The $B \to \pi l \nu$ and $B_s \to K l \nu$ form factors from 2+1 flavor lattice QCD

arXiv:1501.05373

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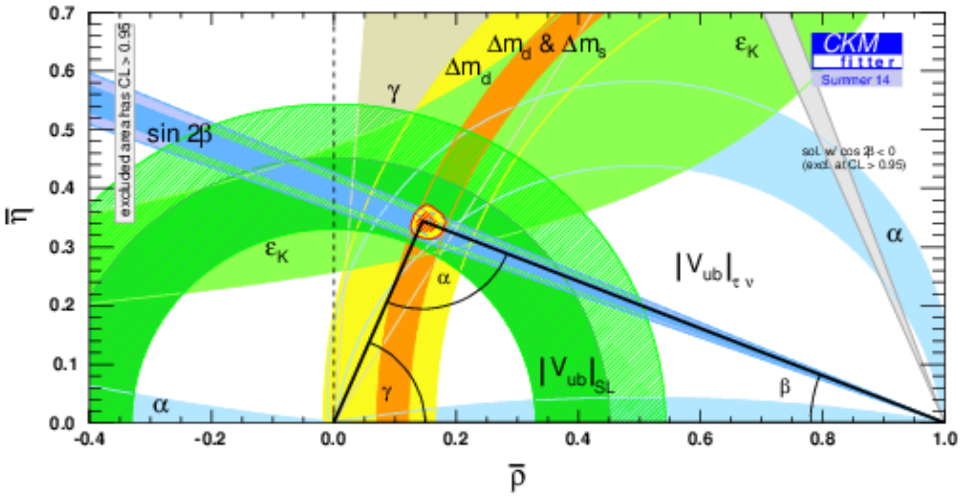
Motivation

A precise determination of $|V_{ub}|$ allows a strong test of the standard model.

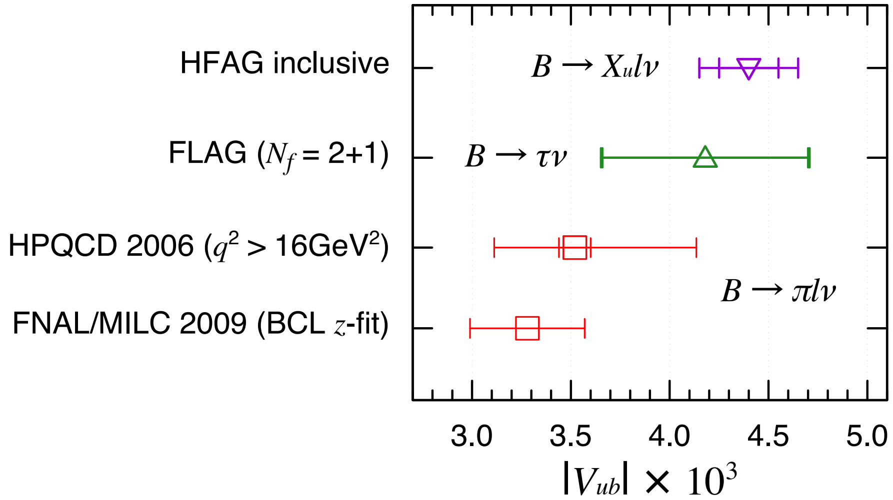
The constraint on the apex $(\bar{\rho},\ \bar{\eta})$ of the CKM triangle from $|V_{ub}|$ will strengthen tests of the Standard-Model CKM framework.



$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2+\bar{\eta}^2}$$

- $\lambda = |V_{ud}|$ known to ~ 1 %
- $|V_{cb}|$ known to ~2 %

Dominant error (green ring) comes from the uncertainty of $|V_{ub}|$ (~10%).

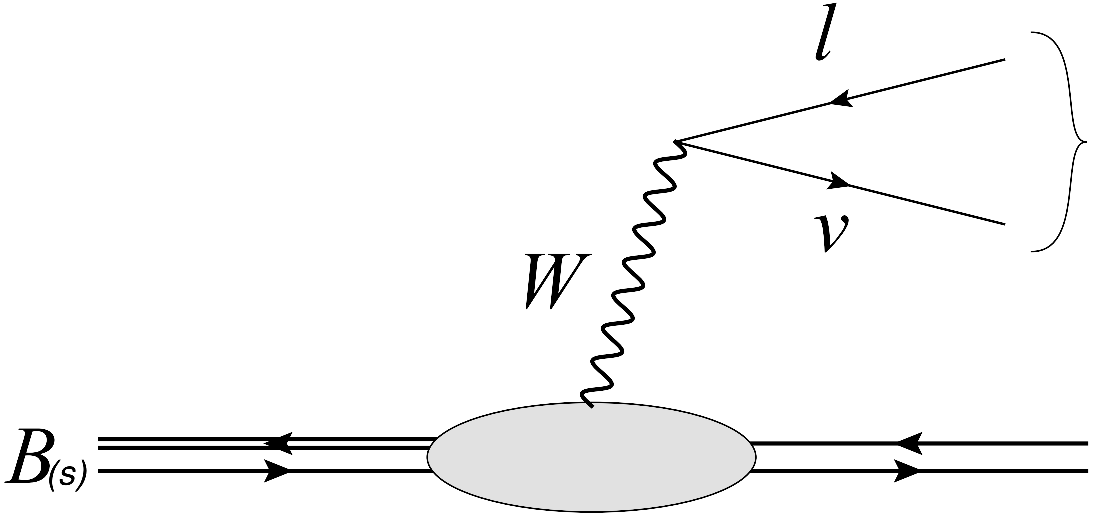


There has been a long standing puzzle in the determination of $|V_{ub}|$.

$\sim 3\sigma$ discrepancy between exclusive ($B \to \pi l \nu$) and inclusive ($B \to X_u l \nu$) determination.

Exclusive determination of $|V_{ub}|$

$f_+(q^2)$ is crucial for the determination of the CKM matrix element $|V_{ub}|$.



$$q^2 = m^2_{B_{(s)}} + m^2_P - 2 m_{B_{(s)}} E_P$$

•The exclusive $B_{(s)} \to P l \nu$ semileptonic decay allows the determination of $|V_{ub}|$ via:

Goal

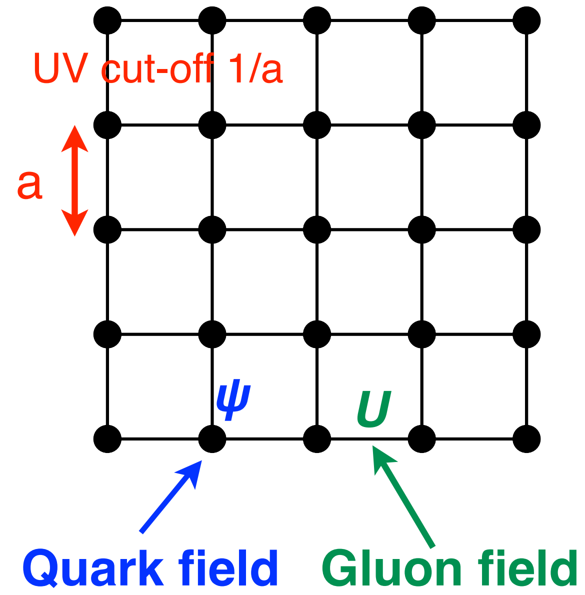
$$\boxed{\frac{d\Gamma}{dq^2}} = \frac{G_F^2}{192\pi^3 m^3_{B_{(s)}}} \left[(m^2_{B_{(s)}} + m^2_P - q^2)^2 - 4 m^2_{B_{(s)}} m^2_P \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|$$

Experiment   Known factor   Hadronic part   CKM matrix

•Experiment can only measure the CKM matrix element times hadronic form factor.

•The hadronic form factor must be computed nonperturbatively via lattice QCD.

Lattice QCD



Recent major progress

- Unquenched QCD simulations (in chiral regime)
- (mostly) Nonperturbative renormalization method
- New approach to heavy quarks on the lattice

$$\langle \mathcal{O}^{\rm cont} \rangle = \lim_{\substack{m_i \to m_i^{\rm phys} \\ a \to 0 \\ m_b \to m_b^{\rm phys}}} \int \mathcal{D}U \, \mathbf{Z}_{\mathcal{O}} \, \mathcal{O}^{\rm lat}[U] \prod_{i=u,d,s} \det K_f[U] \, e^{-S_{YM}[U]}$$

This work

○	1.	Sea quark effects	: $N_f = 0$ (quench) → $N_f = 2$ → $N_f = 2+1$
△	2.	Chiral-continuum limit	: $m_{ud} > m_s/6$ → (physical point)
○	3.	Renormalization factor	: perturbative → non-perturbative
○	4.	heavy quark	: Relativistic heavy quark action
×	5.	Statistics	: All-mode averaging (AMA) method

Lattice actions and parameters

- We use the 2+1 flavor dynamical **domain-wall** fermion gauge field configurations generated by the RBC/UKQCD Collaborations.

C. Allton et al. (RBC/UKQCD Collaboration), Phys. Rev. D78, 114509 (2008)
Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)

	$L \times T$	a [fm]	m_{ud}	m_s	m_π [MeV]	# of configs.	# of sources
Fine Lattice	32×64	≈ 0.08	0.004	0.03	289	628	2
	32×64	≈ 0.08	0.006	0.03	345	445	2
	32×64	≈ 0.08	0.008	0.03	394	544	2
Coarse Lattice	24×64	≈ 0.11	0.005	0.04	329	1636	1
	24×64	≈ 0.11	0.01	0.04	422	1419	1

- Provides important cross-check of existing $N_f = 2+1$ calculations using the MILC staggered ensembles.
- For the b-quark we use the relativistic heavy quark (RHQ) action developed by Li, Lin, and Christ in Refs. N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
  H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)
- We use the nonperturbative determinations of the parameters of the RHQ action obtained in Y.Aoki et. al Phys. Rev. D 86, 116003 (2012).

Form-factor definitions

- Non-perturbative form factors $f_+(q^2)$ and $f_0(q^2)$ parametrize the hadronic matrix element of the $b \to u$ quark flavor-changing vector current V_μ .

$$\langle P|V_\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu$$

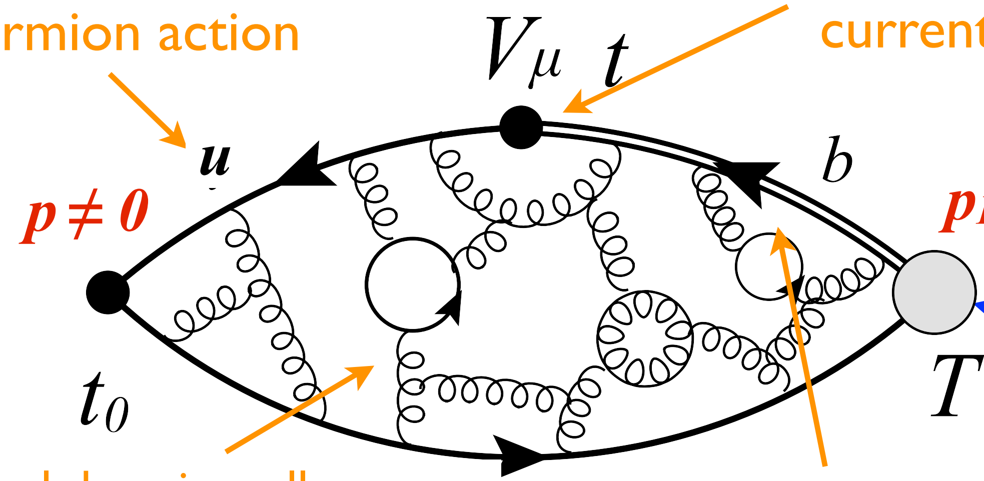
- On the lattice, we calculate the form factors $f_{||}$ and $f_\perp$.
  - Proportional to vector current matrix elements in the $B_{(s)}$ meson rest frame:

$$\begin{aligned} f_{||}(E_P) &= \langle P|V_0|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \\ f_\perp(E_P)p_i &= \langle P|V_i|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \end{aligned}$$

  - Easy to relate to the desired form factor $f_+(q^2)$ and $f_0(q^2)$.

$$\begin{aligned} f_0(q^2) &= \frac{\sqrt{2m_{B_{(s)}}}}{m^2_{B_{(s)}} - m^2_P}\left[(m_{B_{(s)}} - E_P)f_{||}(E_P) + (E^2_P - m^2_P)f_\perp(E_P)\right] \\ f_+(q^2) &= \frac{1}{\sqrt{2m_{B_{(s)}}}}\left[f_{||}(E_P) + (m_{B_{(s)}} - E_P)f_\perp(E_P)\right] \end{aligned}$$

Calculation of lattice form factors



- Extract the lattice form factor from the ratio of the 3pt function to 2pt functions:

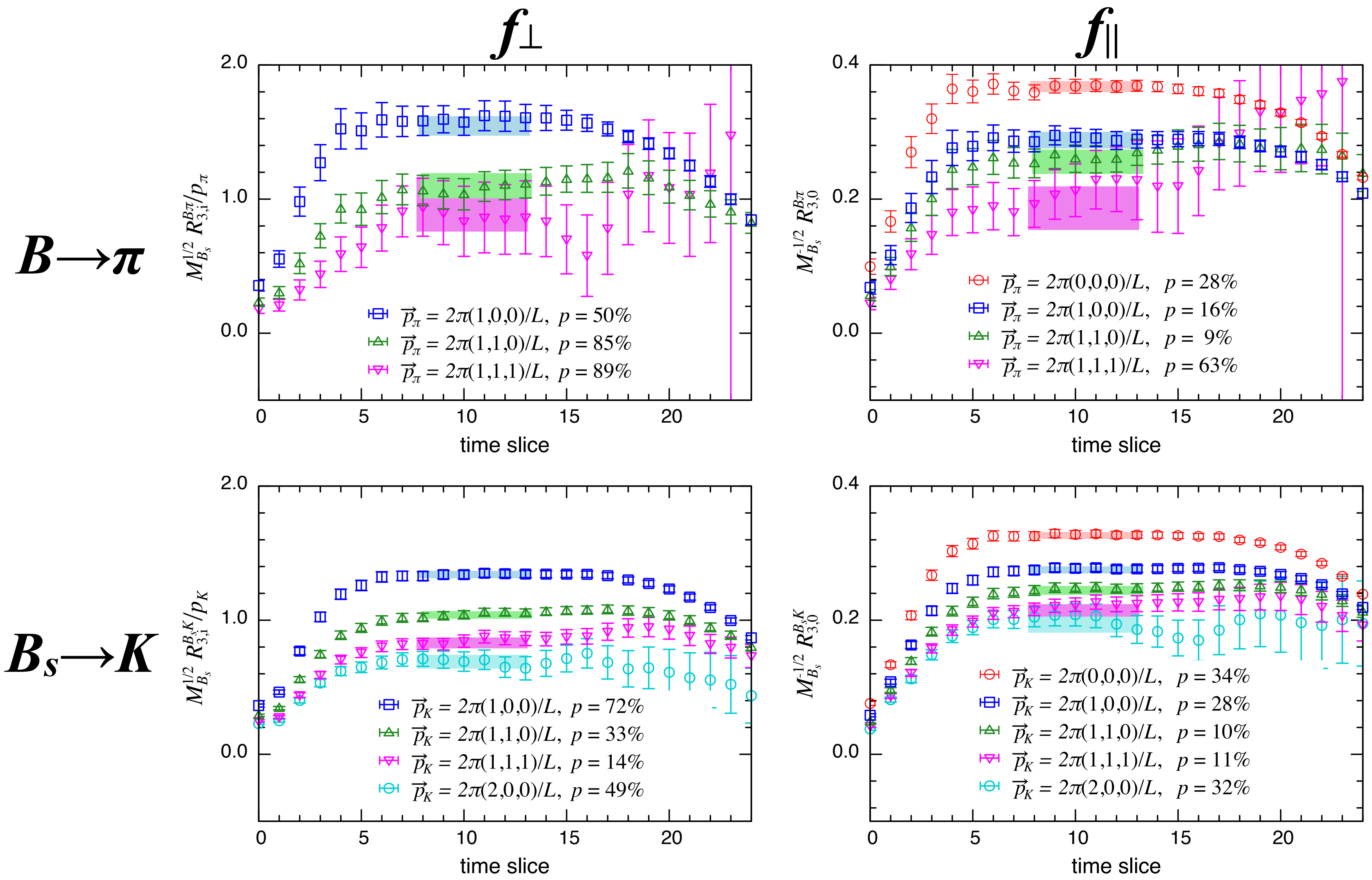
J. A. Bailey et al. (Fermilab Lattice and MILC), Phys. Rev. D79, 054507 (2009).

$$R_{3,\mu}^{B_{(s)}\to P}(t,T) = \frac{C_{3,\mu}^{B_{(s)}\to P}(t,T)}{\sqrt{C_2^P(t)C_2^{B_{(s)}}(T-t)}}\sqrt{\frac{2E_P}{e^{-E_P t}e^{-m_{B_{(s)}}(T-t)}}}$$

$$f_\parallel^{\text{lat}} = \lim_{t,T\to\infty} R_0^{B_{(s)}\to P}(t,T)$$

$$f_\perp^{\text{lat}} = \lim_{t,T\to\infty} \frac{1}{p_P^i} R_i^{B_{(s)}\to P}(t,T)$$

Three-point correlator fits



- We use the lattice data up to $(1,1,1)$ for $B\to\pi$ and $(2,0,0)$ for $B_s\to K$.
- After a careful study, we fix source-sink separations $T - t_0$
- We fit the ratio to a plateau in the region $0 \ll t \ll T$.

Renormalization of lattice form factors

- The continuum form factors are given by

$$\boxed{\begin{aligned} f_{\|}(E_P) &= Z_{V_0}^{bl} \lim_{t,T\to\infty} R_0^{B_{(s)}\to P}(E_P,t,T) \\ f_{\perp}(E_P) &= Z_{V_i}^{bl} \lim_{t,T\to\infty} \frac{1}{p_P^i} R_i^{B_{(s)}\to P}(E_P,t,T) \end{aligned}}$$

- We calculate the heavy-light current renormalization factor Z_V^{bl} using the mostly nonperturbative method. A. X. El-Khadra et al. Phys.Rev. D64, 014502 (2001)

$$Z_{V_\mu}^{bl} = \underbrace{\boxed{\rho_{V_\mu}^{bl}}}^{\approx 1} \sqrt{\boxed{Z_V^{bb} Z_V^{ll}}}$$

compute nonperturbatively

compute with 1-loop mean-field improved lattice perturbation theory

- ρ-factor calculated in PhySyHCAl (framework for automated lattice perturbation theory). C. Lehner arXiv:1211.4013
- Z_V^{ll} obtained by the RBC/UKQCD collaborations by exploiting the fact $Z_A=Z_V$ for domain-wall fermions. Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)
- Z_V^{bb} obtained from the matrix element of the $b\to b$ vector current between two B_s mesons. N. H.Christ et al. (RBC/UKQCD Collaboration), arXiv:1404.4670

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

- Correlated simultaneous chiral-continuum fit ($m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$) to $f_\perp$ and $f_{||}$ data using Hard-pion NLO SU(2) χPT.
  - Strange quark integrated out
  - Applies to regime where $E_P >> m_\pi$

J. Bijnens and I. Jemos, Nucl. Phys. B 840, 54 (2010)

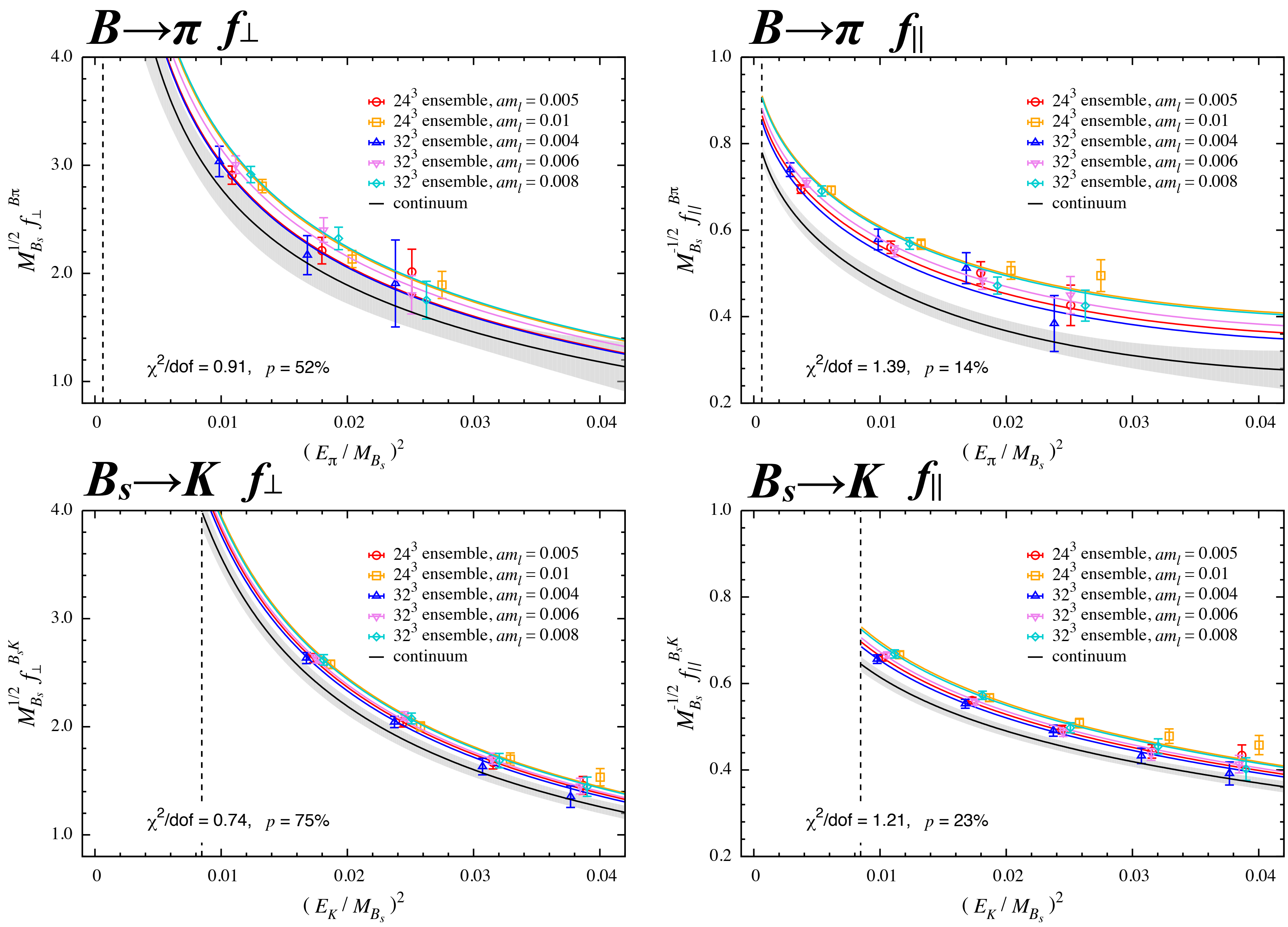
$$\begin{aligned} f_{\|}(m_\pi, E_P, a^2) &= c_{\|}^{(1)}\left(1+(\delta f_{\|})^{\text{Hard-pion}} + c_{\|}^{(2)}\frac{m_\pi^2}{\Lambda^2} + c_{\|}^{(3)}\frac{E_P}{\Lambda} + c_{\|}^{(4)}\frac{E_P^2}{\Lambda^2} + c_{\|}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right) \\ f_{\perp}(m_\pi, E_P, a^2) &= \frac{1}{E+m_B^* - m_B} c_{\perp}^{(1)}\left(1+(\delta f_{\perp})^{\text{Hard-pion}} + c_{\perp}^{(2)}\frac{m_\pi^2}{\Lambda^2} + c_{\perp}^{(3)}\frac{E_P}{\Lambda} + c_{\perp}^{(4)}\frac{E_P^2}{\Lambda^2} + c_{\perp}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right). \end{aligned}$$

The function δf indicate non-analytic "log" functions of the pion mass.

- The hard-pion SU(2) logarithms are given by simply taking the limit $m_\pi/E_P \to 0$.

$$\begin{aligned} (4\pi f_\pi)^2 (\delta f_{\|,\perp}^{B\to\pi})^{\text{Hard-pion}} &= -\frac{3}{4}\left(3g^2+1\right) m_\pi^2 \log\left(\frac{m_\pi^2}{\Lambda^2}\right) \\ (4\pi f_\pi)^2 (\delta f_{\perp,\|}^{B_s\to K})^{\text{Hard-pion}} &= -\frac{3}{4} m_\pi^2 \log\left(\frac{m_\pi^2}{\Lambda^2}\right) \end{aligned}$$

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$



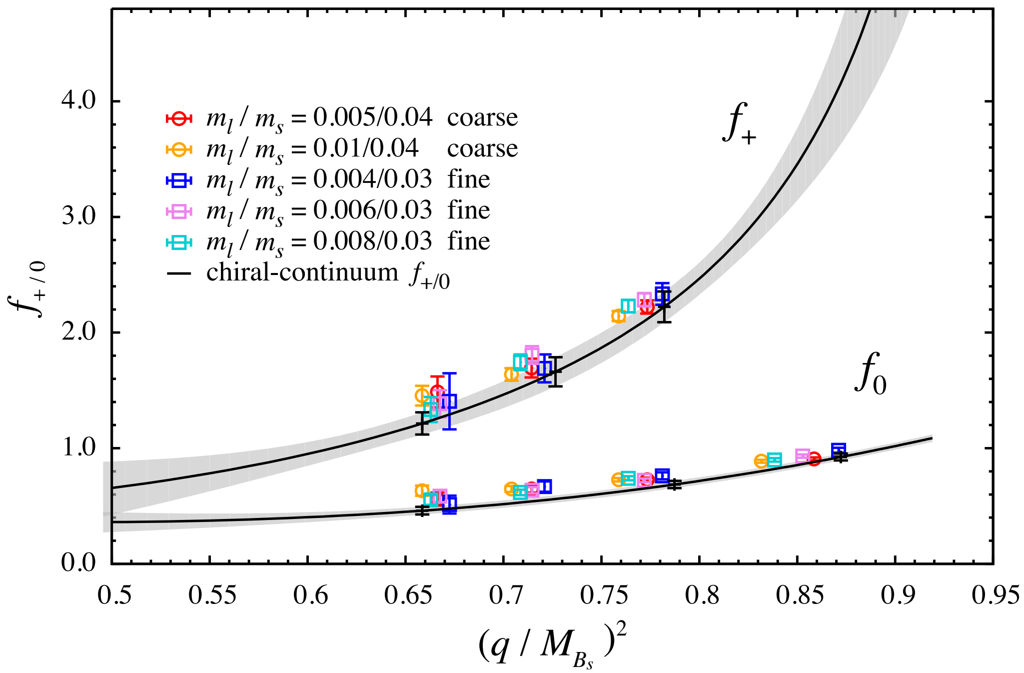
Black curves show chiral-continuum extrapolated $f_{||}$ and $f_\perp$ with statistical errors.

f_+ and f_0

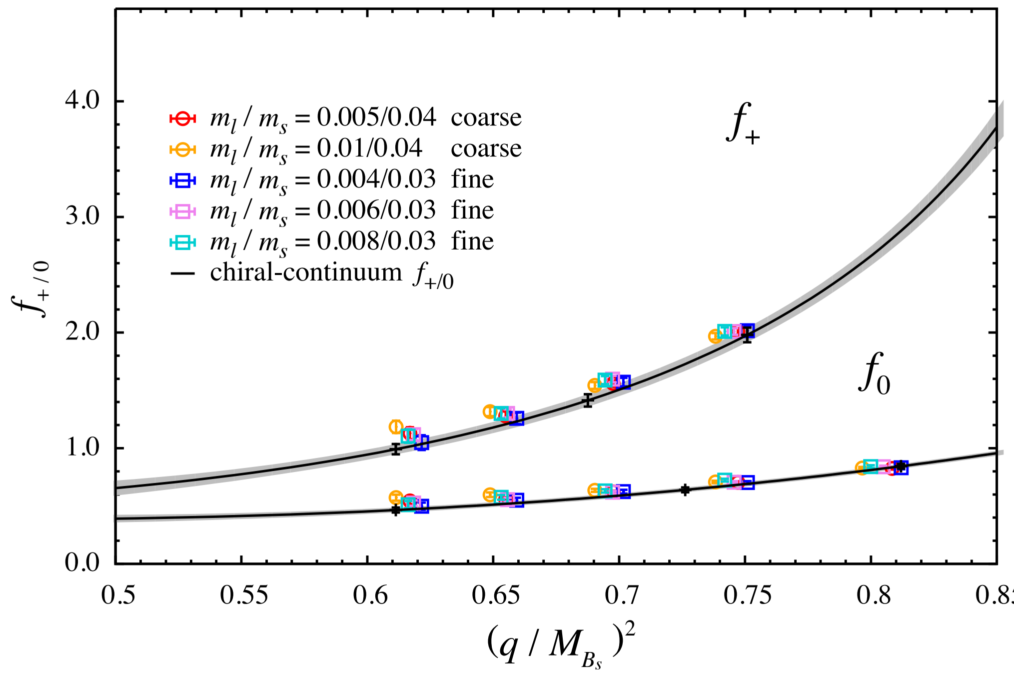
$$f_0(q^2) = \frac{\sqrt{2m_{B_{(s)}}}}{m^2_{B_{(s)}} - m^2_P}\left[(m_{B_{(s)}} - E_P)f_\parallel(E_P) + (E_P^2 - m_P^2)f_\perp(E_P)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2m_{B_{(s)}}}}\left[f_\parallel(E_P) + (m_{B_{(s)}} - E_P)f_\perp(E_P)\right]$$

$B \to \pi$



$B_s \to K$

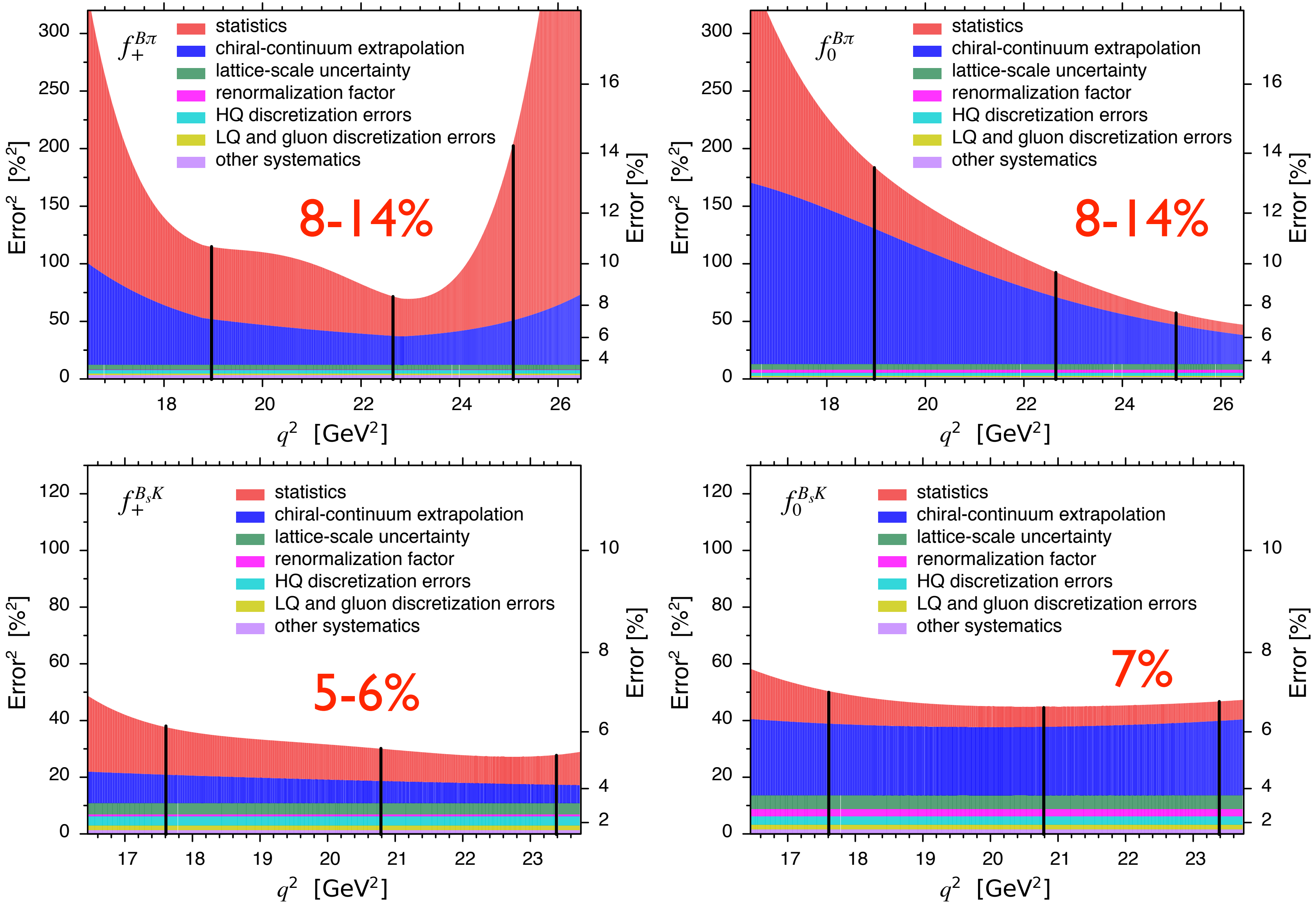


Error budgets

f_+ | f_0

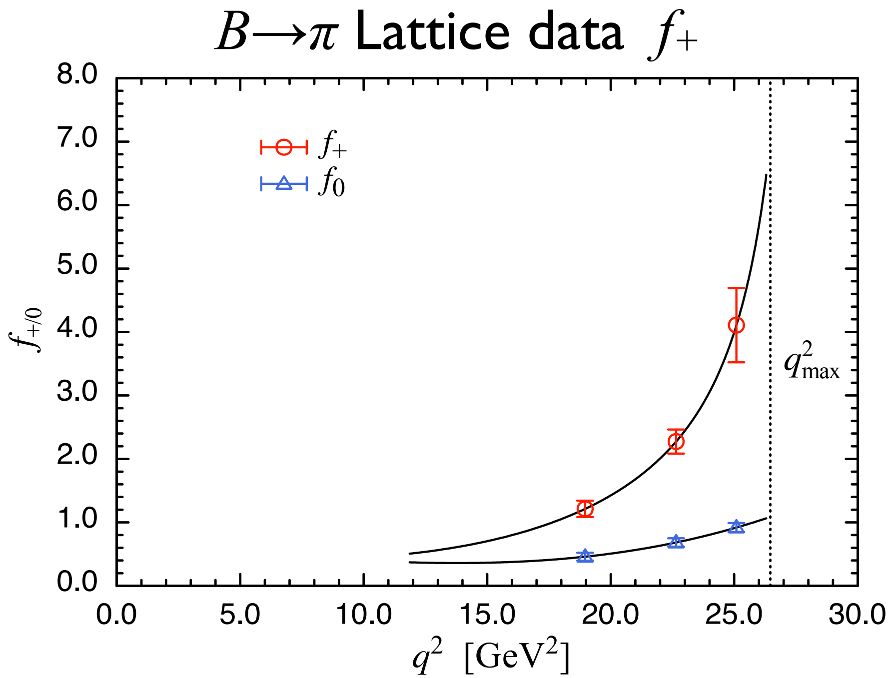
$B \to \pi$

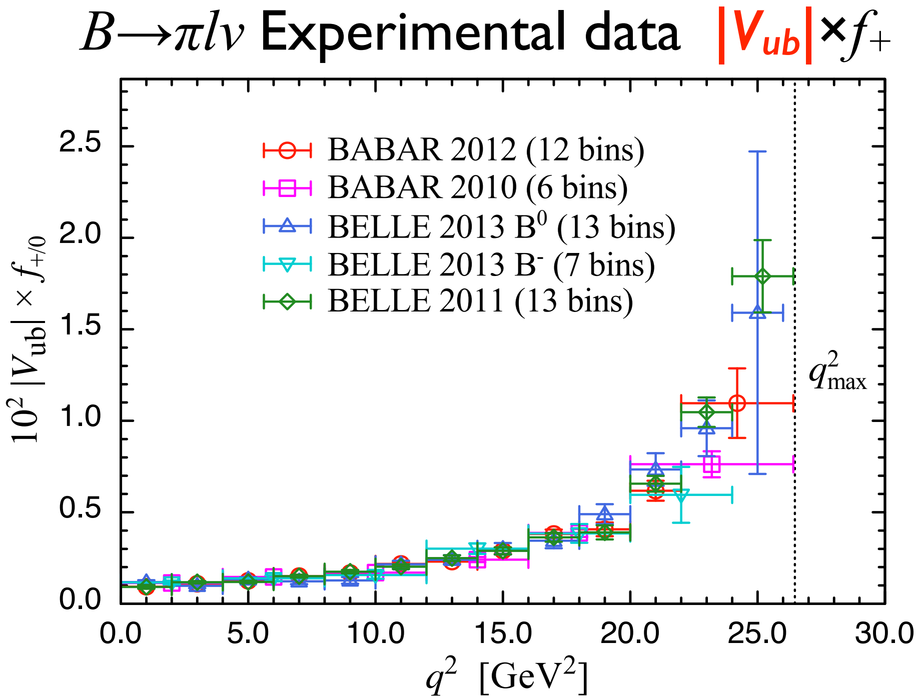
$B_s \to K$



- Dominant uncertainties from **statistics** and **chiral extrapolation.**

Determination of $|V_{ub}|$





- Lattice and experimental data overlap at high q^2 (low E_π^2),

Lattice: $\Delta\mathcal{B}/|V_{ub}|^2 = \frac{\tau_{B_0} G_F^2}{24\pi^2}\int_{(16\mathrm{GeV})^2}^{q^2_{\mathrm{max}}} dq^2 \vec{p}_\pi^{\,3} |f_+(q^2)|^2 = 2.69(52)$

Experiment: $\Delta\mathcal{B} = 3.68(19)\times 10^{-5}$ → $|V_{ub}|$ = 3.69(37) x10^{-3}

- But experimental data is more precise at low q^2.

→ We use **z-expansion fit** to extrapolate lattice data to zero q^2.

z-expansion of f_+ and f_0

Boyd, Grinstein, Lebed, Phys.Rev.Lett. 74 (1995) 4603

We employ the model-independent z-expansion fit to extrapolate lattice results to full kinematic range.

- Consider mapping the variable q^2 onto a new variable z.

  semileptonic region
  $0 < q^2 < t_-$ $\rightarrow$ $|z|<0.28$ (when we choose $t_0 = t_{opt}$)

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

- The form factor $f(q^2)$ is analytic in the semileptonic region except at B^* pole.
  $\rightarrow f(q^2)$ can be expressed as convergent power series.

$$f(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a^{(k)}(t_0) z(q^2, t_0)^k$$

contains subthreshold poles

Arbitrary analytic function which affects the numerical values of the series coefficients

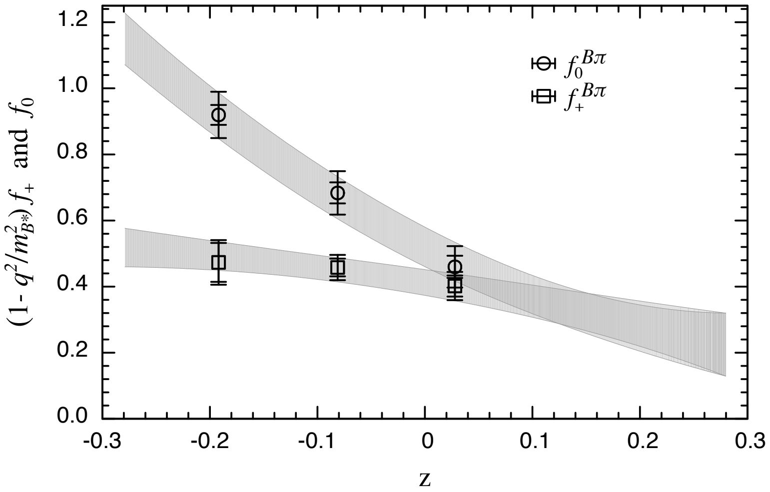
- The sum of the series coefficients is bounded by unitarity.

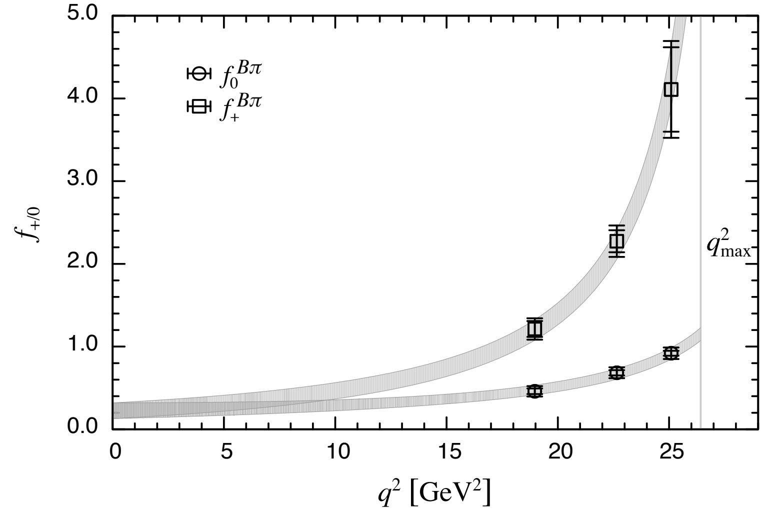
$$\sum_{k=0}^{N} {a^{(k)}}^2 \le 1$$

- Therefore this bound combined with the small $|z|$ ensures that only a small number of terms is needed to accurately describe the shape of the form factor.

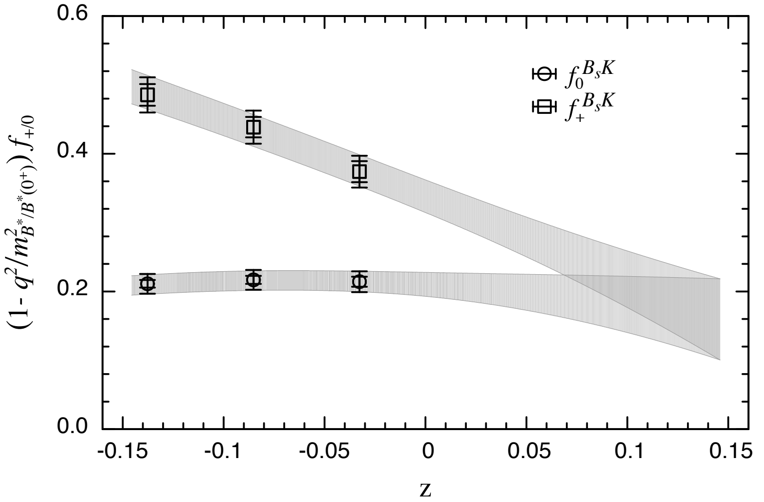
z-expansion of f_+ and f_0

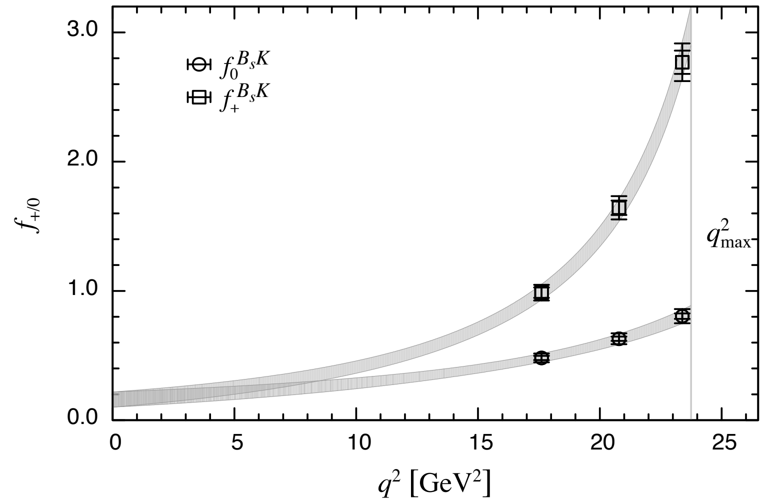
$B\to\pi$





$B_s\to K$

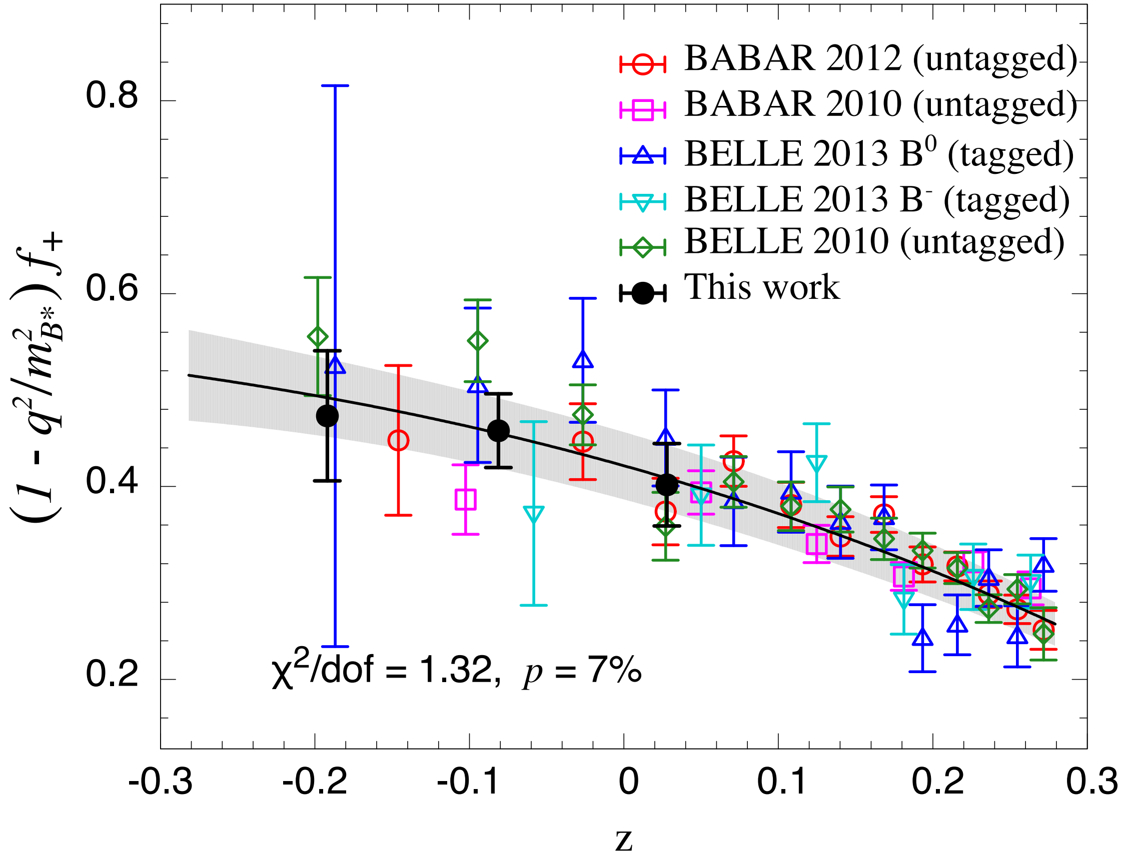




- Kinematic constraint: $f_+(0) = f_0(0)$
- heavy-quark power-counting: $\sum_{k=0}^{N}\left(a_+^{(k)}\right)^2 \sim \left(\frac{\Lambda}{m_b}\right)^3$

Determination of $|V_{ub}|$

Now add experimental data to z-fit to obtain $|V_{ub}|$.



- q^2 dependence of lattice form factor agrees well with experiment.
- Error on normalization (and hence $|V_{ub}|$) saturates with 3-parameter z-fit.

$$\boxed{|V_{ub}| = 3.61(32) \times 10^{-3}}$$

Conclusions and future prospects

- We have calculated the $B \to \pi$ and $B_s \to K$ form factors using 2+1 flavor dynamical domain-wall fermion gauge field configurations with relativistic heavy quark action.

- Provide important independent check on existing calculations using staggered light quarks.

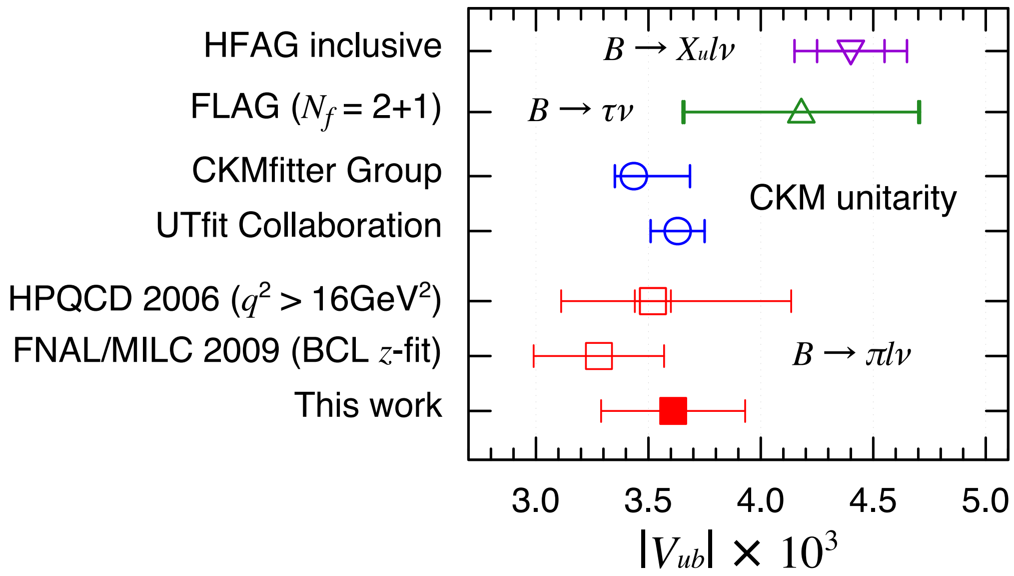
- presented results for $B \to \pi$ and $B_s \to K$ lattice form factors using z-expansion fit.

- $|V_{ub}|$ is determined by combined z-fit with experimental data from Babar and Belle to about 9% precision.

Future prospect

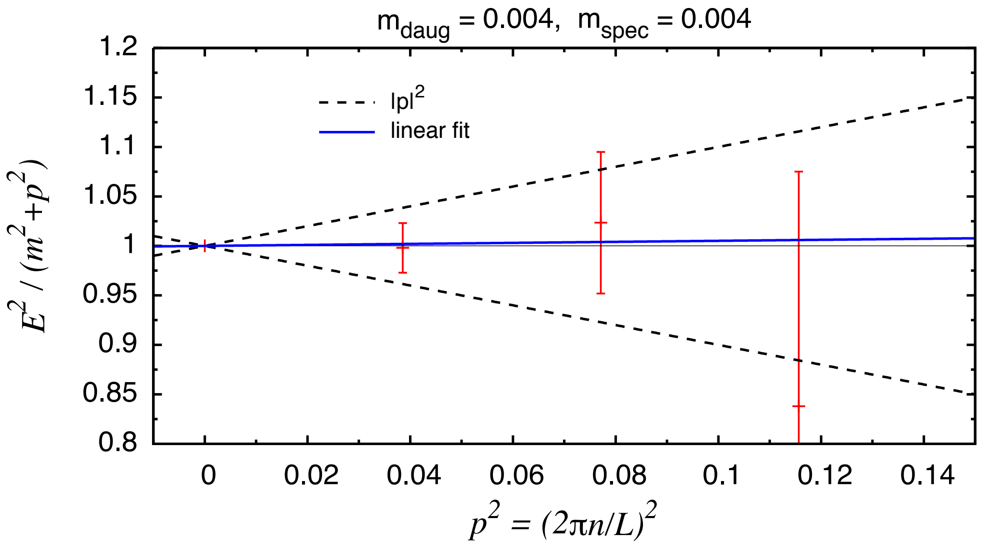
- RBC/UKQCD Möbius domain-wall + Iwasaki ensemble ($M_\pi \sim$ 140MeV).

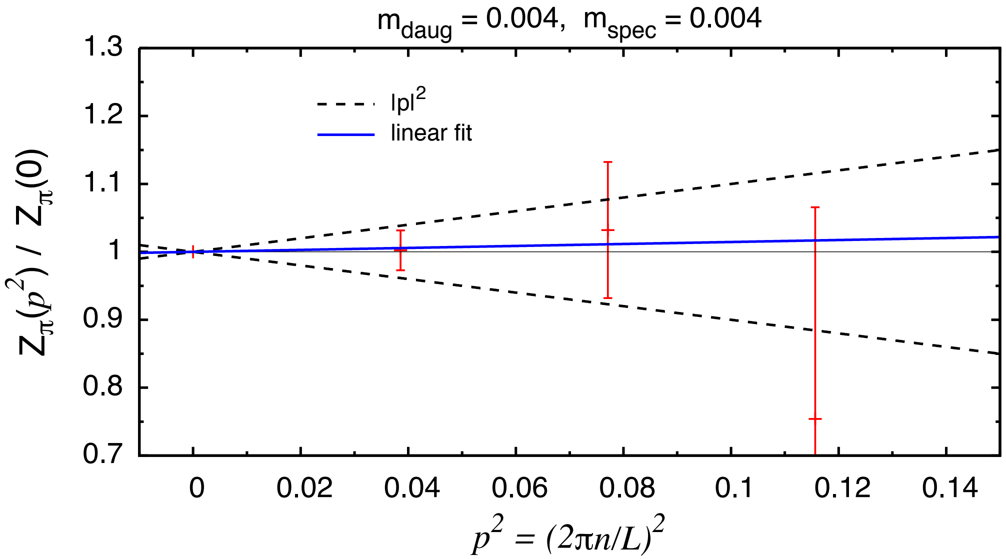
- All-mode averaging method



Backup slides

Dispersion relation and amplitude Z_π





- The pion energies satisfy the continuum dispersion relation: $E_\pi^2 = |\vec{p}_\pi|^2 + m_\pi^2$
- The pion amplitude $Z_\pi = |\langle 0|\mathcal{O}_\pi|\pi\rangle|$ is independent of momentum

$O(a)$ improved vector current operator

The heavy-light current operator at tree level is

$$V_{\mu,0}(x) = \bar{q}(x)\mathcal{O}_{\mu,0}Q(x), \quad \mathcal{O}_{\mu,0} = \gamma_\mu$$

Four single derivative operators are needed for O(a) improvement.

$$\boxed{\begin{array}{rcl} \mathcal{O}_{1,\mu} & = & 2\overrightarrow{D}_\mu \\ \mathcal{O}_{2,\mu} & = & 2\overleftarrow{D}_\mu \\ \mathcal{O}_{3,\mu} & = & 2\gamma_\mu\gamma_i\overrightarrow{D}_i \\ \mathcal{O}_{4,\mu} & = & 2\gamma_\mu\gamma_i\overleftarrow{D}_i \end{array}}$$

The O(a) improved vector current operator is given by

$$\begin{array}{rrcl} \text{temporal } (\mu=0): & \mathcal{O}_0^{\text{imp}} & = & \mathcal{O}_{0,0} + c_3^{V_0}\mathcal{O}_{0,3} + c_4^{V_0}\mathcal{O}_{0,4} \\ \text{spatial } (\mu=i): & \mathcal{O}_i^{\text{imp}} & = & \mathcal{O}_{i,0} + c_1^{V_i}\mathcal{O}_{i,1} + c_2^{V_i}\mathcal{O}_{i,2} + c_3^{V_i}\mathcal{O}_{i,3} + c_4^{V_i}\mathcal{O}_{i,4} \end{array}$$

Coefficients are determined by 1-loop lattice perturbation theory.

Relativistic heavy quark action for b-quarks

Heavy quark mass introduces discretization errors of $O((ma)^n)$.

- At bottom quark mass, it becomes severe: $m_b \sim 4$ GeV and $1/a \sim 2$ GeV, then $m_b a > O(1)$.

Relativistic heavy quark action (RHQ action)

$$S^{\mathrm{RHQ}} = \sum_{n,n'} \bar{\psi}_n \left\{ \mathbf{m_0} + \gamma_0 D_0 - \frac{a D_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a \vec{D}^2}{2} \right] - a \sum_{\mu\nu} \frac{i \mathbf{c_P}}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \psi'_n$$

- The Fermilab group showed that you can remove all errors of $O((ma)^n)$ by appropriately tuning the parameters of the anisotropic clover action

  A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D55, 3933 (1997)

- Errors are of $O(a^2p^2)$.

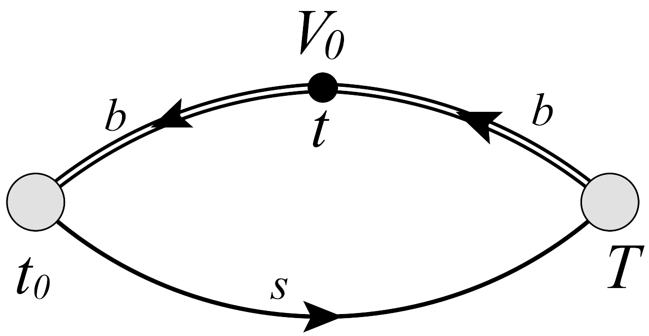
- Li, Lin, and Christ showed that the parameters $\{m_0, \zeta, c_P\}$ can be tuned nonperturbatively.

  N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
  H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

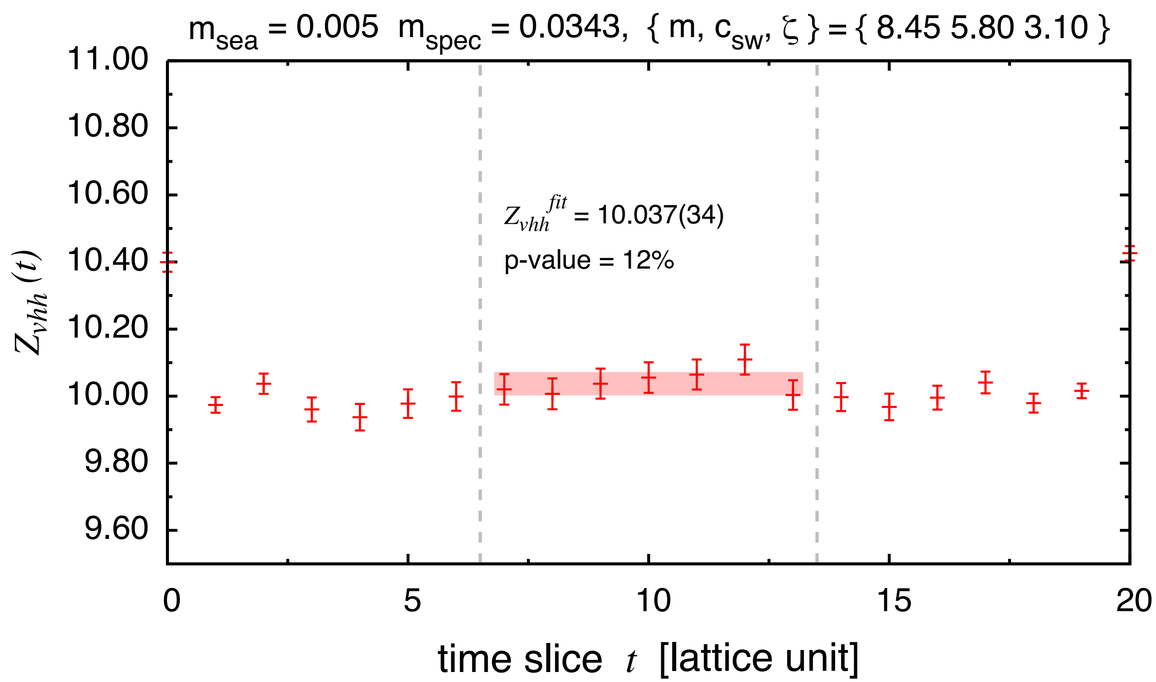
- We use the results for the parameters of the RHQ action obtained for b-quarks in Y. Aoki et. al Phys. Rev. D 86, 116003 (2012)

Renormalization factor $Z_V{}^{bb}$

Norman H.Christ et al., arXiv:1404.4670



$$
\begin{aligned}
Z_V^{bb} \times \langle m_0^B | V_0^{bb} | m_0^B \rangle &= 2m_B \\
\frac{C_2^B(T)}{C_{3,\mu}^{B\to B}(t,T)} &\xrightarrow{t,T\to\infty} Z_V^{bb}
\end{aligned}
$$

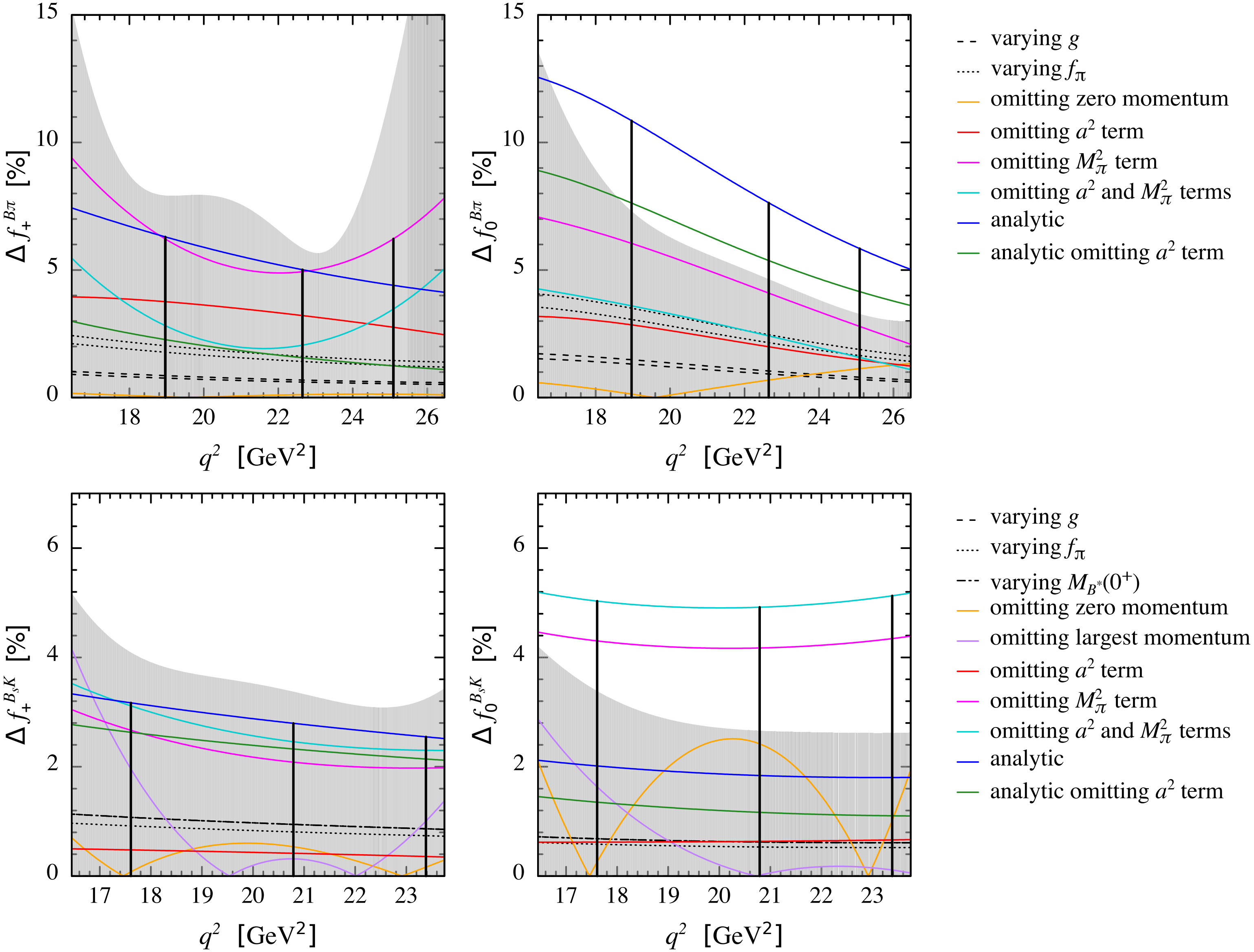


At tree level, the expression of $Z_V{}^{bb}$ is given by

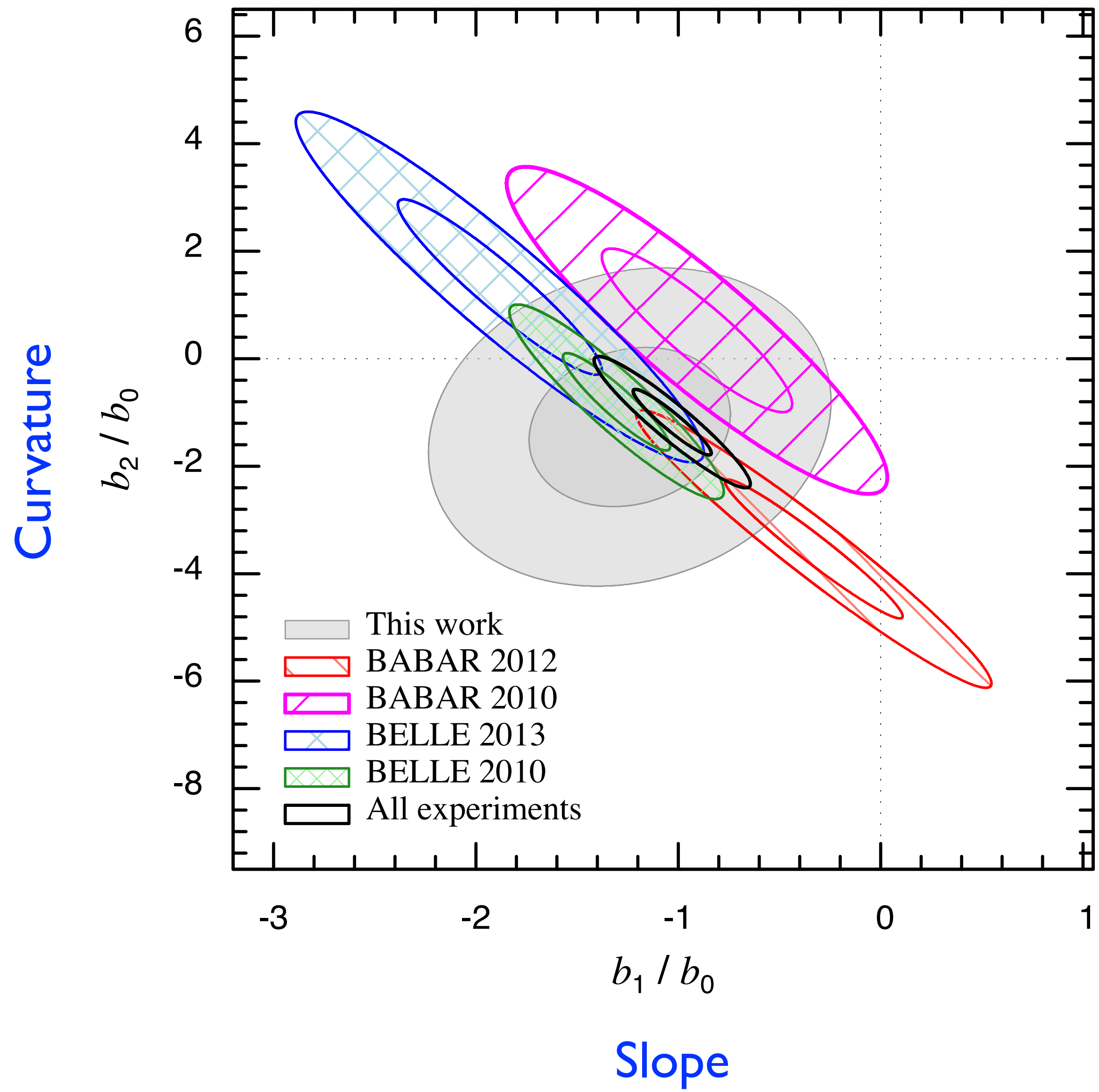
$$
Z_V^{bb} \quad = \quad u_0 \exp(M_1), \quad M_1 = \log[1 + \tilde{m}_0], \quad \tilde{m}_0 = \frac{m_0}{u_0} - (1+3\zeta)(1 - \frac{1}{u_0})
$$

Here $m_0 = 7.80$, $\zeta = 3.20$, $u_0 = 0.8757$.

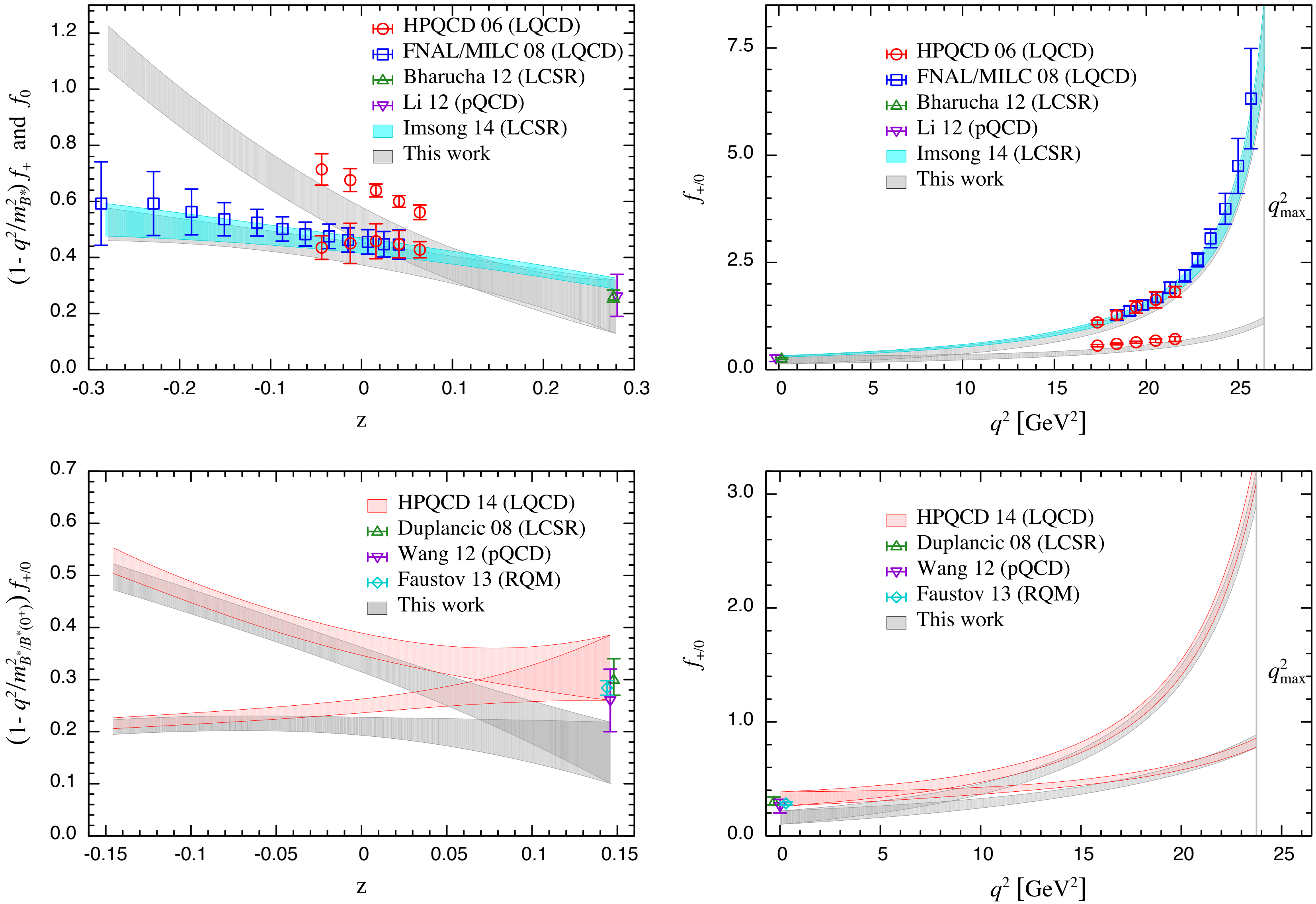
$$
\begin{aligned}
\textbf{NP} &: \quad Z_V^{bb} = 10.037(34) \\
\textbf{tree level} &: \quad Z_V^{bb} = 9.993
\end{aligned}
$$



varying g
varying f_π
omitting zero momentum
omitting a^2 term
omitting M_π^2 term
omitting a^2 and M_π^2 terms
analytic
analytic omitting a^2 term
$\Delta f_+^{B\pi}$ [%]
$\Delta f_0^{B\pi}$ [%]
q^2 [GeV^2]
varying g
varying f_π
varying $M_{B^*}(0^+)$
omitting zero momentum
omitting largest momentum
omitting a^2 term
omitting M_π^2 term
omitting a^2 and M_π^2 terms
analytic
analytic omitting a^2 term
$\Delta f_+^{B_sK}$ [%]
$\Delta f_0^{B_sK}$ [%]
q^2 [GeV^2]



Curvature
b_2 / b_0
6
4
2
0
-2
-4
-6
-8
-3
-2
-1
0
1
b_1 / b_0
Slope
This work
BABAR 2012
BABAR 2010
BELLE 2013
BELLE 2010
All experiments


HPQCD 06 (LQCD)
FNAL/MILC 08 (LQCD)
Bharucha 12 (LCSR)
Li 12 (pQCD)
Imsong 14 (LCSR)
This work
$(1-q^2/m^2_{B^*})f_+$ and f_0
z
$f_{+/0}$
q^2 [GeV^2]
q^2_{max}
HPQCD 14 (LQCD)
Duplancic 08 (LCSR)
Wang 12 (pQCD)
Faustov 13 (RQM)
This work
$(1-q^2/m^2_{B^*/B^*(0^+)})f_{+/0}$