Proton Decay in (a minimal) Non-SUSY $SO(10)$ Model

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Overview

1. Introduction
2. Motivation
3. Building the Model
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6. Detailed Analysis of Higgs Potential
7. Fermion Masses, Mixings and Leptogenesis
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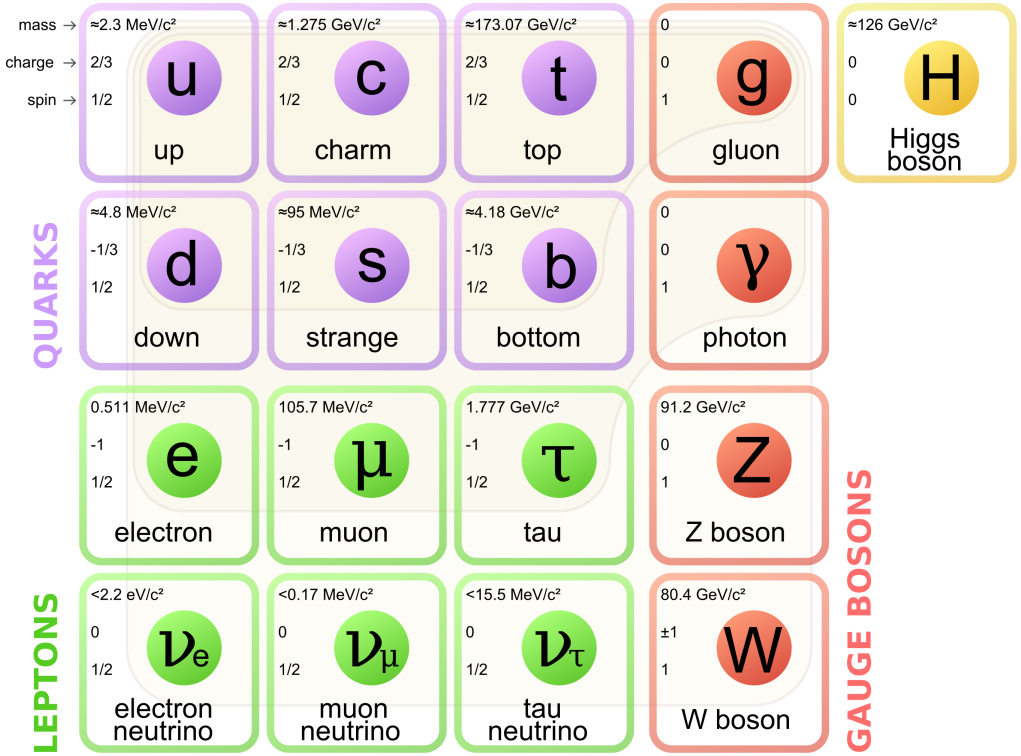
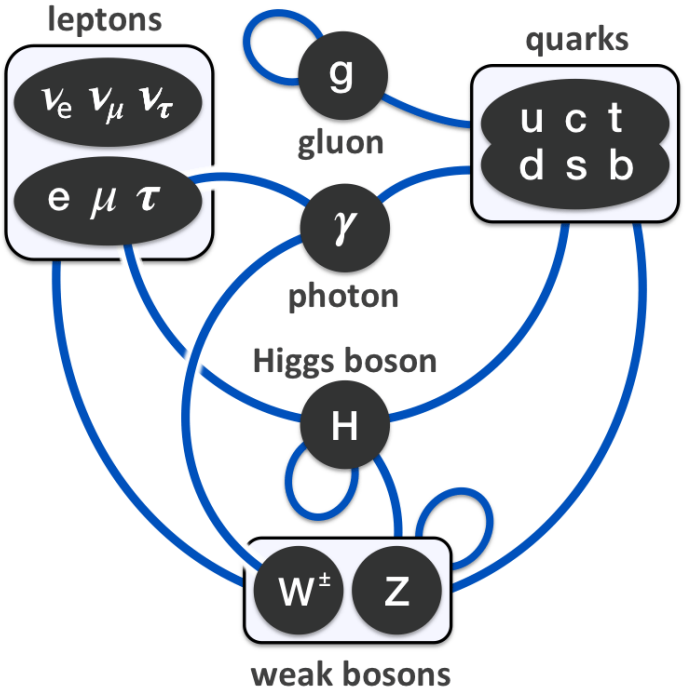
Standard Model of Particle Physics



Standard Model of Elementary Particles(by MissMJ - Wikipedia);



Elementary particle interactions in the SM(by Eric Drexler);

Unanswered Questions:

- **Charge Equality** : $|1 + \frac{Q_e}{Q_p}| < 10^{-21}$
- **Strong CP Problem**: Strong interaction sector admits a CP violating term, leading to a physical observable $\overline{\theta}$. Neutron Electric Dipole Moment limits $\overline{\theta} < 10^{-10}$.
- **Dark Matter**: The total massenergy of the known universe contains 4.9% ordinary matter, 26.8% dark matter (arXiv:1303.5062). Yet SM does not have any candidate for Dark Matter.
- **Neutrino Oscillation**: Neutrinos of different flavor (ν_e, ν_μ, ν_τ) can oscillate into each other due to non-zero neutrino mass and mixing angles. Flavor eigenstates of neutrinos are linear combination of field of three (or more) neutrinos (ν_j) with non-zero mass.
- **Stability of Higgs potential**: In SM, the Higgs Quartic Coupling becomes negative around 10^{11} GeV. This instabity in the electro-weak vacuum indicates that we might be living in a metable universe.

$SO(10)$ Grand Unification

- $SO(10)$ is a group of rank 5 with the extra diagonal generator of $SO(10)$ being $B - L$ as in the left-right symmetric groups. So, the gauge interactions of $SO(10)$ conserve parity thus making parity a part of a continuous symmetry.
- 16-dimensional spinor representation of $SO(10)$ can accommodate **ALL** fermions of one generation

$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

The first 3 indicates color spin and last two weak spin.

$$Y = \frac{1}{3}\sum(C) - \frac{1}{2}\sum(W)$$

- Unification of three couplings (α_s, α_{2L} and α_Y) into one coupling constant α_{GUT} even in Non-SUSY scenario with the help of an intermediate scale.
- Existence of ν_R and thus neutrino mass via seesaw.
- Baryon asymmetry.

Breaking of Non-SUSY $SO(10)$

One Step Breaking chains:

$$
\begin{aligned}
\text{chain 1a}: G_I &= \{2_L 2_R 4_C\}, \\
\text{chain 1b}: G_I &= \{2_L 2_R 4_C \times P\}, \\
\text{chain 2a}: G_I &= \{2_L 2_R 1_X 3_c\}, \\
\text{chain 2b}: G_I &= \{2_L 2_R 1_X 3_c \times P\}, \\
\text{chain 3}: G_I &= \{2_L 1_R 4_C\}, \\
\text{chain 4}: G_I &= \{2_L 1_R 1_X 3_c\},
\end{aligned}
$$

Chain	G_I	$\log_{10}(M/1\text{ GeV})$		ω_U
		M_I	M_U	
1a	$2_L 2_R 4_C$	10.75	16.3 ± 0.3	45.9 ± 0.6
1b	$2_L 2_R 4_C \times P$	13.65	15.1 ± 0.4	40.7 ± 0.5
2a	$2_L 2_R 1_X 3_c$	8.7	16.6 ± 0.3	46.2 ± 0.4
2b	$2_L 2_R 1_X 3_c \times P$	10.0	15.6 ± 0.3	43.7 ± 0.4
3	$2_L 1_R 4_C$	11.0	14.5 ± 0.2	44.3 ± 0.4

Deshpande, Keith, Pal (1992)

Breaking of Non-SUSY $SO(10)$

Two Step Breaking chains:

$$
\begin{aligned}
\text{I}: &\quad SO(10) \xrightarrow[210]{} \{2_L 2_R 4_C\} \xrightarrow[45]{} \{2_L 2_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{II}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[210]{} \{2_L 2_R 1_X 3_c P\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{III}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[45]{} \{2_L 2_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{IV}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 1_X 3_c P\} \xrightarrow[210]{} \{2_L 2_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{V}: &\quad SO(10) \xrightarrow[210]{} \{2_L 2_R 4_C\} \xrightarrow[45]{} \{2_L 1_R 4_C\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{VI}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[45]{} \{2_L 1_R 4_C\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{VII}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[210]{} \{2_L 2_R 4_C\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{VIII}: &\quad SO(10) \xrightarrow[45]{} \{2_L 2_R 1_X 3_c\} \xrightarrow[45]{} \{2_L 1_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{IX}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 1_X 3_c P\} \xrightarrow[45]{} \{2_L 1_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{X}: &\quad SO(10) \xrightarrow[210]{} \{2_L 2_R 4_C\} \xrightarrow[210]{} \{2_L 1_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
\text{XI}: &\quad SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[210]{} \{2_L 1_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\} \\
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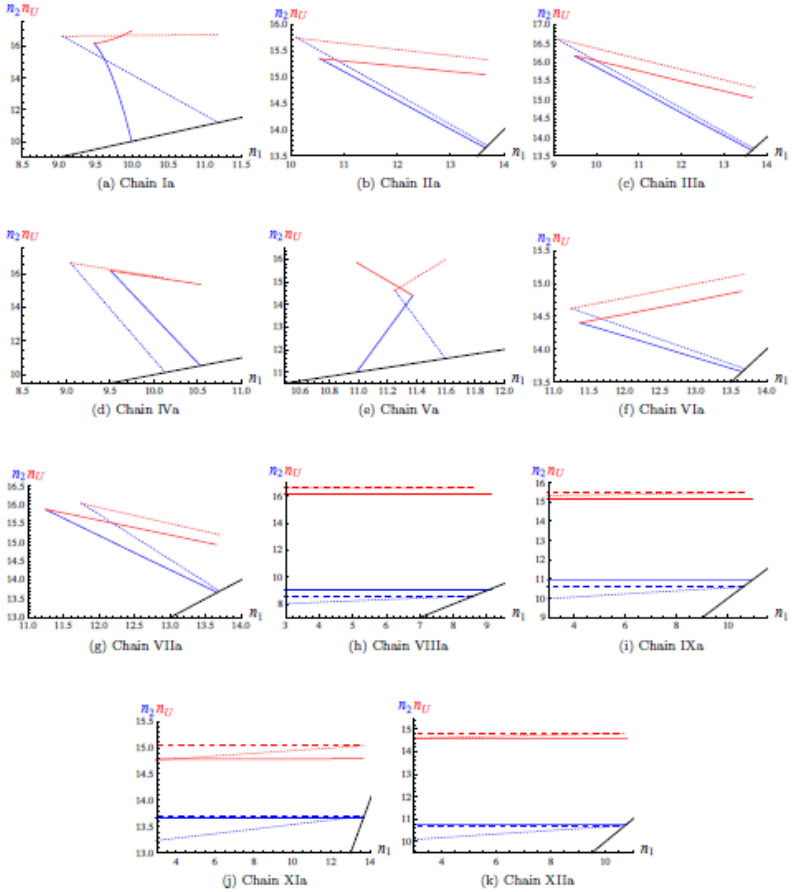
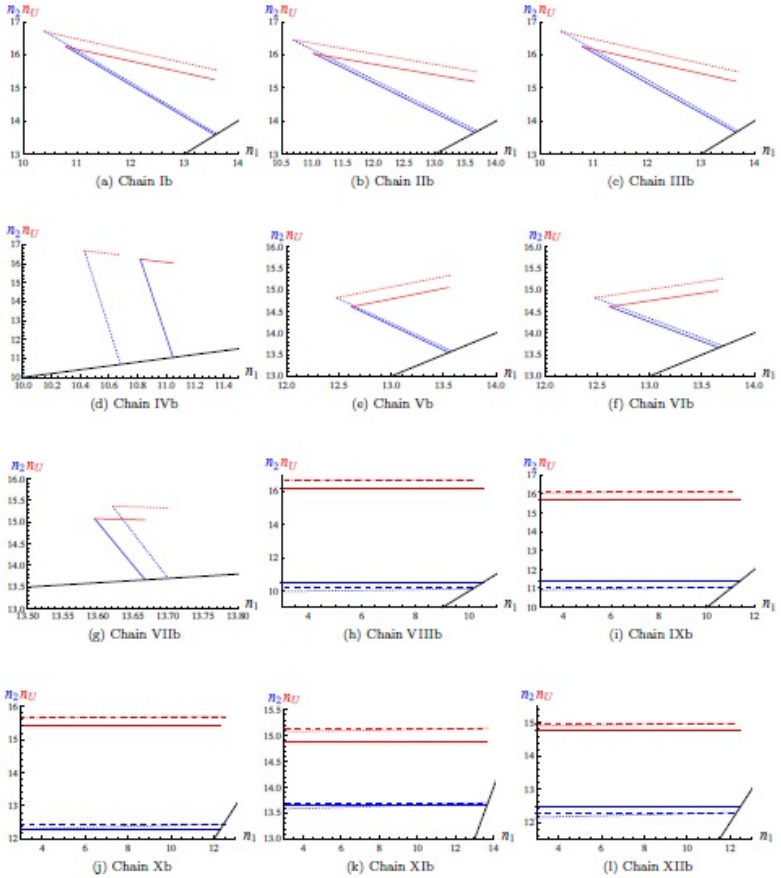
\end{aligned}
$$

Deshpande, Keith, Pal (1992)
Bertolini, Luzio, Malinsky (2009)

Breaking of Non-SUSY $SO(10)$

Two Step Breaking chains:





Bertolini, Luzio, Malinsky (2009)

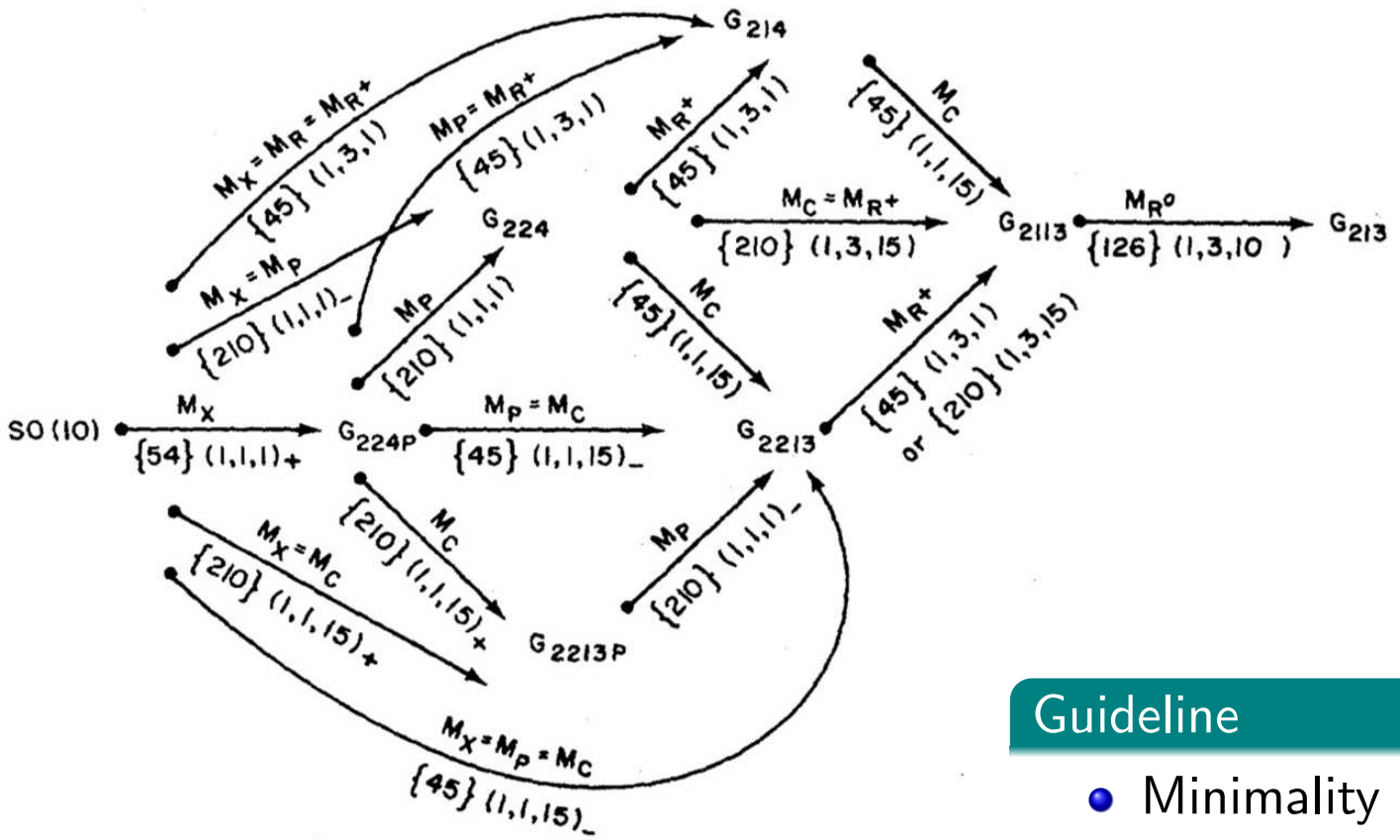
Motivation

Looking for a Minimal Realistic Unified Model with the properties:

- Unification of the coupling happens at large enough energy scale which is compatible with the proton-lifetime
- Some kind of particle spectrum which can modify the higgs quartic coupling so that stability issue of the electro weak vacuum can be addressed
- A realistic Yukawa sector which can generate realistic fermion masses and Mixings including neutrino data
- Can also generate the baryon asymmetry (most probably via leptogenesis)
- An axion suitable to so solve Strongs CP problem and account for the observed Dark Matter.

G. Altarelli and D. Meloni, 2013
Malinsky et al, 2011, 2012, 2013

$SO(10) \to \ldots \to$ SM



Guideline

- Minimality
- Predictivity
- Phenomenological issues

Chang, Mohapatra, Gipson,Marshak,Parida 1985.
Pal, Keith Deshpande 1993.
Bertolini, Luzio, Malinsky 2009

Possible Higgs Sector

Let's try:

- $45 + 16$ or $126 \rightarrow$ tends to go through $SU(5)$ breaking channel. If it is forced to go through L-R symmetric channel, one gets tachyonic masses.(1) Quantum salvation of these type of model has been shown by assuming that the loop level contribution to the Higgs masses surpass the tree level ones.(2)

  (1) Yasue 1981, Anastaze, Buccella 1983, Babu,Ma 1985, (2) Bertolini,Luzio, Malinsky 2010

- $54 + 16 \rightarrow$ 54 and 16 do not have any nontrivial cross couplings, so the global symmetry is SO(10) x SO(10). When this symmetry breaks down, there are Goldstone bosons, belonging to $\{(3, 2, 1/6) + h.c.\}$

- $54 + 126 \rightarrow$ possible candidate!!!

The Model

$$SO(10) \xrightarrow{\text{Real} \quad 54} SU(4)_C \times SU(2)_L \times SU(2)_R \times D$$

$$SU(4)_C \times SU(2)_L \times SU(2)_R \times D \xrightarrow{\text{Complex} \quad 126} SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{Complex} \quad 10} SU(3)_C \times U(1)_Q$$

Higgs Mass Scale

Extended Survival Hypothesis(ESH): Higgs acquire the maximum mass compatible with the symmetry breaking.[1] A more relaxed version of the hypothesis states "Higgs multiplet remains at the maximum energy scale compatible with the symmetry breaking and phenomenological constraints."

- $54 = (1,3,3) + (20',1,1) + (6,2,2) + (1,1,1)$
  - $< 54 >$ breaks $SO(10)$ $\Rightarrow$ All the components of 54 **@** M_U.
- $126 = (10,1,3) + (\overline{10},3,1) + (15,2,2) + (6,1,1)$
  - $< (10,1,3) >$ breaks $PS \times D$ $\Rightarrow$ $(10,1,3)$ **@** M_i.
  - D-parity $\Rightarrow$ $(\overline{10},3,1)$ **@** M_i.
  - Realistic fermion mass spectrum $\Rightarrow$ $(15,2,2)$ **@** M_i.
  - Detailed potential analysis $\Rightarrow$ $(6,1,1)$ **@** M_i.
- $10 = (1,2,2) + (6,1,1) = (1,2,+1/2) + (1,2,-1/2) + (3,1,-1/3) + (\overline{3},1,+1/3)$
  - $< (1,2,-1/2) >$ breaks EW $\Rightarrow$ $(1,2,-1/2)$ **@** M_w
  - ESH $\Rightarrow$ all other **@** Higher Scale

[1] H. Georgi (1979), F. del Aguila and L. E. Ibanez (1981), R. N. Mohapatra and G. Senjanovic(1983)

The Higgs Sector

SO(10)	$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$SU(5) \times U(1)_X$	Scale
10	$H_1(6,1,1)$	$T_1(3,1,+\frac{1}{3})$	$(5,+2)$	M_U
		$T_2(\bar{3},1,-\frac{1}{3})$	$(\bar{5},-2)$	M_U
	$H_2(1,2,2)$	$H(1,2,+\frac{1}{2})$	$(5,+2)$	M_W
		$R(1,2,-\frac{1}{2})$	$(\bar{5},-2)$	M_I
54	$\zeta_1(1,3,3)$	$\phi_1(1,3,+1)$	$(15,4)$	M_U
		$\phi_2(1,3,0)$	$(24,0)$	M_U
		$\phi_3(1,3,-1)$	$(\overline{15},-4)$	M_U
	$\zeta_2(6,2,2)$	$\phi_4(3,2,+\frac{5}{6})$	$(24,0)$	M_U
		$\phi_5(3,2,-\frac{1}{6})$	$(15,4)$	M_U
		$\phi_6(\bar{3},2,+\frac{1}{6})$	$(15,4)$	M_U
		$\phi_7(\bar{3},2,-\frac{5}{6})$	$(24,0)$	M_U
	$\zeta_3(20',1,1)$	$\phi_8(\bar{6},1,-\frac{2}{3})$	$(\overline{15},-4)$	M_U
		$\phi_9(6,1,+\frac{2}{3})$	$(15,4)$	M_U
		$\phi_{10}(8,1,0)$	$(24,0)$	M_U
	$\zeta_0(1,1,1)$	$\phi_0(1,1,0)$	$(24,0)$	M_U

Contd...

SO(10)	$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$SU(5) \times U(1)_X$	Scale
126	$\Sigma_1(6, 1, 1)$	$\Sigma_{11}(3, 1, +\frac{1}{3})$	$(45, 2)$	M_U
		$\Sigma_{12}(\bar{3}, 1, -\frac{1}{3})$	$(\bar{5}, -2)$	M_U
	$\Sigma_2(10, 3, 1)$	$\Sigma_{21}(1, 3, +1)$	$(\overline{15}, 6)$	M_I
		$\Sigma_{22}(3, 3, +\frac{1}{3})$	$(45, 2)$	M_I
		$\Sigma_{23}(6, 3, -\frac{1}{3})$	$(\overline{50}, 2)$	M_I
	$\Sigma_3(\overline{10}, 1, 3)$	$\Sigma_{31}(1, 1, 0)$	$(1, -10)$	M_I
		$\Sigma_{32}(1, 1, -1)$	$(10, -6)$	M_I
		$\Sigma_{33}(1, 1, -2)$	$(\overline{50}, -2)$	M_I
		$\Sigma_{34}(\bar{3}, 1, -\frac{4}{3})$	$(10, -6)$	M_I
		$\Sigma_{35}(\bar{3}, 1, -\frac{1}{3})$	$(\overline{50}, -2)$	M_I
		$\Sigma_{36}(\bar{3}, 1, +\frac{2}{3})$	$(45, 2)$	M_I
		$\Sigma_{37}(\bar{6}, 1, +\frac{4}{3})$	$(\overline{50}, -2)$	M_I
		$\Sigma_{38}(\bar{6}, 1, +\frac{1}{3})$	$(45, 2)$	M_I
		$\Sigma_{39}(\bar{6}, 1, -\frac{2}{3})$	$(\overline{15}, 6)$	M_I
	$\Sigma_4(15, 2, 2)$	$\Sigma_{41}(1, 2, +\frac{1}{2})$	$(\bar{5}, -2)$	M_I
		$\Sigma_{42}(1, 2, -\frac{1}{2})$	$(45, 2)$	M_I
		$\Sigma_{43}(3, 2, -\frac{7}{6})$	$(\overline{50}, -2)$	M_I
		$\Sigma_{44}(3, 2, -\frac{1}{6})$	$(10, -6)$	M_I
		$\Sigma_{45}(\bar{3}, 2, +\frac{1}{6})$	$(\overline{15}, 6)$	M_I
		$\Sigma_{46}(\bar{3}, 2, +\frac{7}{6})$	$(45, 2)$	M_I
		$\Sigma_{47}(8, 2, +\frac{1}{2})$	$(\overline{50}, -2)$	M_I
		$\Sigma_{48}(8, 2, -\frac{1}{2})$	$(45, 2)$	M_I

Evolution of the Gauge Couplings

$$\beta(g) = \mu\frac{dg}{d\mu} = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}C_2(G) - \frac{4}{3}\kappa S_2(F) - \frac{1}{6}\eta S_2(S)\right\}$$
$$-\frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}\left[C_2(G)\right]^2 - \kappa\left[4C_2(F) + \frac{20}{3}C_2(G)\right]S_2(F) - \left[2C_2(S) + \frac{1}{3}C_2(G)\right]\eta S_2(S)\right\} + \cdots$$

Two- Loop RG Equation:

$$\frac{d\alpha^{-1}(\mu)}{d\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}}$$

where,

$$a_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\kappa S_2(F_i) + \frac{1}{6}\eta S_2(S_i)$$
$$b_{ii} = -\frac{34}{3}\left[C_2(G_i)\right]^2 + \kappa\left[4C_2(F_i) + \frac{20}{3}C_2(G_i)\right]S_2(F_i)$$
$$-\left[2C_2(S_i) + \frac{1}{3}C_2(G_i)\right]\eta S_2(S_i)$$
$$b_{ij} = 4\kappa C_2(F_j)S_2(F_i) + 2\eta C_2(S_j)S_2(S_i)$$

Here $i \neq j$.

Contd...

β coefficients of the theory:

$$a_{SM}^T = \begin{pmatrix} \frac{41}{10} & -\frac{19}{6} & -7 \end{pmatrix}; \qquad b_{SM} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix};$$

$$a_{PS}^T = \begin{pmatrix} \frac{26}{3} & \frac{26}{3} & 1 \end{pmatrix}; \qquad b_{PS} = \begin{pmatrix} \frac{779}{3} & 48 & \frac{1245}{2} \\ 48 & \frac{779}{3} & \frac{1245}{2} \\ \frac{249}{2} & \frac{249}{2} & \frac{1209}{2} \end{pmatrix};$$

While the matching conditions are:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_G(\mu)} - \lambda_i(\mu)$$

Evolution of Gauge Couplings

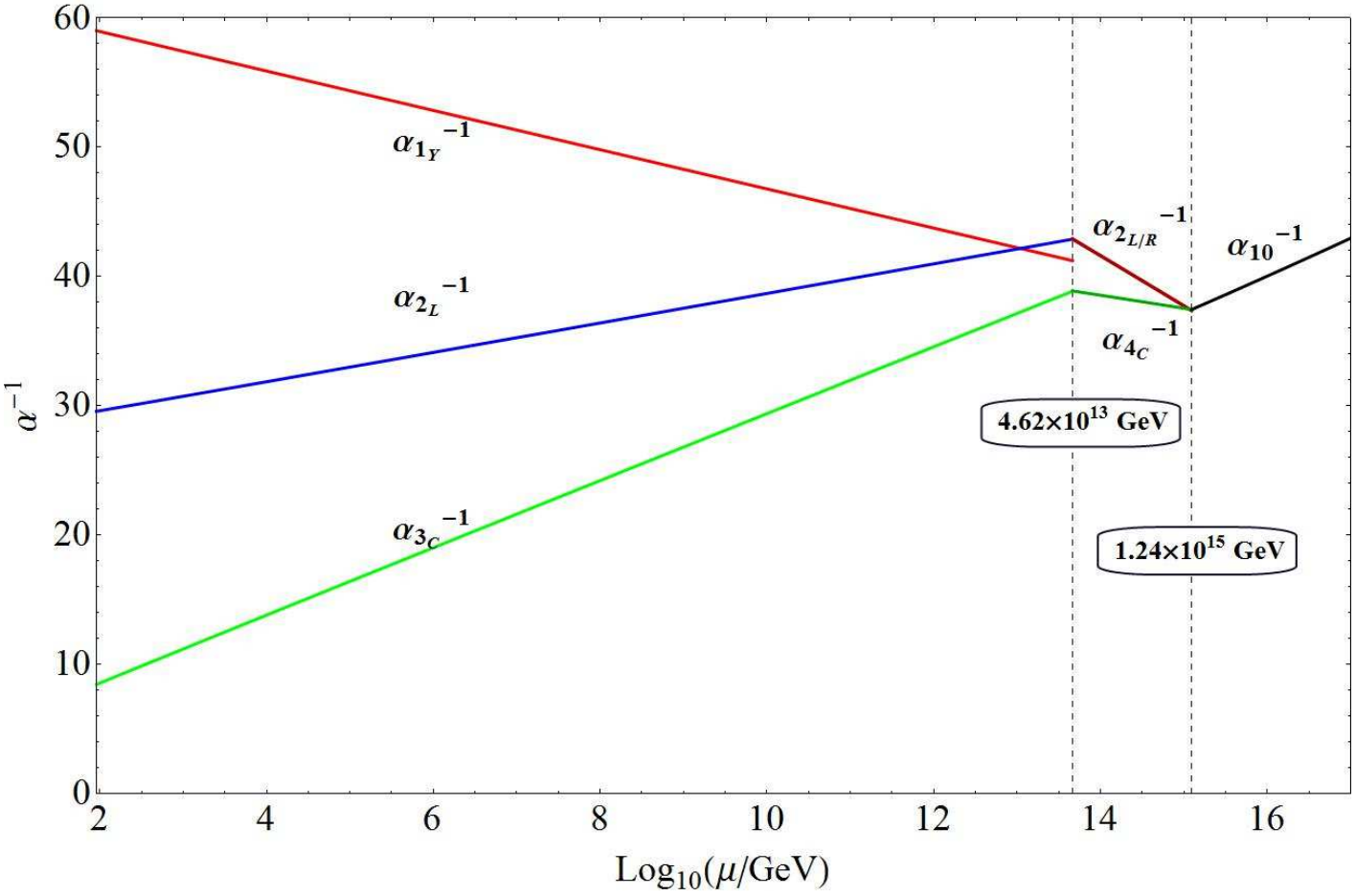


Figure : Running of gauge couplings without threshold correction using two-loop RGE. Pati-Salam symmetry with D parity is assumed as the intermediate scale. The dotted vertical line corresponds to intermediate scale and unification scale.

Proton Lifetime

$$\tau_P \approx \frac{m_X^4}{g^4 m_P^5}$$

Here,

$$\begin{aligned} g^2 &\approx \frac{4\pi}{35} \\ m_X &\approx 1.20 \times 10^{15} GeV \end{aligned}$$

So,

$$\tau_P \approx 5 \times 10^{29} yrs$$

Current Limit on proton life time:

$$\tau_P > 1.29 \times 10^{34} yrs$$

⇒ Let's be a little bit more careful!

⇒ There are lots of scalar particles.

⇒ All of them cannot be degenerate!

⇒ Proton Decay equation is too crude!

Threshold Corrections are Important!

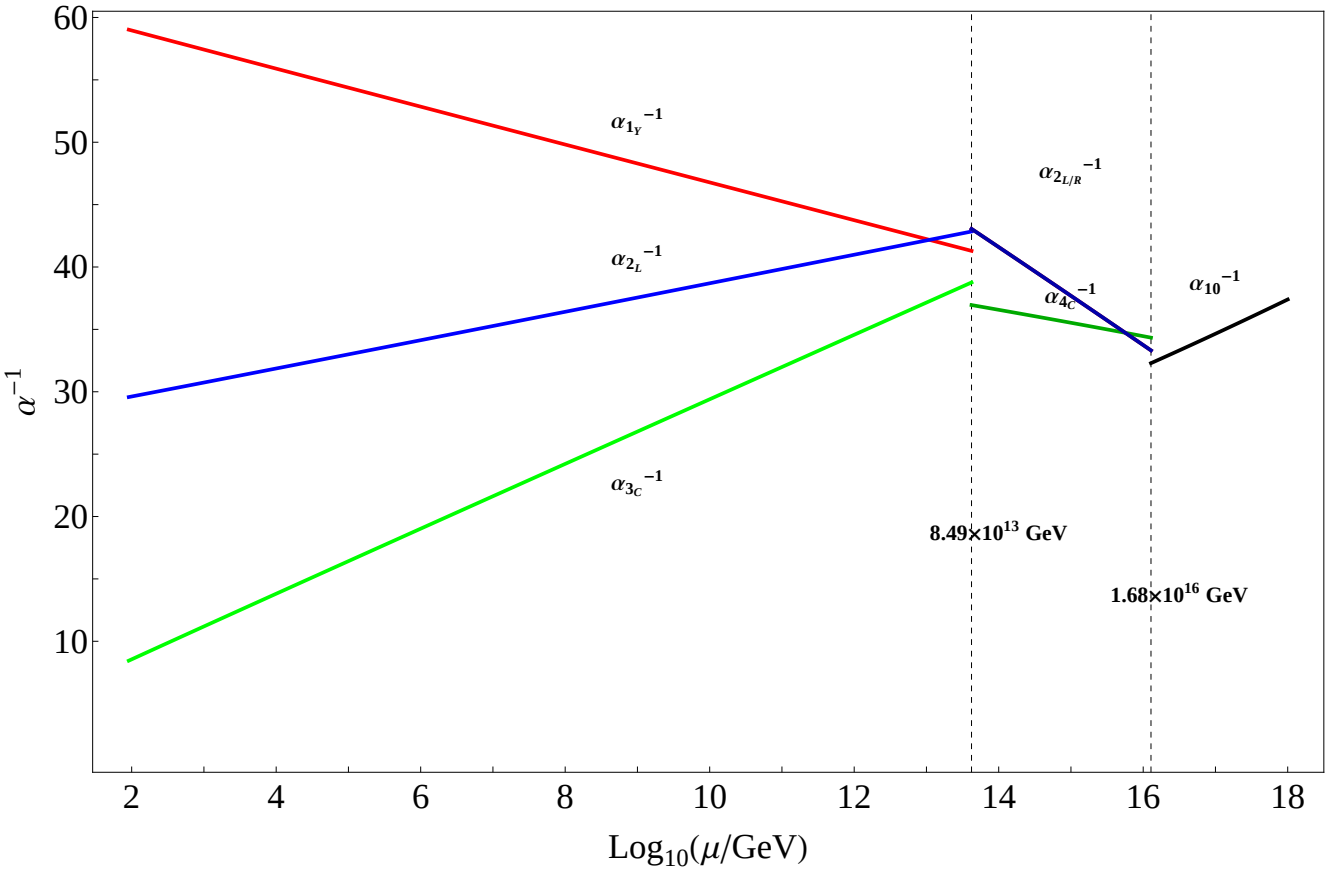


Figure : Running of gauge couplings with one-loop threshold corrections using two-loop RGE. This sample point corresponds to a case where some of the Higgs masses are taken to be twice of the corresponding scale determined without the threshold corrections and the others are one tenth of the scale. Special attention was given to the color triplet masses, so that they are heavier than 10^{13} GeV. In this extreme scenario, the mass of the leptoquark guage boson (the one responsible of proton decay) is maximized. Then the scales were updated with an iteration process so that, the scales corresponds to the masses of the respective gauge bosons.

Proton Lifetime (Revisited)

$$\tau_P \approx \frac{\pi}{4} R_L^2 (1+F+D) \frac{|\alpha|^2}{f_\pi^2} m_p \alpha_G^2 \left[A_{SR}^2 \left(\frac{1}{M_{(X,Y)}^2} + \frac{1}{M_{(X',Y')}^2} \right)^2 + \frac{4A_{SL}^2}{M_{(X,Y)}^4} \right]^{-1}$$

Here,

$$A_{SL(R)} = \prod_{i=1}^{n} \prod_{sc}^{Mz \leq m_{sc} < M_G} \left[\frac{\alpha_i(m_{sc+1})}{\alpha_i(m_{sc})} \right]^{\frac{\gamma_{L(R)i(sc)}}{b_i(m_{sc+1} - m_{sc})}}$$

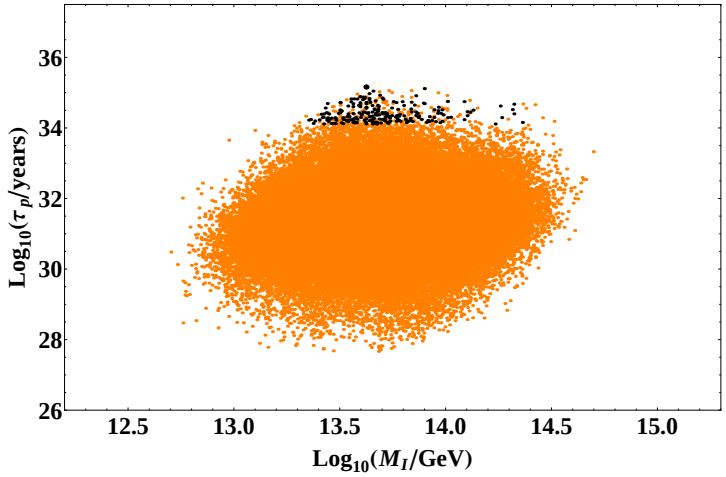
where,

$$\gamma_{L(sm)} = \left\{ \frac{23}{20}, \frac{9}{4}, 2 \right\}; \qquad \gamma_{R(sm)} = \left\{ \frac{11}{20}, \frac{9}{4}, 2 \right\}; \qquad \gamma_{L/R(ps)} = \left\{ \frac{15}{4}, \frac{9}{4}, \frac{9}{4} \right\}$$

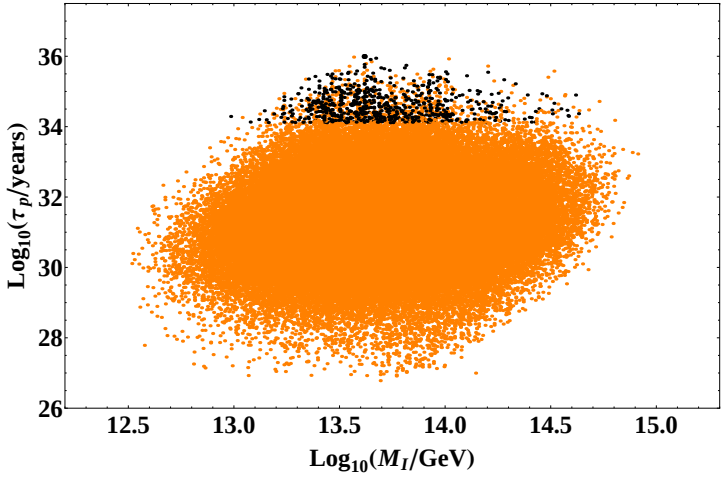
Proton Life Time

$$\begin{aligned} \Gamma^{-1}(p \to e^+ \pi^0) &\approx (8.2 \times 10^{34} \text{yr}) \\ &\times \left(\frac{\alpha_H}{0.0122 \text{GeV}^3} \right)^{-2} \left(\frac{\alpha_G}{1/34.7} \right)^{-2} \left(\frac{A_R}{3.35} \right)^{-2} \left(\frac{M_X}{10^{16} \text{GeV}} \right)^4 \end{aligned}$$

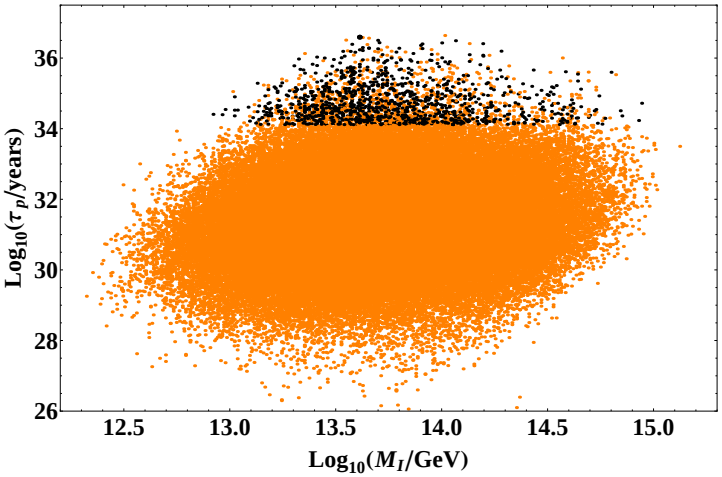
Scatter plot in the parameter space



(a) R=[10^{-1}, 2]



(b) R=[20^{-1}, 2]



(c) R=[33^{-1}, 2]

Figure: Proton lifetime as a function of the intermediate scale for different levels of threshold correction. The ratio of the mass of each Higgs boson to the corresponding scale is kept in between $R = \{10^{-1}, 20^{-1}, 33^{-1}\}$ and 2, with $R = \left(\frac{M_{\text{Higgs Boson}}}{\text{Corresponding Scale}}\right)$. All the black points are phenomenologically viable ones. Points in the orange shaded region either goes through gauge boson mediated proton decay with a lifetime shorter than the experimental limit ($1.29 \times 10^{34} yrs$), or they correspond to scenarios where at least one of the color triplet Higgs boson acquires a mass less than 10^{13} GeV.

Next Step

- Even though we cannot predict the scalar masses, all of them cannot be treated as free parameter.
- Scalar mass spectrum for the model will generate certain mass relationship.
- The process will put constraints on the parameter space.

The Higgs potential

The possible invariants are:

1. $54 \cdot 54$
2. $54 \cdot 54 \cdot 54$
3. $54 \cdot 54 \cdot 54 \cdot 54$ $(\times 2)$
4. $126 \cdot \overline{126}$
5. $126 \cdot 126 \cdot \overline{126} \cdot \overline{126}$ $(\times 4)$
6. $10 \cdot \overline{10}$
7. $10 \cdot 10 \cdot \overline{10} \cdot \overline{10} \times$ $(\times 2)$
8. $S \cdot S^*$
9. $S \cdot S \cdot S^* \cdot S^*$

1. $54 \cdot 54 \cdot 126 \cdot \overline{126}$ $(\times 2)$
2. $126 \cdot \overline{126} \cdot 10 \cdot \overline{10}$ $(\times 2)$
3. $\overline{126} \cdot \overline{126} \cdot 126 \cdot 10$
4. $54 \cdot 54 \cdot 10 \cdot \overline{10}$ $(\times 2)$
5. $126 \cdot 126 \cdot 10 \cdot 10$
6. $126 \cdot \overline{126} \cdot S \cdot S^*$
7. $10 \cdot \overline{10} \cdot S \cdot S^*$
8. $54 \cdot 54 \cdot S \cdot S^*$
9. $126 \cdot 126 \cdot 54 \cdot S$
10. $10 \cdot 10 \cdot S$

Here we have assumed a PQ symmetry whose natural scale will be below M_I. Under this PQ symmetry

$$\mathbf{10_H} \to e^{-2i\alpha}\mathbf{10_H}; \qquad \mathbf{126_H} \to e^{2i\alpha}\mathbf{126_H}; \qquad \mathbf{S_H} \to e^{-4i\alpha}\mathbf{S_H}; \qquad \mathbf{16_F} \to e^{i\alpha}\mathbf{16_F}$$

Scalar Potential

The most general potential for the 54 and 126 representation can be written as

$$
\begin{aligned}
V(\phi,\Sigma) \quad = \quad & -\frac{\mu^2}{2}\Phi_{ij}\Phi_{ij} + \frac{c}{3}\Phi_{ij}\Phi_{jk}\Phi_{ki} + \frac{a}{4}\Phi_{ij}\Phi_{ij}\Phi_{kl}\Phi_{kl} + \frac{b}{2}\Phi_{ij}\Phi_{jk}\Phi_{kl}\Phi_{li} \\
& -\frac{\nu^2}{2\cdot 5!}\Sigma_{ijklm}\overline{\Sigma}_{ijklm} \\
& +\frac{\lambda_0}{2!^2 5!^2}\Sigma_{ijklm}\overline{\Sigma}_{ijklm}\Sigma_{nopqr}\overline{\Sigma}_{nopqr} + \frac{\lambda_2}{4!^2}\Sigma_{ijklm}\overline{\Sigma}_{ijkln}\Sigma_{opqrm}\overline{\Sigma}_{opqrn} \\
& +\frac{\lambda_4}{3!^2 2!^2}\Sigma_{ijklm}\overline{\Sigma}_{ijkno}\Sigma_{pqrlm}\overline{\Sigma}_{pqrno} + \frac{\lambda_4'}{3!^2}\Sigma_{ijklm}\overline{\Sigma}_{ijkno}\Sigma_{pqrln}\overline{\Sigma}_{pqrmo} \\
& +\frac{\alpha}{2!5!}\Phi_{ij}\Phi_{ij}\Sigma_{pqrlm}\overline{\Sigma}_{pqrlm} + \frac{\beta}{3!}\Phi_{ij}\Phi_{kl}\Sigma_{mnoik}\overline{\Sigma}_{mnojl}
\end{aligned}
$$

Scalar Potential(Contd...)

The interaction part of the potential with the 10 can be written as

$$
\begin{aligned}
V(\Phi,\Sigma,\phi) \quad = \quad & -\xi_0^2\phi_i\phi_i^* + \xi_1\phi_i\phi_i^*\phi_j\phi_j^* + \xi_2\phi_i\phi_i\phi_j^*\phi_j^* + \xi_3\Phi_{i,j}\phi_i\phi_j^* + \frac{\gamma_1}{4!}\Sigma_{ijklm}\overline{\Sigma}_{ijkln}\phi_m\phi_n^* \\
& + \frac{\gamma_2}{4!}\Sigma_{ijklm}\overline{\Sigma}_{ijkln}\phi_n\phi_m^* + \frac{\eta_0}{2}\Phi_{ij}\Phi_{ij}\phi_k\phi_k^* + \frac{\eta_1}{(3!)^2(2!)^2}\Sigma_{ijklm}\overline{\Sigma}_{ijkpq}\overline{\Sigma}_{lmpqn}\phi_n \\
& + \frac{\eta_1^*}{(3!)^2(2!)^2}\overline{\Sigma}_{ijklm}\Sigma_{ijkpq}\Sigma_{lmpqn}\phi_n^* + \eta_2\Phi_{ij}\Phi_{ik}\phi_j\phi_k^* + \frac{\eta_3}{4!}\Sigma_{ijklm}\Sigma_{ijkln}\phi_m\phi_n \\
& + \frac{\eta_3^*}{4!}\overline{\Sigma}_{ijklm}\overline{\Sigma}_{ijkln}\phi_m^*\phi_n^*
\end{aligned}
$$

Part of the potential with the Singlet S can be written as

$$
\begin{aligned}
V(S) \quad = \quad & -\mu_s^2 SS^* + \chi_1(SS^*)^2 + \chi_2\Sigma_{ijklm}\overline{\Sigma}_{ijklm}SS^* + \chi_3\Phi_{ij}\Phi_{ij}SS^* + \frac{\chi_4}{4!}\Sigma_{ijklm}\Sigma_{ijkln}\Phi_{mn}S \\
& + \frac{\chi_4^*}{4!}\overline{\Sigma}_{ijklm}\overline{\Sigma}_{ijkln}\Phi_{mn}S^* + \chi_5\phi_i\phi_i^*SS^* + \chi_6\phi_i\phi_iS^* + \chi_6^*\phi_i^*\phi_i^*S
\end{aligned}
$$

Scalar Mass Spectrum

From this we can find out the relevant scalar masses as,

$$M^2(1,3,0) = \frac{8}{5} b\omega_s^2 + c\omega_s;$$
$$M^2(8,1,0) = \frac{2}{5} b\omega_s^2 - c\omega_s;$$
$$M^2(3,3,-\frac{1}{3}) = 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)\sigma^2;$$
$$M^2(6,3,+\frac{1}{3}) = 8\left(\lambda_2 + \lambda_4 + 4\lambda_4'\right)\sigma^2;$$
$$M^2(1,1,+2) = 8\left(\lambda_2 + \lambda_4 + 4\lambda_4'\right)\sigma^2;$$
$$M^2(\overline{3},1,+\frac{4}{3}) = 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)\sigma^2;$$
$$M^2(\overline{6},1,-\frac{4}{3}) = 8\left(\lambda_2 + \lambda_4 + 4\lambda_4'\right)\sigma^2;$$
$$M^2(\overline{6},1,-\frac{1}{3}) = 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)\sigma^2;$$

Contd...

$$M^2(1,3,+1) = \begin{pmatrix} \frac{8}{5}b\omega_s^2 + c\omega_s + \frac{1}{2}\beta\sigma^2 & 2\chi_4\sigma v_s \\ 2\chi_4\sigma v_s & 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)\sigma^2 \end{pmatrix};$$

$$M^2(\overline{6},1,+\frac{2}{3}) = \begin{pmatrix} \frac{2}{5}b\omega_s^2 - c\omega_s + \frac{1}{2}\beta\sigma^2 & 2\chi_4\sigma v_s \\ 2\chi_4\sigma v_s & 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)\sigma^2 \end{pmatrix};$$

$$M^2(3,2,+\frac{7}{6}) = \begin{pmatrix} 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)\sigma^2 + \beta\omega_s^2 & -2\sqrt{2}\chi_4\omega_s v_s \\ -2\sqrt{2}\chi_4\omega_s v_s & 8\left(\lambda_2 + \lambda_4 + 2\lambda_4'\right)\sigma^2 + \beta\omega_s^2 \end{pmatrix};$$

$$M^2(8,2,+\frac{1}{2}) = \begin{pmatrix} 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)\sigma^2 + \beta\omega_s^2 & -2\sqrt{2}\chi_4\omega_s v_s \\ -2\sqrt{2}\chi_4\omega_s v_s & 8\left(\lambda_2 + \lambda_4 + 2\lambda_4'\right)\sigma^2 + \beta\omega_s^2 \end{pmatrix};$$

$$M^2[3,2,+\frac{1}{6}) = \begin{pmatrix} 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)\sigma^2 + \beta\omega_s^2 & 2\sqrt{2}\chi_4\omega_s v_s & -2\chi_4\sigma v_s \\ 2\sqrt{2}\chi_4\omega_s v_s & \beta\omega_s^2 & \frac{1}{\sqrt{2}}\beta\sigma\omega_s \\ -2\chi_4\sigma v_s & \frac{1}{\sqrt{2}}\beta\sigma\omega_s & \frac{1}{2}\beta\sigma^2 \end{pmatrix};$$

The Mass Matrix of $M^2(3,2,+\frac{1}{6})$ has a zero eigenvalue which corresponds to the Goldstone boson.

Contd...

$$M^2(3,1,-\frac{1}{3}) = \begin{pmatrix} 4\left(3\lambda_2+3\lambda_4+4\lambda_4'\right)\sigma^2+4\beta\omega_s^2 & 4\sqrt{2}\chi_4\omega_s v_s & 0 & 0 & 0 \\ 4\sqrt{2}\chi_4\omega_s v_s & 8\left(\lambda_2+\lambda_4+2\lambda_4'\right)\sigma^2+4\beta\omega_s^2 & 16\sqrt{2}\lambda_4'\sigma^2 & 0 & 4\eta_1\sigma^2 \\ 0 & 16\sqrt{2}\lambda_4'\sigma^2 & 8\left(\lambda_2+\lambda_4\right)\sigma^2 & 0 & 4\sqrt{2}\eta_1\sigma^2 \\ 0 & 0 & 0 & A1 & \sqrt{2}\chi_6 v_s \\ 0 & 4\eta_1\sigma^2 & 4\sqrt{2}\eta_1\sigma^2 & \sqrt{2}\chi_6 v_s & B1 \end{pmatrix};$$

$$M^2(1,2,+\frac{1}{2}) = \begin{pmatrix} 8\left(\lambda_2+\lambda_4-2\lambda_4'\right)\sigma^2+\beta\omega_s^2 & 2\sqrt{2}\chi_4\omega_s v_s & 0 & 4\sqrt{3}\eta_1\sigma^2 \\ 2\sqrt{2}\chi_4\omega_s v_s & 4\left(3\lambda_2+3\lambda_4+4\lambda_4'\right)\sigma^2+\beta\omega_s^2 & 0 & 0 \\ 0 & 0 & A2 & \sqrt{2}\chi_6 v_s \\ 4\sqrt{3}\eta_1\sigma^2 & 0 & \sqrt{2}\chi_6 v_s & B2 \end{pmatrix};$$

$$M^2(1,1,0) = \begin{pmatrix} \frac{1}{10}c\omega_s+\frac{12}{5}a\omega_s^2+\frac{14}{25}b\omega_s^2 & -\sqrt{\frac{3}{5}}(\alpha-\beta)\sigma\omega_s & -\sqrt{\frac{3}{5}}\chi_3\omega_s v_s \\ -\sqrt{\frac{3}{5}}(\alpha-\beta)\sigma\omega_s & \frac{1}{4}\lambda_0\sigma^2 & \frac{1}{2}\chi_2\sigma v_s \\ -\sqrt{\frac{3}{5}}\chi_3\omega_s v_s & \frac{1}{2}\chi_2\sigma v_s & \chi_1 v_s^2 \end{pmatrix}.$$

Here in the triplet and doublet matrices, we have defined

$$\begin{aligned} A1 &= -\frac{2}{5}\xi_3\omega_s+\frac{6}{5}\eta_0\omega_s^2+\frac{4}{25}\eta_2\omega_s^2+\gamma_1\sigma^2+\frac{1}{2}\chi_5 v_s^2+m^2 \\ B1 &= -\frac{2}{5}\xi_3\omega_s+\frac{6}{5}\eta_0\omega_s^2+\frac{4}{25}\eta_2\omega_s^2+\gamma_2\sigma^2+\frac{1}{2}\chi_5 v_s^2+m^2 \\ A2 &= \frac{3}{5}\xi_3\omega_s+\frac{6}{5}\eta_0\omega_s^2+\frac{9}{25}\eta_2\omega_s^2+\gamma_1\sigma^2+\frac{1}{2}\chi_5 v_s^2+m^2 \\ B2 &= \frac{3}{5}\xi_3\omega_s+\frac{6}{5}\eta_0\omega_s^2+\frac{9}{25}\eta_2\omega_s^2+\gamma_2\sigma^2+\frac{1}{2}\chi_5 v_s^2+m^2 \end{aligned}$$

Yukawa sector of the model

Yukawa part of the Lagrangian is given as:

$$\mathcal{L} = 16_F \left(Y_{10} 10_H + Y_{\overline{126}} \overline{126}_H \right) 16_F \tag{1}$$

For such a case, the quark and lepton mass matrices become:

$$M_u = h v_u + f \kappa_u; \qquad M_d = h v_d + f \kappa_d;$$
$$M_\nu^D = h v_u - 3 f \kappa_u; \qquad M_l = h v_d - 3 f \kappa_d;$$
$$M_\nu^M = f \sigma. \tag{2}$$

A more compact form which is more popular for a fit to masses and mixing angles:

$$M_u = r(H + sF); \qquad M_d = H + F;$$
$$M_\nu^D = r(H - 3sF); \qquad M_l = H - 3F;$$
$$M_\nu^M = r_R F \tag{3}$$

where $H = h v_d, F = f \kappa_d$ are complex symmetric matrices and $r = {}^{v_u}/_{v_d}, s = {}^{\kappa_u}/(r \kappa_d), r_R = {}^{\kappa_d}/_{\sigma}$ are dimonsionless parameter.

Babu, Mohapatra (1993) , Bertolini, Frigerio, Malinsky (2004), Fukuyama, Okada (2002)
Babu, Macesanu (2005), Bajc, Melfo, Senjanovic, Vissani (2004), Bertolini, Malinsky, Schwetz (2006)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004), Dutta, Mimura, Mohapatra (2007), Aulakh et al (2004)
Bajc, Dorsner, Nemevsek (2009), A. S. Joshipura et al (2011), Dueck, Rodejohann (2013)

Yukawa sector of the model

The term $126_H \cdot \overline{126}_H \cdot 126_H \cdot 10_H$ with the coefficient η_1 is important for the fermion mass fitting, as this is term which generates the induced vev for the electroweak doublets contained in the 126_H, or more precisely in the $(15,2,2)_{PS}$[2]

$$\langle 10_H \rangle \sim \eta_1 \left(\frac{\langle 126_H \rangle^2}{M^2_{(15,2,2)_{PS}}} \right) \langle (15,2,2)_{PS} \rangle \,. \tag{4}$$

From the structure of the doublet mass matrix ($\mathcal{D}$), we see that the massless SM Higgs doublet h_{SM} becomes

$$h_{SM} = \alpha_H H_u + \beta_H H_d^* + \alpha_h h_u + \beta_h h_d^* \tag{5}$$

[2] K. S. Babu and R. N. Mohapatra (1993)

Fitting the experimental data

The input at GUT Scale:

$$
\begin{aligned}
m_u &= 0.0006745 & m_c &= 0.3308 & m_t &= 97.335 \\
m_d &= 0.0009726 & m_s &= 0.02167 & m_b &= 1.1475 \\
m_e &= 0.000344 & m_\mu &= 0.0726 & m_\tau &= 1.350 \\
s_{12} &= 0.2248 & s_{23} &= 0.03278 & s_{13} &= 0.00216 \\
& & \delta_{CKM} &= 1.193 \ . & &
\end{aligned}
$$

The output for Neutrino:

$$\sin^2\theta_{12} \simeq 0.27 \ , \ \sin^2 2\theta_{23} \simeq 0.90 \ , \ \sin^2 2\theta_{13} \simeq 0.08 \ .$$

K. S. Babu and C. Macesanu, [hep-ph/0505200].

Fitting the experimental data

Best fit parameter values: The minimal nonsusy $SO(10)$

Parameters obtained for best fit solution.

$$r = 69.1739; \quad s = 0.362941 - 0.0463175i$$

$$M_l = \begin{pmatrix} 0.000469652 & 0 & 0 \\ 0 & 0.0991466 & 0 \\ 0 & 0 & 1.68558 \end{pmatrix}$$

$$M_d = \begin{pmatrix} -0.00110182 + 0.00164125i & 0.0040374 - 0.00434507i & 0.0145011 + 0.0480084i \\ 0.0040374 - 0.00434507i & -0.0331074 + 0.00870484i & -0.0187112 - 0.158707i \\ 0.0145011 + 0.0480084i & -0.0187112 - 0.158707i & 0.754282 + 0.611126i \end{pmatrix}$$

Results:

$$M_u = \begin{pmatrix} -0.0575896 + 0.0967087i & 0.231322 - 0.25593i & 0.881792 + 2.78041i \\ 0.231322 - 0.25593i & -0.826159 + 0.612181i & -1.21531 - 9.21493i \\ 0.881792 + 2.78041i & -1.21531 - 9.21493i & 62.9262 + 36.2872i \end{pmatrix}$$

$$M_\nu = r_R r^2 \begin{pmatrix} -0.0000981928 + 0.00131563i & 0.0000421662 - 0.003853i & 0.0276444 + 0.0202258i \\ 0.0000421662 - 0.003853i & -0.0640659 + 0.0272358i & -0.0653091 - 0.0653272i \\ 0.0276444 + 0.0202258i & -0.0653091 - 0.0653272i & 0.054234 - 0.0680089i \end{pmatrix}$$

Joshipura and Patel., arXiv:1102.5148 [hep-ph]

Benchmark point using one-loop RGE

Parameter	Value	Parameter	Value
b	1.70	a	0.31
λ_2	-0.17	λ_0	0.90
λ_4	0.48	α	-0.23
λ_4'	0.17	χ_1	0.10
β	1.25×10^{-5}	χ_2	0.12
η_1	-0.002	χ_3	-0.01
η_2	0.90	c	9.36×10^{15} GeV
χ_4	-0.55	ξ_3	-3.15×10^{14} GeV
χ_5	0.32	χ_6	-2.67×10^{14} GeV
γ_1	-0.38	v_s	9.36×10^{10} GeV
γ_2	0.52	σ	8.65×10^{14} GeV
η_0	-0.15	ω_s	1.38×10^{16} GeV

Table : Sample parameters and vev's to generate a benchmark point using one-loop RGE. The initial parameter and the vev values were updated through the iteration processes described in the text, and the listed values correspond to the final stable point.

Higgs spectrum

Multiplet	Mass [GeV]	Multiplet	Mass [GeV]
$(1,3,0)$	2.54×10^{16}	$(8,1,0)$	1.11×10^{12}
$(3,3,-\frac{1}{3})$	2.17×10^{14}	$(6,3,+\frac{1}{3})$	2.42×10^{14}
$(1,1,+2)$	2.42×10^{14}	$(\overline{3},1,+\frac{4}{3})$	2.17×10^{14}
$(\overline{6},1,-\frac{4}{3})$	2.42×10^{14}	$(\overline{6},1,-\frac{1}{3})$	2.17×10^{14}
$(1,3,+1)$	2.54×10^{16}	$(\overline{6},1,+\frac{2}{3})$	1.13×10^{12}
	2.91×10^{14}		2.91×10^{14}
$(3,2,+\frac{7}{6})$	2.47×10^{14}	$(8,2,+\frac{1}{2})$	2.47×10^{12}
	2.22×10^{14}		2.22×10^{14}
$(3,2,+\frac{1}{6})$	4.83×10^{13}	$(1,2,+\frac{1}{2})$	2.23×10^{14}
	2.95×10^{14}		8.12×10^{13}
$(3,1,-\frac{1}{3})$	2.86×10^{14}		1.13×10^{12}
	2.37×10^{14}		≈ 0
	8.22×10^{14}	$(1,1,0)$	1.66×10^{16}
	6.99×10^{13}		3.90×10^{13}
	1.28×10^{13}		2.69×10^{10}

Table : Sample scalar mass spectrum corresponding to the benchmark point

Gauge coupling evolution using one-loop RGE

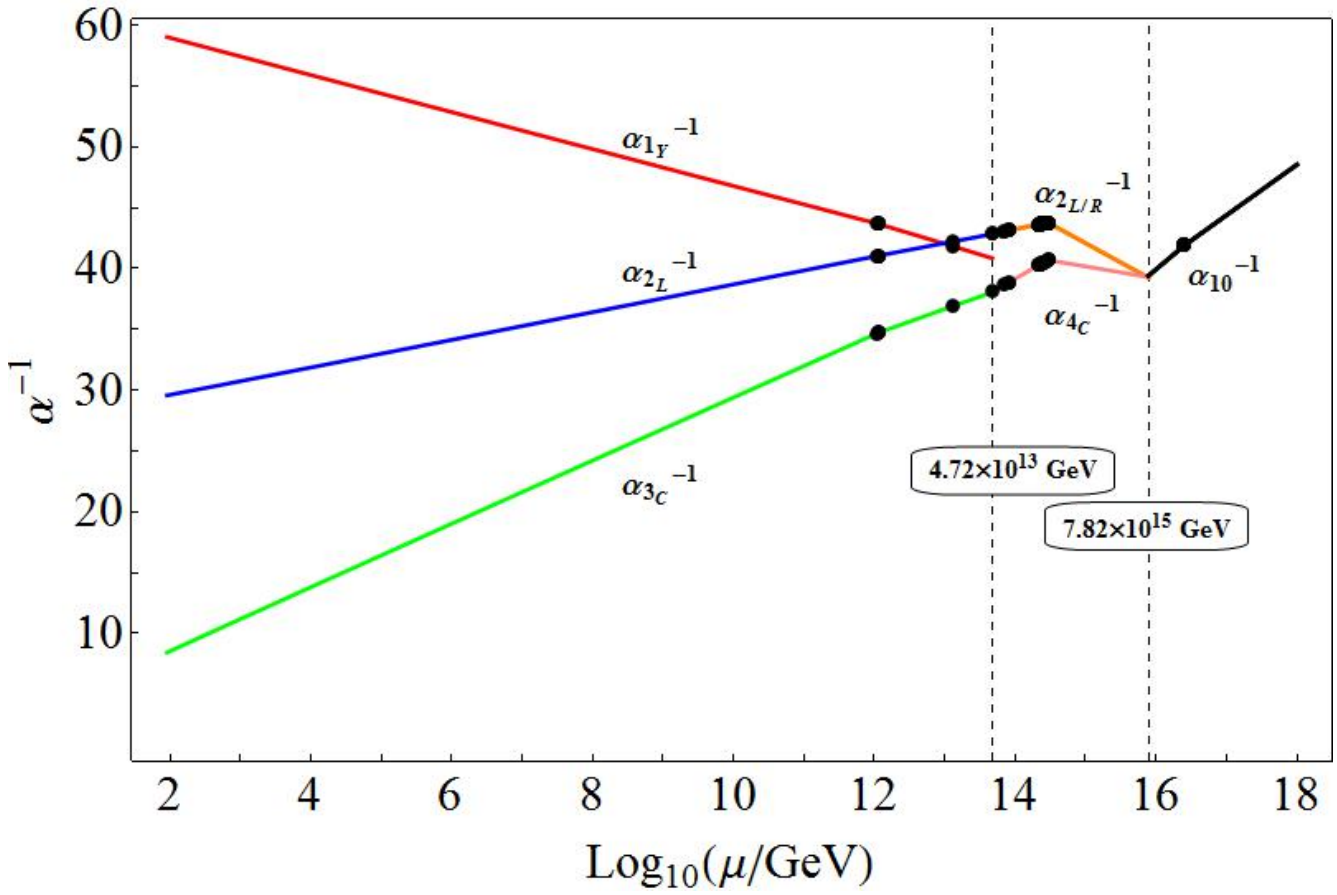


Figure : Evolution of gauge couplings using one-loop RGE with threshold corrections determined by the scalar mass spectrum given in previous table. The unification scale determined here is compatible with the current experimental limit on proton lifetime. The small black circles correspond to the various scalar masses changing the beta function coefficient and inflicting changes in the slope of the graphs. The vertical dashed lines

Result

- Proton Life-time, $\tau = 5.72 \times 10^{34}$ yrs, beyond the current upper-limit (1.29×10^{34} yrs)
- Mass of the X,Y gauge boson is $7.82 \times 10^{15} GeV$
- Mass of the triplet $(3, 1, -\frac{1}{3})$ is $\gtrsim 10^{13}$ GeV
- One of the Singlet Mass is near 10^{10} GeV

Benchmark point using two-loop RGE

Parameter	Value	Parameter	Value
b	1.70	a	0.31
λ_2	-0.17	λ_0	0.90
λ_4	0.49	α	-0.23
λ_4'	0.17	χ_1	0.10
β	1.25×10^{-5}	χ_2	0.12
η_1	-0.002	χ_3	-0.01
η_2	-0.73	c	8.50×10^{15} GeV
χ_4	-0.60	ξ_3	1.83×10^{15} GeV
χ_5	0.32	χ_6	-2.67×10^{14} GeV
γ_1	-0.38	v_s	9.36×10^{10} GeV
γ_2	0.52	σ	8.56×10^{13} GeV
η_0	-0.15	ω_s	1.25×10^{16} GeV

Table : Sample parameters and vev's to generate benchmark point using two-loop RGE. The initial parameters and the vev's were updated through the iteration processes described in the text, and the listed values correspond to the final stable point.

Higgs Spectrum

Multiplet	Mass [GeV]	Multiplet	Mass [GeV]
$(1,3,0)$	2.31×10^{16}	$(8,1,0)$	1.11×10^{12}
$(3,3,-\frac{1}{3})$	2.18×10^{14}	$(6,3,+\frac{1}{3})$	2.42×10^{14}
$(1,1,+2)$	2.42×10^{14}	$(\bar{3},1,+\frac{4}{3})$	2.18×10^{14}
$(\bar{6},1,-\frac{4}{3})$	2.42×10^{14}	$(\bar{6},1,-\frac{1}{3})$	2.18×10^{14}
$(1,3,+1)$	2.31×10^{16}	$(\bar{6},1,+\frac{2}{3})$	1.13×10^{12}
	2.92×10^{14}		2.92×10^{14}
$(3,2,+\frac{7}{6})$	2.47×10^{14}	$(8,2,+\frac{1}{2})$	2.47×10^{12}
	2.22×10^{14}		2.22×10^{14}
$(3,2,+\frac{1}{6})$	2.96×10^{14}	$(1,2,+\frac{1}{2})$	2.23×10^{14}
	4.38×10^{13}		8.12×10^{13}
$(3,1,-\frac{1}{3})$	2.83×10^{14}		1.13×10^{12}
	2.35×10^{14}		≈ 0
	8.22×10^{13}	$(1,1,0)$	1.66×10^{16}
	7.06×10^{13}		3.90×10^{13}
	1.28×10^{13}		2.69×10^{10}

Table : Sample scalar mass spectrum corresponding to the benchmark point generated.

Gauge coupling evolution using two-loop RGE

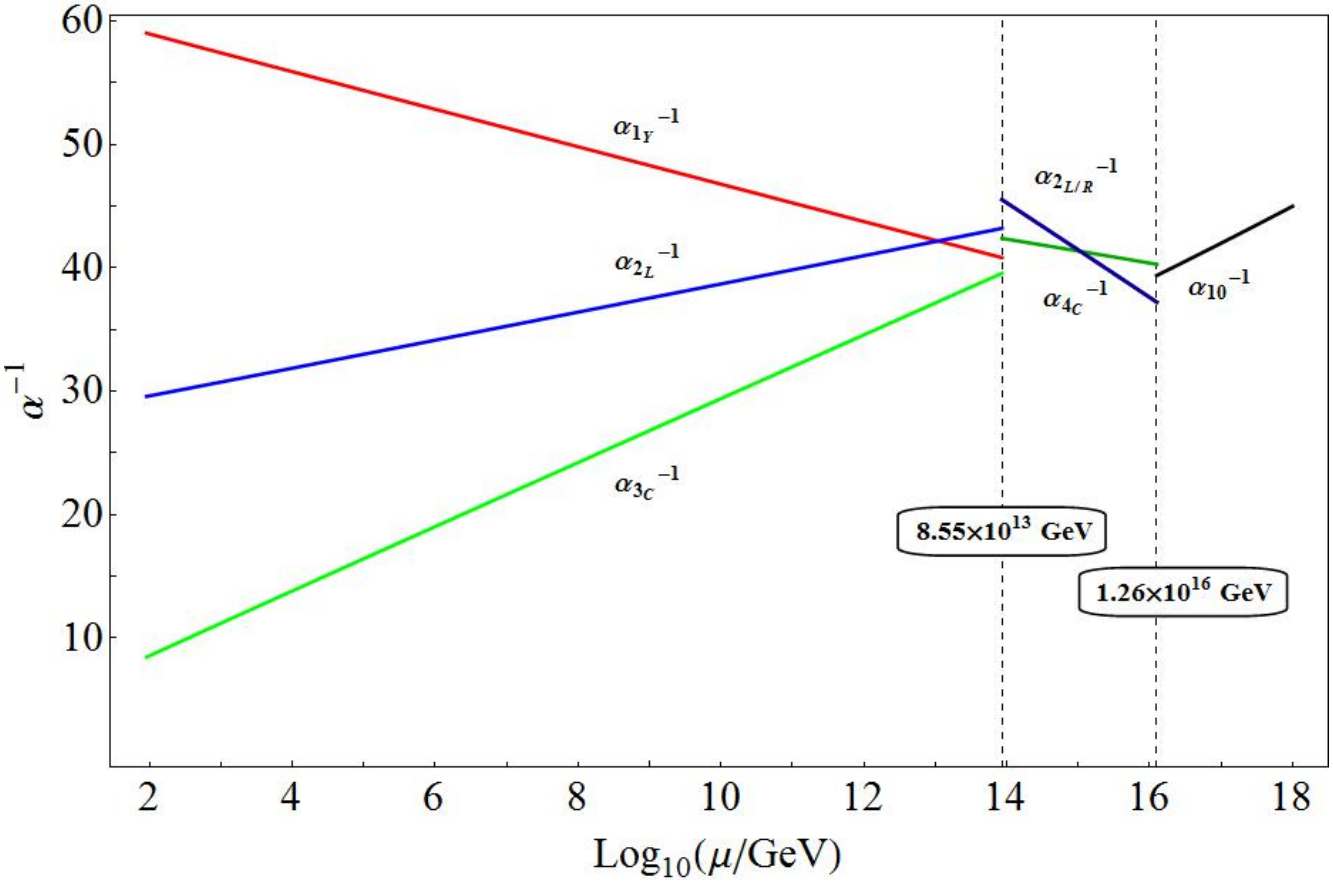
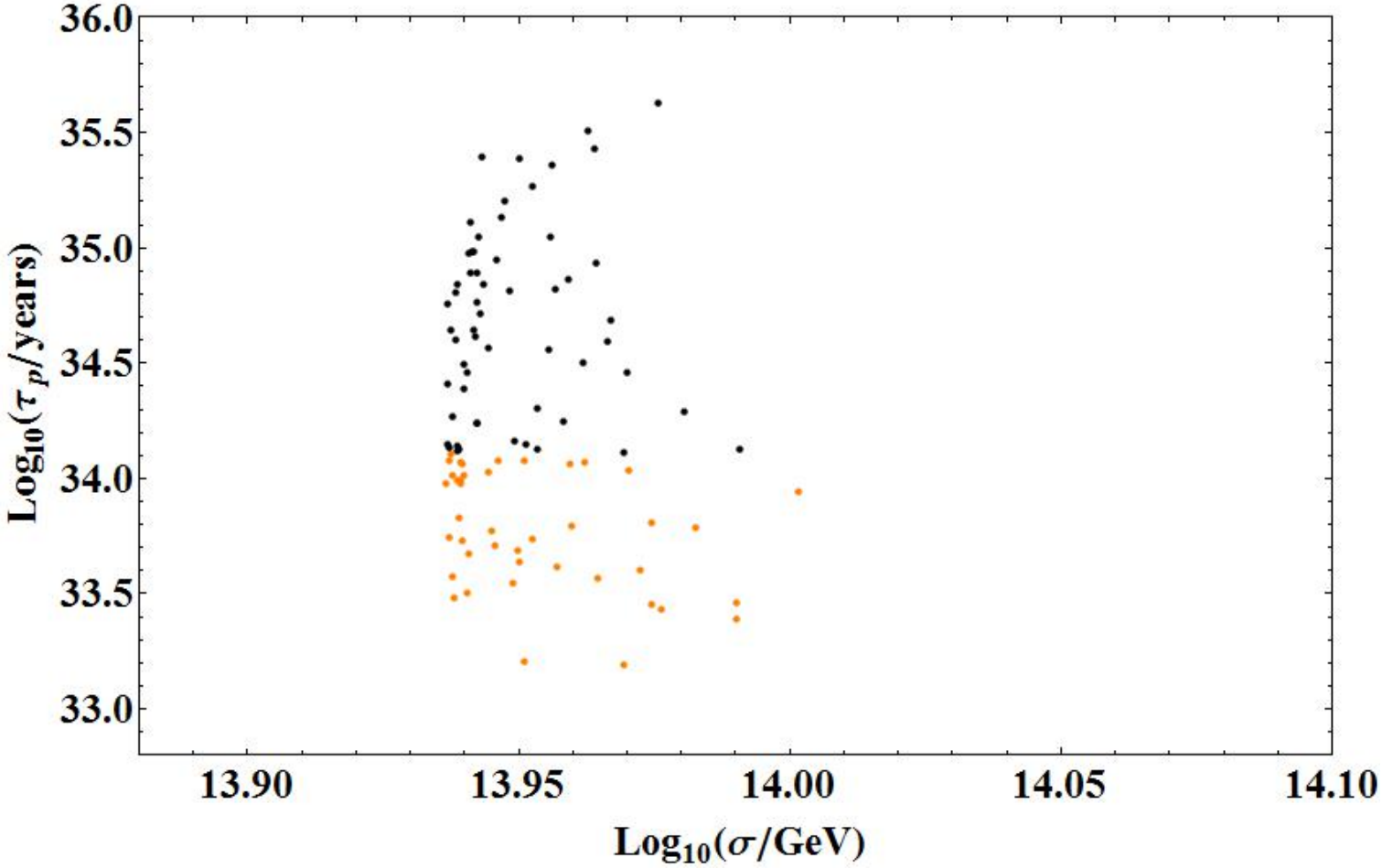


Figure : Evolution of gauge couplings using two-loop RGE with threshold correction. The unification scale determined here is compatible with the current experimental limit on proton lifetime. The discontinuity in the running of the gauge couplings is due to the threshold correction determined using the scalar mass spectrum given in Table. The vertical dashed lines correspond to the intermediate scale and unification scale.

Result

- Proton Life-time, $\tau = 2.21 \times 10^{34}$ yrs, beyond the current upper-limit (1.29×10^{34} yrs)
- Mass of the X,Y gauge boson is $7.11 \times 10^{15} GeV$
- Mass of the triplet $(3, 1, -\frac{1}{3})$ is $\gtrsim 10^{13}$ GeV
- One of the Singlet Mass is near 10^{10} GeV

Scatter Plot



Proton branching ratio

Proton Decay operators in physical basis

$$
\begin{aligned}
O(e^C_\alpha, d_\beta) &= c(e^C_\alpha, d_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}_L\ \gamma^\mu\ u_{jL}\ \overline{e^C_\alpha}_L\ \gamma_\mu\ d_{k\beta L} \\
O(e_\alpha, d^C_\beta) &= c(e_\alpha, d^C_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}_L\ \gamma^\mu\ u_{jL}\ \overline{d^C_{k\beta}}_L\ \gamma_\mu\ e_{\alpha L} \\
O(\nu_l, d_\alpha, d^C_\beta) &= c(\nu_l, d_\alpha, d^C_\beta)\ \epsilon_{ijk}\ \overline{u^C_i}_L\ \gamma^\mu\ d_{j\alpha L}\ \overline{d^C_{k\beta}}_L\ \gamma_\mu\ \nu_{lL} \\
O(\nu^C_l, d_\alpha, d^C_\beta) &= c(\nu^C_l, d_\alpha, d^C_\beta)\ \epsilon_{ijk}\ \overline{d^C_{i\beta}}_L\ \gamma^\mu\ u_{jL}\ \overline{\nu^C_l}_L\ \gamma_\mu\ d_{k\alpha L}
\end{aligned}
$$

where:

$$
\begin{aligned}
c(e^C_\alpha, d_\beta) &= k_1^2[V_1^{11}V_2^{\alpha\beta} + (V_1V_{UD})^{1\beta}(V_2V_{UD}^\dagger)^{\alpha 1}] \\
c(e_\alpha, d^C_\beta) &= k_1^2\ V_1^{11}V_3^{\beta\alpha} +\ k_2^2\ (V_4V_{UD}^\dagger)^{\beta 1}(V_1V_{UD}V_4^\dagger V_3)^{1\alpha} \\
c(\nu_l, d_\alpha, d^C_\beta) &= k_1^2\ (V_1V_{UD})^{1\alpha}(V_3V_{EN})^{\beta l} \\
&+\ \ k_2^2\ V_4^{\beta\alpha}(V_1V_{UD}V_4^\dagger V_3V_{EN})^{1l} \\
c(\nu^C_l, d_\alpha, d^C_\beta) &= k_2^2[(V_4V_{UD}^\dagger)^{\beta 1}(U_{EN}^\dagger V_2)^{l\alpha} + V_4^{\beta\alpha}(U_{EN}^\dagger V_2V_{UD}^\dagger)^{l1}]; \\
\alpha = \beta &\neq 2.
\end{aligned}
$$

Nath and Perez., arXiv:0601023 [hep-ph]

Proton branching ratio

Proton decay rates

$$\Gamma(p \to K^+\bar{\nu}) = \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2} A_L^2 \left|\alpha\right|^2 \times$$

$$\times \sum_{i=1}^{3} \left| \frac{2m_p}{3m_B} D\ c(\nu_i, d, s^C) + [1 + \frac{m_p}{3m_B}(D+3F)] c(\nu_i, s, d^C) \right|^2 \qquad (28)$$

$$\Gamma(p \to \pi^+\bar{\nu}) = \frac{m_p}{8\pi f_\pi^2} A_L^2 \left|\alpha\right|^2 (1+D+F)^2 \sum_{i=1}^{3} \left| c(\nu_i, d, d^C) \right|^2 \qquad (29)$$

$$\Gamma(p \to \eta e_\beta^+) = \frac{(m_p^2 - m_\eta^2)^2}{48\pi f_\pi^2 m_p^3} A_L^2 \left|\alpha\right|^2 (1+D-3F)^2 \times \{ \left| c(e_\beta, d^C) \right|^2 + \left| c(e_\beta^C, d) \right|^2 \} \qquad (30)$$

$$\Gamma(p \to K^0 e_\beta^+) = \frac{(m_p^2 - m_K^2)^2}{8\pi f_\pi^2 m_p^3} A_L^2 \left|\alpha\right|^2 [1 + \frac{m_p}{m_B}(D-F)]^2 \times \{ \left| c(e_\beta, s^C) \right|^2 + \left| c(e_\beta^C, s) \right|^2 \} \qquad (31)$$

$$\Gamma(p \to \pi^0 e_\beta^+) = \frac{m_p}{16\pi f_\pi^2} A_L^2 \left|\alpha\right|^2 (1+D+F)^2 \{ \left| c(e_\beta, d^C) \right|^2 + \left| c(e_\beta^C, d) \right|^2 \} \qquad (32)$$

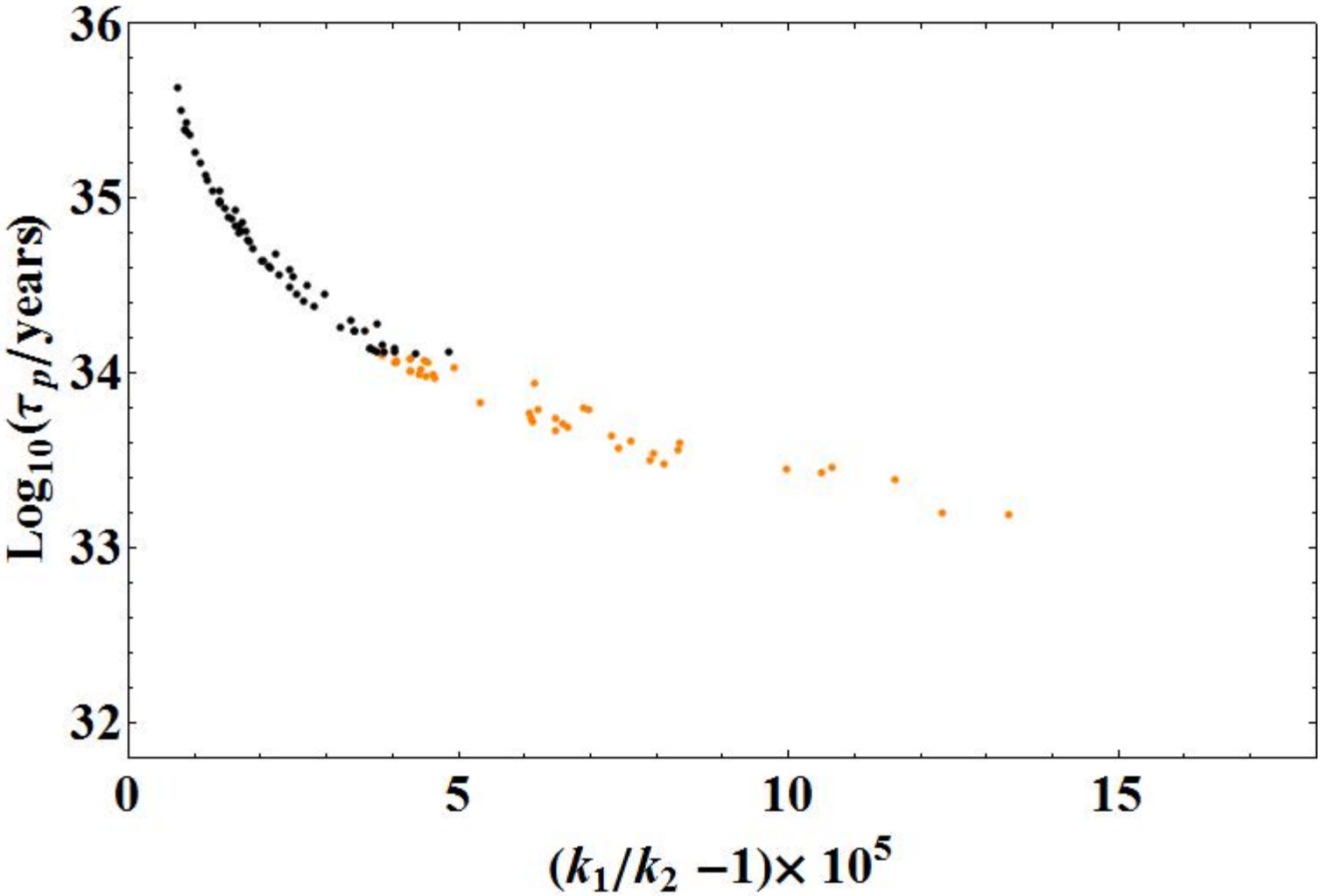
Nath and Perez., arXiv:0601023 [hep-ph]

Prediction of branching ratios

The branching ratios for $d = 6$ gauge mediated proton decay operators :

$$\Gamma(p \to \pi^0 e^+) \to 47\%$$
$$\Gamma(p \to \pi^0 \mu^+) \to 1\%$$
$$\Gamma(p \to \eta^0 e^+) \to 0.20\%$$
$$\Gamma(p \to \eta^0 \mu^+) \to 0.00\%$$
$$\Gamma(p \to K^0 e^+) \to 0.16\%$$
$$\Gamma(p \to K^0 \mu^+) \to 3.62\%$$
$$\Gamma(p \to \pi^+ \overline{\nu}) \to 48\%$$
$$\Gamma(p \to K^+ \overline{\nu}) \to 0.22\%$$

Proton decay branching ratio



The Strong CP Problem and Axion

Strong interaction sector admits a term that violates both CP and P:

$$\mathcal{L}_{QCD} = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

This leads to a physical observable

$$\overline{\theta} = \theta + Arg[detM_q]$$

Neutron Electric Dipole Moment limits $\overline{\theta} < 10^{-10}$
Most popular solution to the problem is suggested by Peccei and Quinn which provides a dynamic solution.

DFSZ Axion:

- One complex scalar ϕ
- One uptype Higgs doublet ϕ_u
- One downtype Higgs doublet ϕ_d

Axion field is given by

$$A \approx -Im[\phi] + \frac{2v_u v_d}{v_w^2 v_s}(v_u Im[\phi_d + v_d Im[\phi_u])$$

Peccei, Quinn Phys Rev. Lett **38** (1977);
Dine, Fischler, Srednicki 1981, Zhitnitskii 1980

Axion in Minimal $SO(10)$

Axion in this model

- One complex scalar with vev v_s
- One uptype Higgs doublet ϕ_1^u from 10_H with vev v_u
- One downtype Higgs doublet ϕ_1^d from 10_H with vev v_d
- One uptype Higgs doublet ϕ_2^u from 126_H with induced vev k_u
- One downtype Higgs doublet ϕ_2^d from 126_H with induced vev k_d

Axion mass is given by,

$$m_a = \frac{z^{1/2}}{1+z} \frac{f_\pi m_\pi}{f_a}$$

where $z = {}^{m_u}/_{m_d}$ and f_a is the axion decay constant. For the numerical analysis we took $m_\pi = 135$ MeV and $f_\pi \approx 135$ MeV and kept the range of $z = 0.35 - 0.60$. Then for $f_a = v_s$, we get m_a $(8 - 175)$ μeV, which is compatible with both the laboratory experimental limit and astrophysical bounds.

Axion in Minimal $SO(10)$ GUT

PQ-symmetry breaking before or during inflation

$$\Omega_a h^2 \approx 0.7 \left(\frac{f_a}{10^{12}\ GeV} \right)^{7/6} \left(\frac{\overline{\Theta}_i}{\pi} \right)^2 ,$$

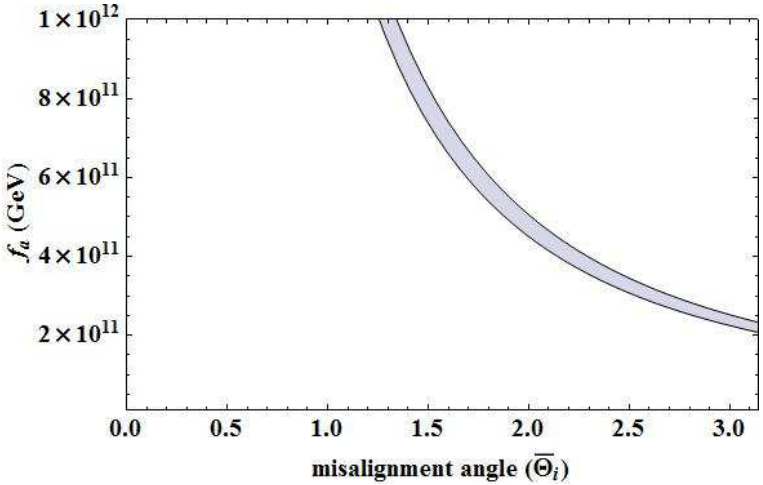


Figure : The region in the parameter space where cold ADM saturates DM abundance.

As the model allows the axion decay constant as low as 5×10^{10} GeV and as high as 10^{12} GeV, the misalignment angle can take any value beyond 1.26, ie $1.26 < \left|\overline{\Theta}_i\right| < \pi$. We also see that, for f_a smaller than 2.33×10^{11} GeV, axionic dark matter fails to explain the entirety of dark matter abundance.

Axion in Minimal $SO(10)$ GUT

PQ-symmetry breaking after inflation

$$\Omega_{a,mis} h^2 = 2.07 \left(\frac{f_a}{10^{12}\ GeV} \right)^{7/6}$$

while

$$\Omega_a h^2 = 2.07(1 + \alpha_{dec}) \left(\frac{f_a}{10^{12}\ GeV} \right)^{7/6}$$

Under such consideration, one finds that the Planck data corresponds to a axion mass $m_a \approx 80\ \mu$eV and axion decay constant $f_a \approx 8 \times 10^{10}$ GeV, which are perfectly admissible in the GUT under scrutiny.

E. Di Valentino, E. Giusarma, M. Lattanzi, A. Melchiorri and O. Mena [arXiv:1405.1860 [astro-ph.CO]];

Summary

- Here, we have constructed a Minimal Non-SUSY $SO(10)$ GUT model which is capable of explaining all the unanswered questions of standard model yet none of the experimental data can exclude the model.
- Besides the fact that the model has to rely on Fine-tuning for problems like hierarchy, the model is quite a natural one. We did not have to abandon ESH (Extended Survival Hypothesis) and no new fermions were needed.
- The Higgs sector was bare minimum to generate realistic fermion masses and that was good enough for solve other issues.
- The axion found in the model can explain Dark Matter.
- One issue that we did not touch yet in this model is "Inflation". But we realize that two field inflation has a natural place here.

Thank You

Fitting the experimental data(Contd.)

m_u (MeV)	0.495 ± 0.185	$\lvert V_{us}\rvert$	0.2254 ± 0.0006
m_d (MeV)	1.155 ± 0.495	$\lvert V_{cb}\rvert$	0.04194 ± 0.0006
m_s (MeV)	22.0 ± 7.0	$\lvert V_{ub}\rvert$	0.00369 ± 0.00013
m_c (GeV)	0.235 ± 0.035	J	$(3.16 \pm 0.1) \times 10^{-5}$
m_b (GeV)	1.00 ± 0.04	$\sin^2\theta^l_{12}$	0.308 ± 0.017
m_t (GeV)	74.15 ± 3.85	$\sin^2\theta^l_{23}$	0.3875 ± 0.0225
r	0.031 ± 0.001	$\sin^2\theta^l_{13}$	0.0241 ± 0.0025

Table 4: *Input values at the scale* $M_{GUT} = 2 \times 10^{16}$ *GeV.*

obs.	*fit*	*pull*	*obs.*	*fit*	*pull*
m_u(MeV)	0.49	0.03	$\lvert V_{us}\rvert$	0.225	0.038
m_d(MeV)	0.78	0.75	$\lvert V_{cb}\rvert$	0.042	-0.208
m_s(MeV)	32.5	-1.50	$\lvert V_{ub}\rvert$	0.0038	-0.659
m_c(GeV)	0.287	-1.49	J	3.1×10^{-5}	0.589
m_b(GeV)	1.11	-2.77	$\sin^2\theta^l_{12}$	0.318	0.611
m_t(GeV)	71.4	0.70	$\sin^2\theta^l_{23}$	0.353	-1.548
r	0.031	0.10	$\sin^2\theta^l_{13}$	0.0222	-0.758
η_B	5.699×10^{-10}	-0.001			

Table 5: *Best fit solutions for the fermion observables at the scale* $M_{GUT} = 2 \cdot 10^{16}$

light ν *masses (eV)*	*heavy* ν *masses* (10^{11} *GeV*)	*phases* (°)	m_{ee} *(eV)*
.0046	1.00	$\delta = 88.6$	5×10^{-4}
.0098	1.09	$\phi_1 = -33.2$	
.0504	21.4	$\phi_2 = 15.7$	

Table 6: *Predicted values of the light neutrino masses, of the heavy right-handed masses, of the leptonic CP-violating phases and of* m_{ee}.

Altarelli and Meloni., arXiv:1305.1001v2 [hep-ph]

Fitting the experimental data(Contd.)

A Best fit parameter values

Here we list the best fit values of the 15 parameters used in our fit procedure. The 12 elements in M_d are as follows:

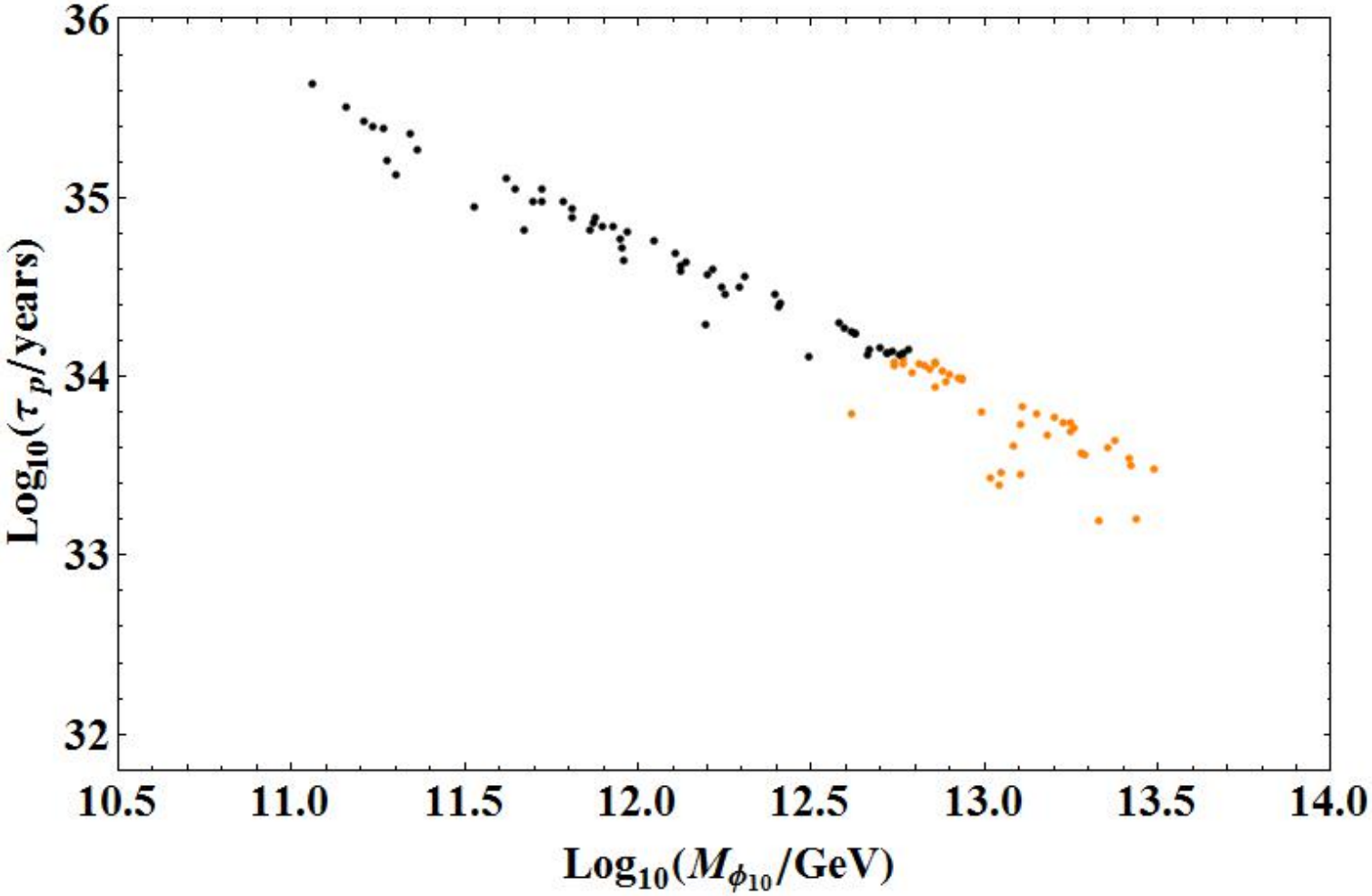
$$M_d\,(\mathrm{GeV}) = \begin{pmatrix} (-.0034, .0004) & (-7.7\times 10^{-6}, -.0098) & (-.0112, -.0712) \\ (-7.7\times 10^{-6}, -.0098) & (.0108, .0010) & (.2162, .0060) \\ (-.0112, -.0712) & (.2162, .0060) & (1.062, -.0584) \end{pmatrix},$$

The complex parameter s and the real parameter r_v are:

$$s = (.37, -.079) \qquad r_v = 60.03\,. \tag{30}$$

- The model is completely consistent with the Fermin mass fitting generated for Non-SUSY SO(10) models*. For example the sample point has $r_v = 60.9$ and $s = (0.36, 0)$

*Altarelli and Meloni., arXiv:1305.1001v2 [hep-ph]
*Joshipura and Patel., arXiv:1102.5148 [hep-ph]


36
35
34
33
32
10.5
11.0
11.5
12.0
12.5
13.0
13.5
14.0
$\mathrm{Log}_{10}(\tau_p/\mathrm{years})$
$\mathrm{Log}_{10}(M_{\phi_{10}}/\mathrm{GeV})$