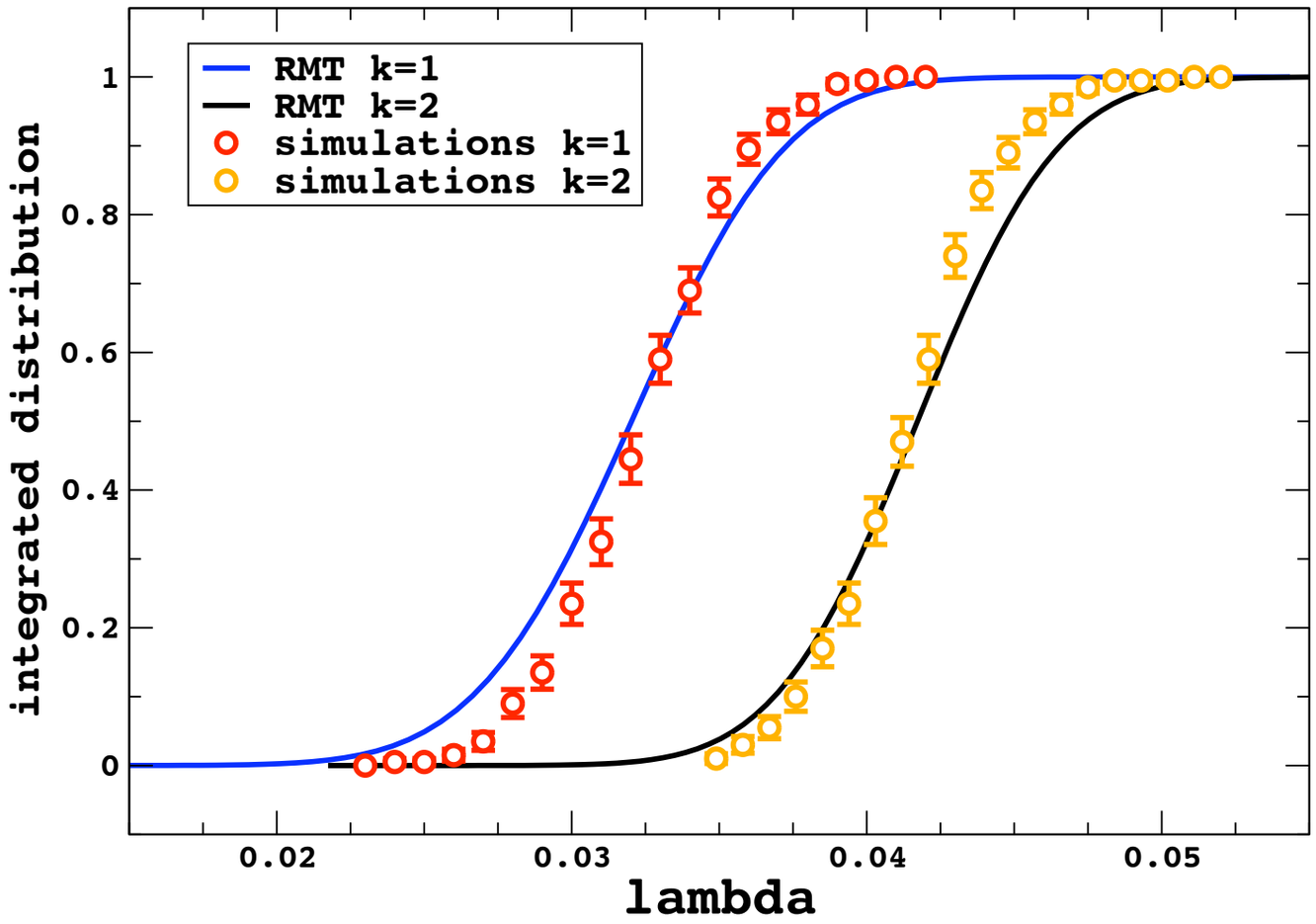
good agreement with RMT

looks like chiral symmetry spontaneously broken

theory looks non-conformal

consistent with Appelquist & co., Pallante & co.

12 flavors, fundamental



same lattice action

integrated distributions, k = 1,2

$$\int_0^z p_k(z')dz'$$

again, good agreement with RMT, looks like chiral symmetry broken

theory looks non-conformal

not consistent with Appelquist & co.

this is a surprise

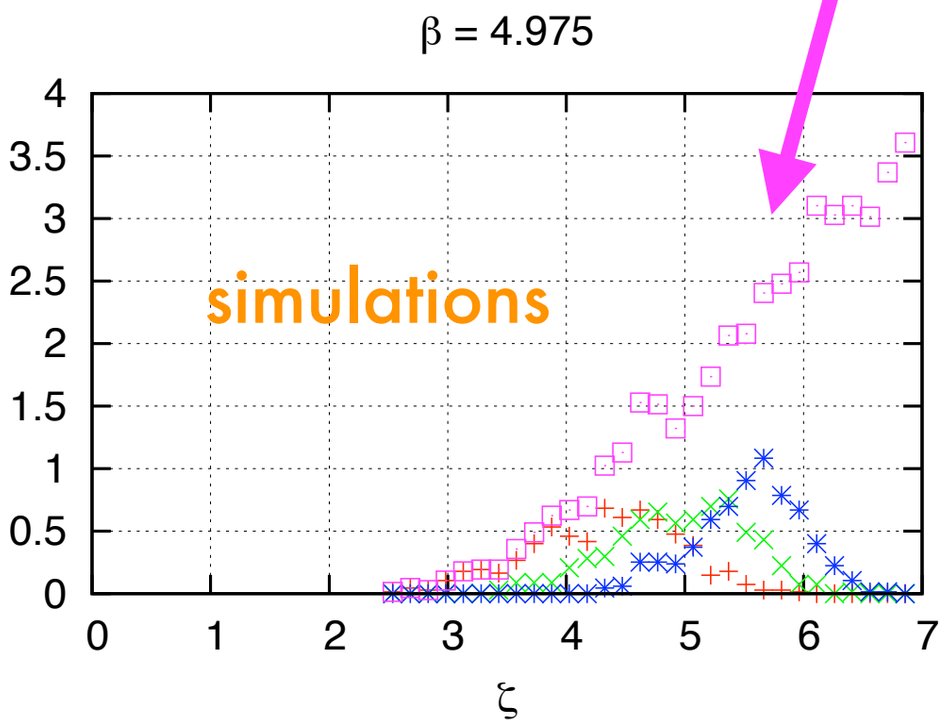
2 flavors, 2 index symmetric

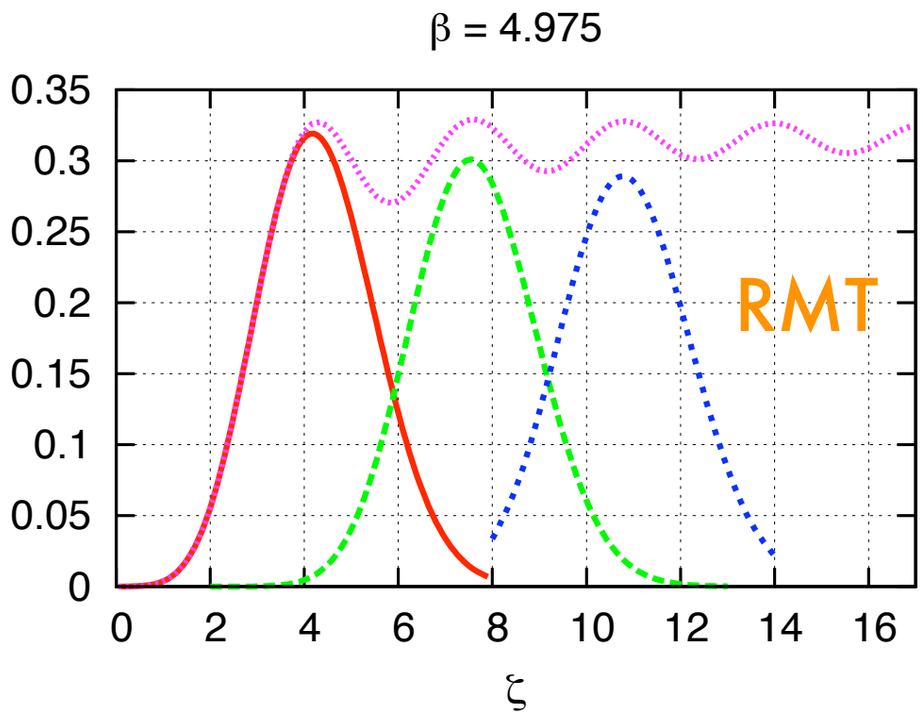
chiral fermion simulations - expensive

small volume 6^4

$SU(3)$ color

fit condensate, predict $\rho(z) = \sum_{k=1}^{\infty} p_k(z)$





no chiral SB, theory conformal? consistent with DeGrand, Svetitsky, Shamir

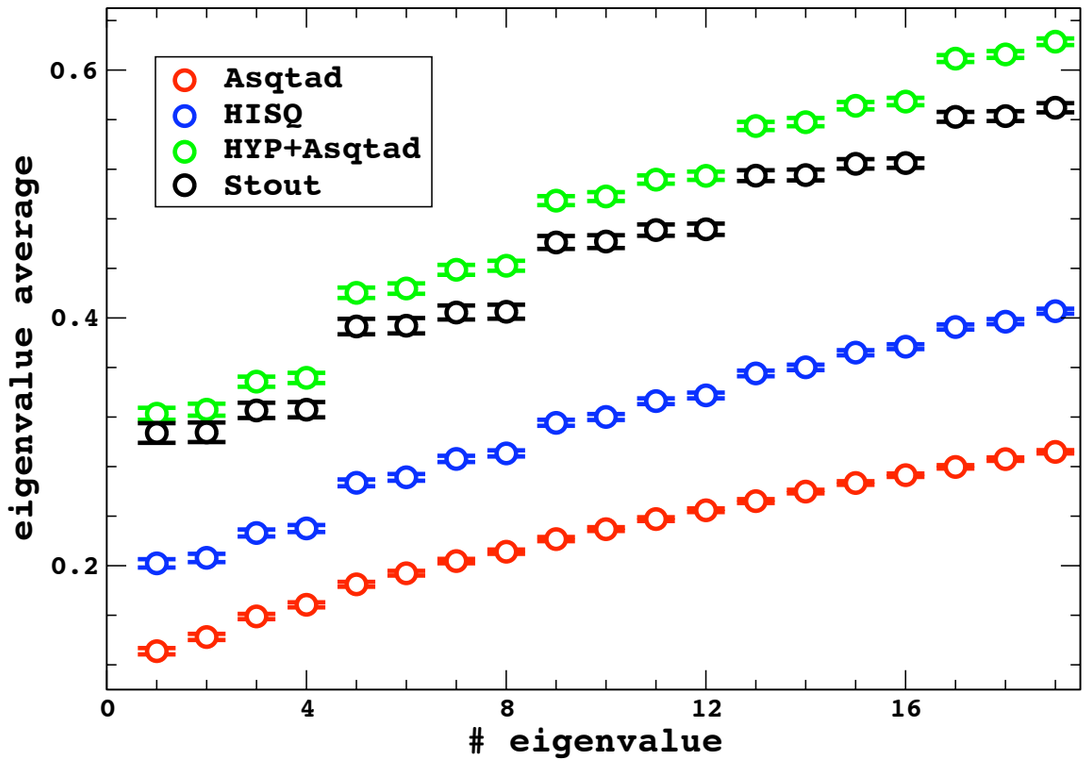
danger: physical volume too small? - force theory to be almost free

caveats

- Monte Carlo algorithm, quark mass, volume, statistics
- lattice artifacts: cheap method has flavor violations
- eigenvalue quartets form when flavor symmetry restored
- lesson: use further improved actions

various improved actions

Asqtad, HISQ,
HYP+Asqtad, Stout



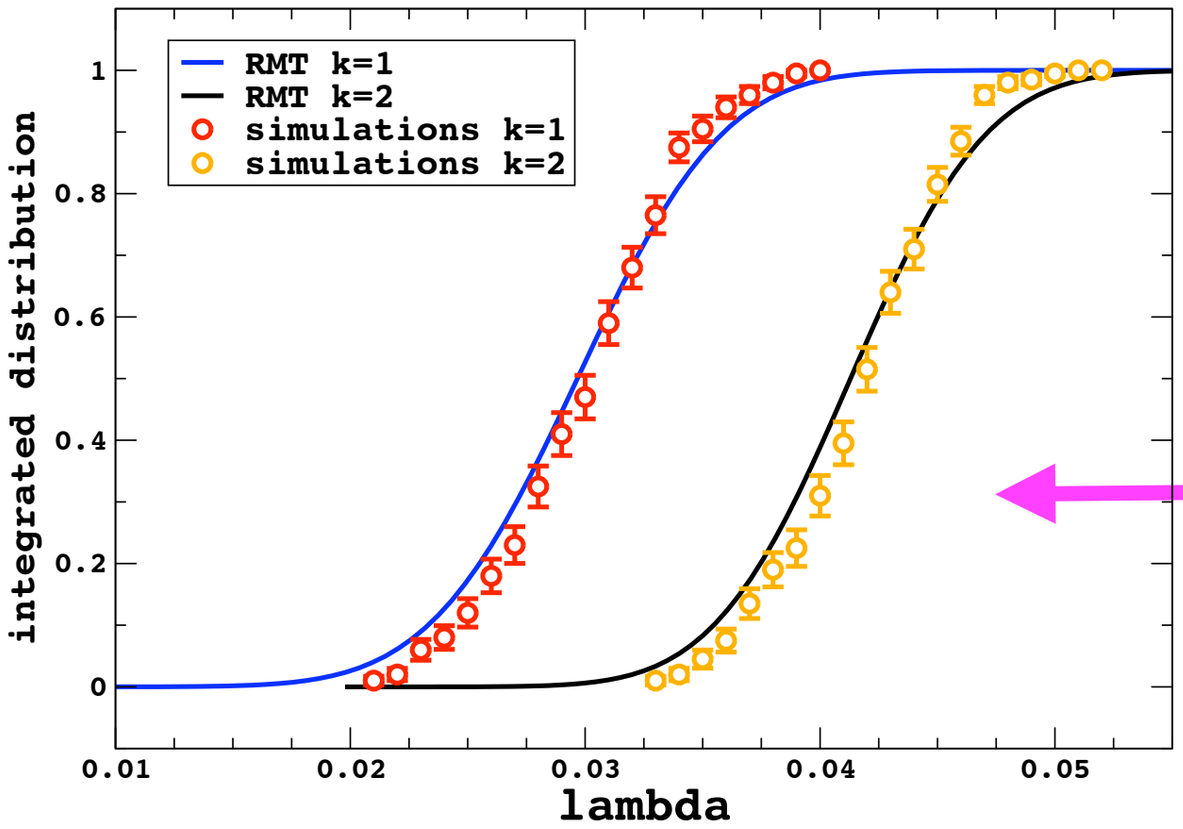
outlook

- eigenvalue method complements beta function, spectrum of masses, finite temperature transitions,...
- hint that $SU(3), N_F = 8, 12$ fundamental not conformal
- improved staggered action essential, ongoing
- $SU(3), N_F = 2$ 2-index Symmetric looks conformal
- expensive 2S simulations at larger volume needed
- developing quickly, large computational resources

many talks at Lattice 2008

NVIDIA graphics card

flavor breaking

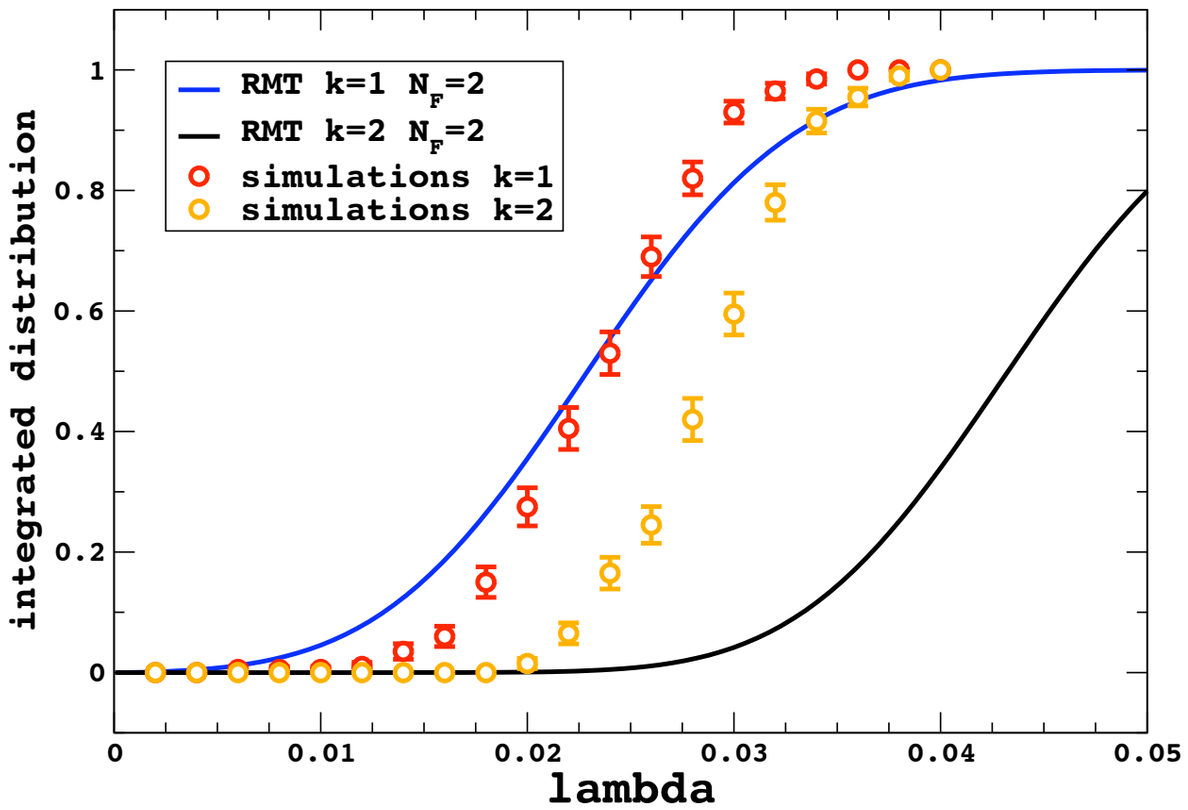


$N_F = 8$ fit good

$N_F = 2$ fit poor

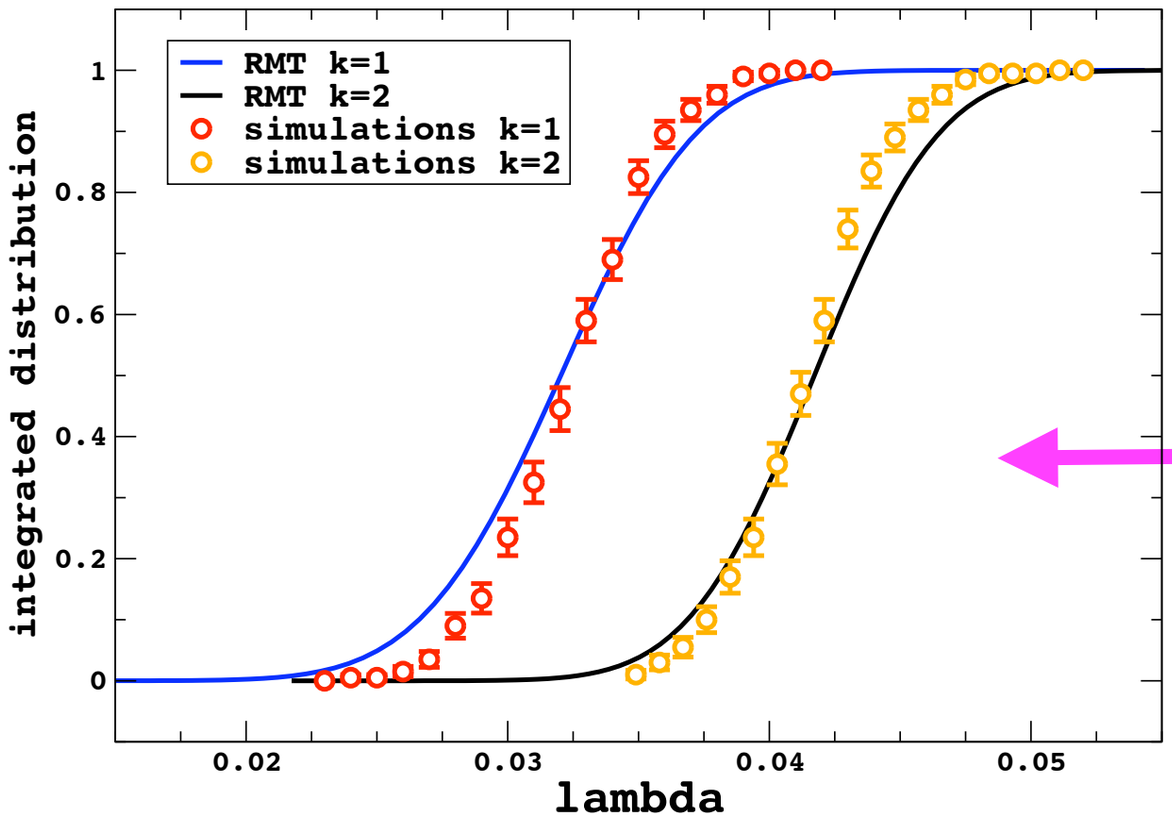
if flavor breaking large,
simulations should agree
with RMT with smaller # flavors

8 flavor simulations



flavor breaking appears to be small

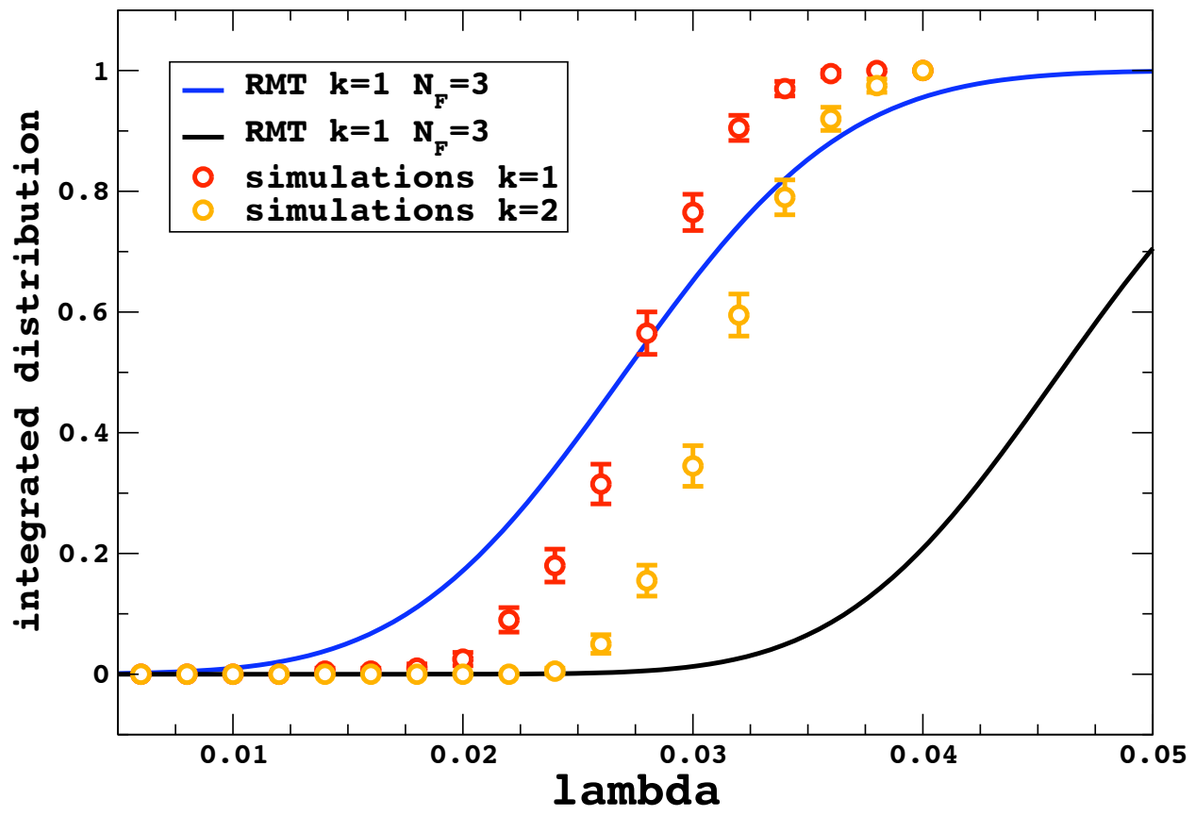
flavor breaking



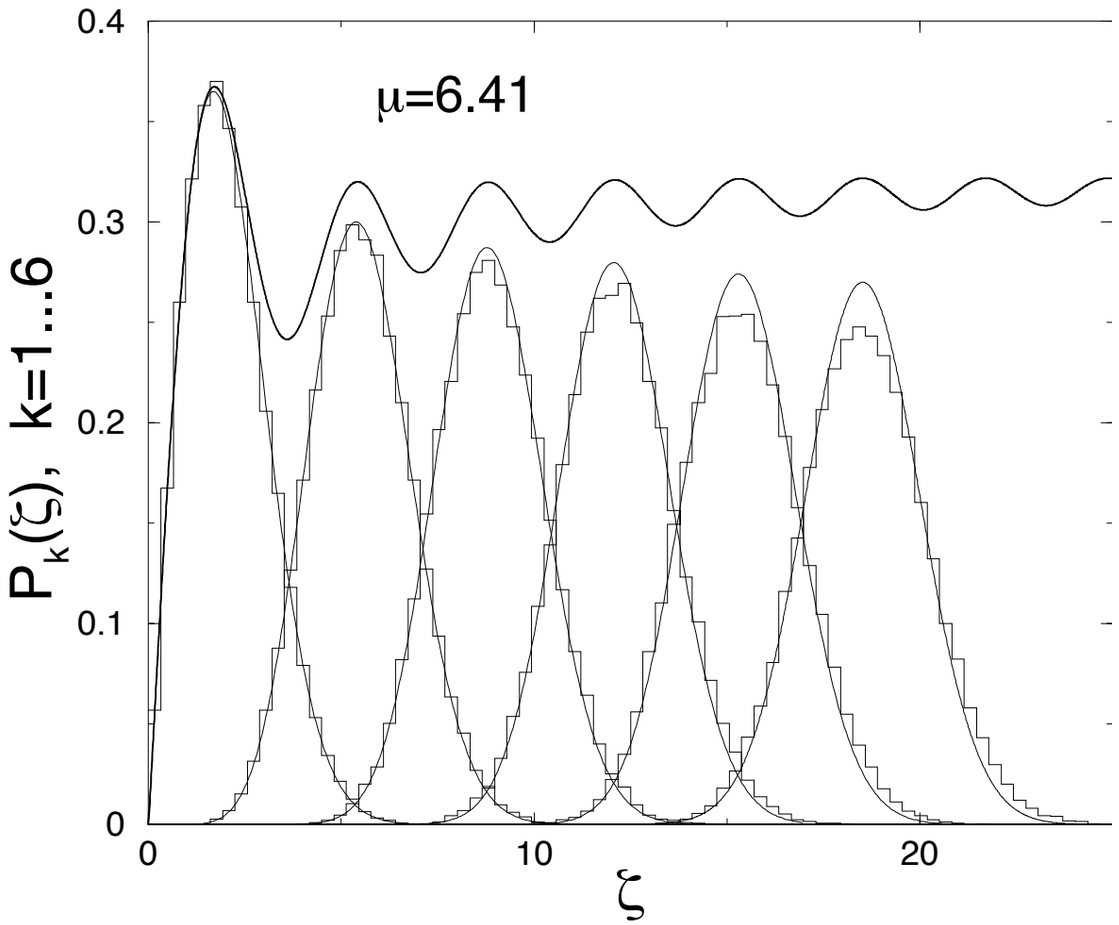
$N_F = 12$ fit good

flavor breaking also appears to be small for 12 flavor simulations

$N_F = 3$ fit poor



flavor breaking danger



Damgaard et al (2000)

$SU(3)$ color

1 staggered flavor, fundamental

dynamical simulations

lowest 6 eigenvalue distributions

superb agreement
with $N_F = 1$ RMT NOT $N_F = 4$

flavor symmetry badly broken - can only tune 1 pion to be light

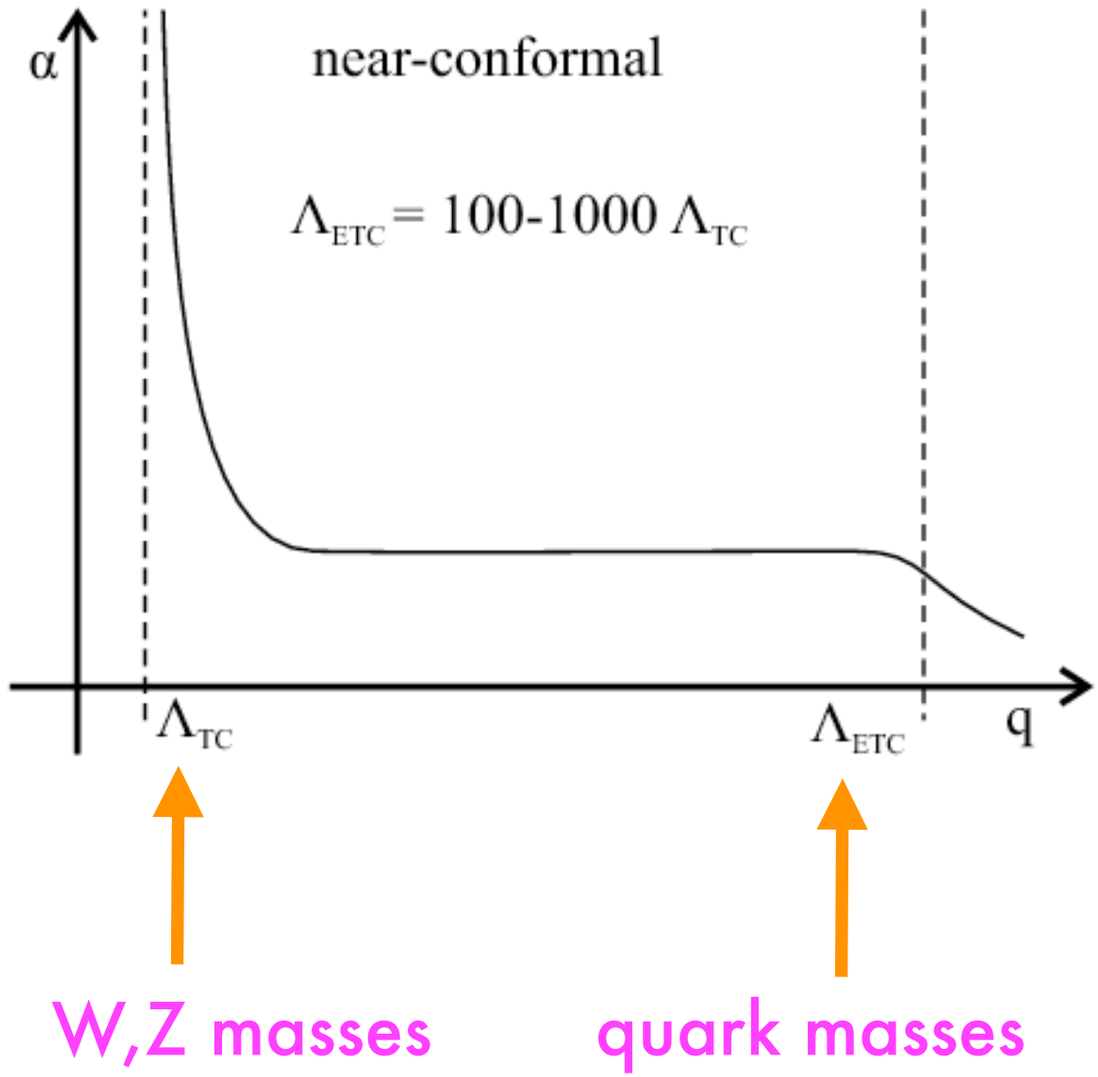
want # light flavors same as in continuum theory - beware artifacts

walking technicolor

if coupling walks, separate scales
fix FCNC's
light composite Higgs?

techniquark fundamental rep.
need large N_F

bad for EW precision



lattice fermions

Wilson

add term to action, doublers infinitely heavy

break chiral symmetry explicitly

Staggered

use only 1/4 of Dirac spinor components

4 flavors in continuum (taste)

flavor symmetry badly broken at finite lattice spacing

fractional power of determinant for e.g. 2 flavors

Chiral (overlap)

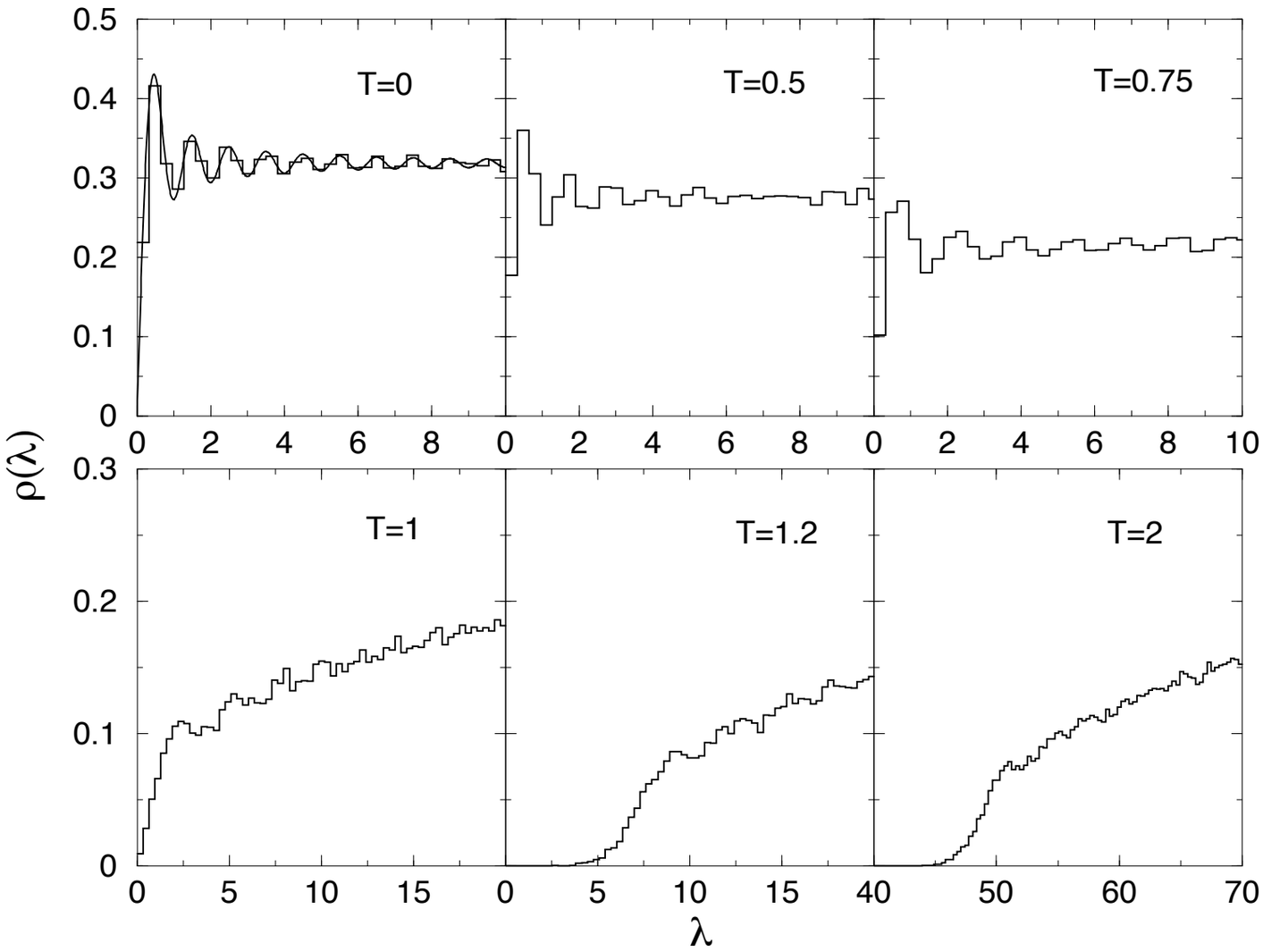
exact chiral symmetry on lattice

arbitrary number flavors possible

$D_{xy} \neq 0 \ \ \forall \, x, y$ lattice positions

Monte Carlo simulations much more difficult

finite temperature eigenvalues



Damgaard et al (2000)

pure Yang-Mills simulations

eigenvalue density of staggered Dirac operator

eigenvalue density smoothly changes as condensate vanishes at high T