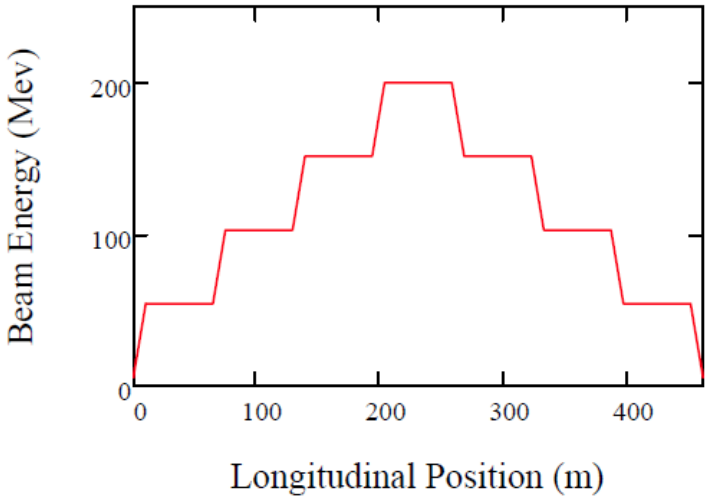
Beam Losses due to Gass Scattering in C-BETA

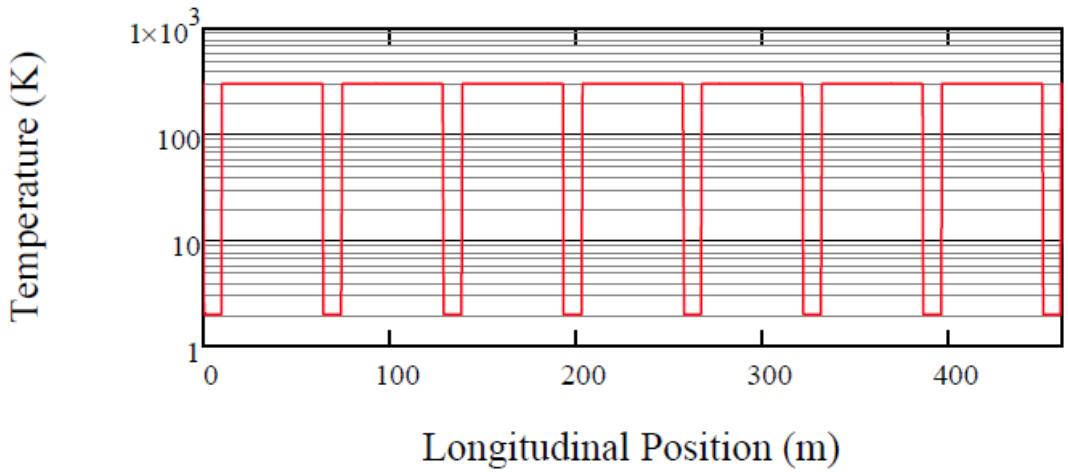
Analytical estimates using averaging beta functions

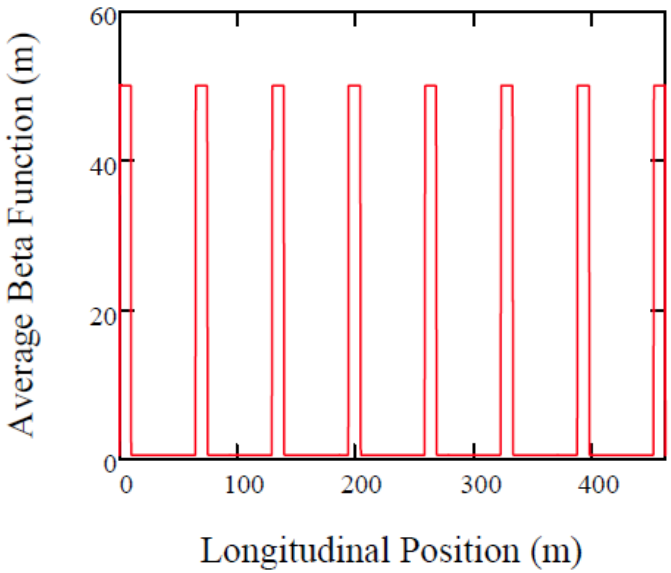
Parameters to be used for for C-BETA

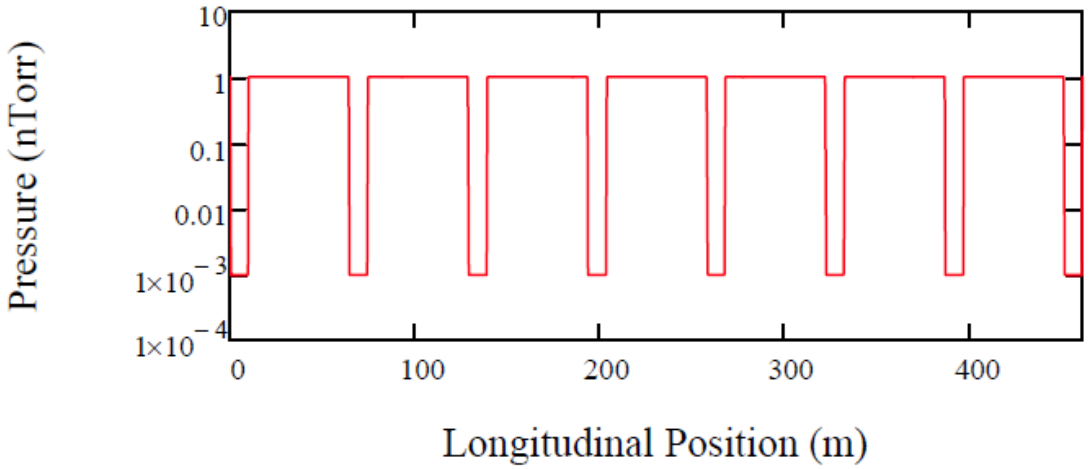
Avg current	0.13 ~ 1.3 A	Energy gain per pass	48.5 MeV
Bunch charge	256 pC	Number of FFAG passes	7
Rep. frequency	1.3 GHz	Cold temp. (linac)	2 K
Warm temp. (arcs+straight)	300 K	Linac pressure	1.E-12 Torr
Warm section pressure	1E-9 Torr	Avg. beta function in arcs	0.5 m
Ion species	H_2 , CO, H_2O	Norm. Emittance, RMS	2 um
Accelerator circumference	64.34 m	Linac length	10 m
Avg. beta function in linac	50 m		

Beam parameters along the ring









Beam Loss due to Elastic Scattering

$$I_{loss} = I_{avg} \int_0^{s_{coll}} n_{gas}(s)\sigma_c(\theta_{ape}(s))ds$$

Angle Aperture Due To Collimeter at Certain Location

$$\theta_{ape}(z_{scatter}) > \frac{y_{coll}}{\sqrt{\beta_{scatter}\beta_{coll}}\sin\Delta\psi}\sqrt{\frac{E_{coll}}{E_{scatter}}}$$

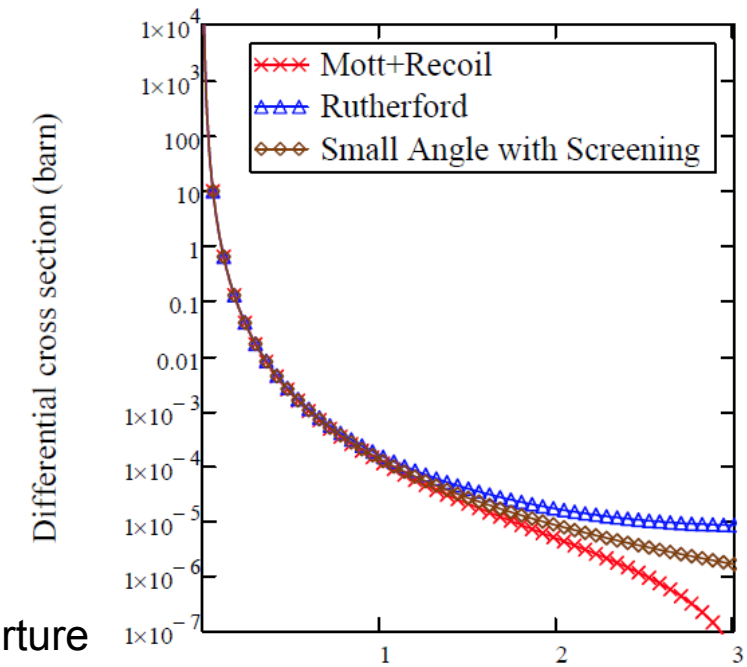
Averaging over initial phases and considering only 1-D aperture

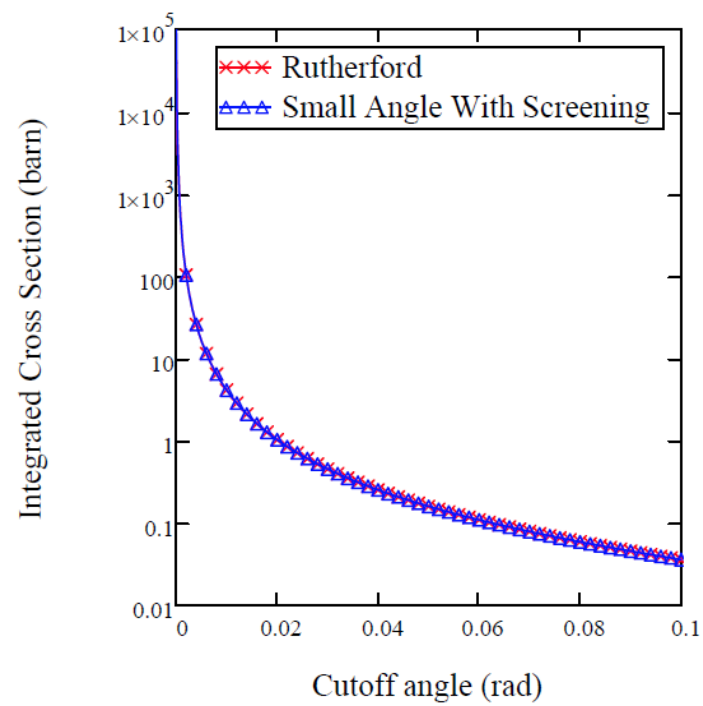
$$\theta_{ape}(z_{scatter}) > \frac{2y_{coll}}{\sqrt{\beta(z_{scatter})\beta(z_{coll})}}\sqrt{\frac{E(z_{coll})}{E(z_{scatter})}}$$

$$\sigma_c(\theta_{ape}) = \int_{\theta_{ape}}^{\theta_{max}} \left(\frac{d\sigma}{d\Omega}\right)_{\theta<<1} d\Omega = 4\pi Z_i^2 r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \left[\frac{1}{\theta_{ape}^{\ 2}+\theta_{min}^2} - \frac{1}{\theta_{max}^2+\theta_{min}^2}\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta<<1+screening} = 4Z_i^2 r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{(\theta^2+\theta_{min}^2)^2}$$

$$\theta_{min} = \frac{\hbar}{pa} \approx \frac{Z^{1/3}}{192}\frac{m_e c}{p} \approx \frac{2.6\times10^{-6}}{p(Gev)} \qquad \theta_{max} = \frac{\hbar}{pR} \approx \frac{Z^{1/3}}{192}\frac{m_e c}{p} \approx \frac{0.138}{p(Gev)}$$





Beam Loss Due to Bremsstrahlung

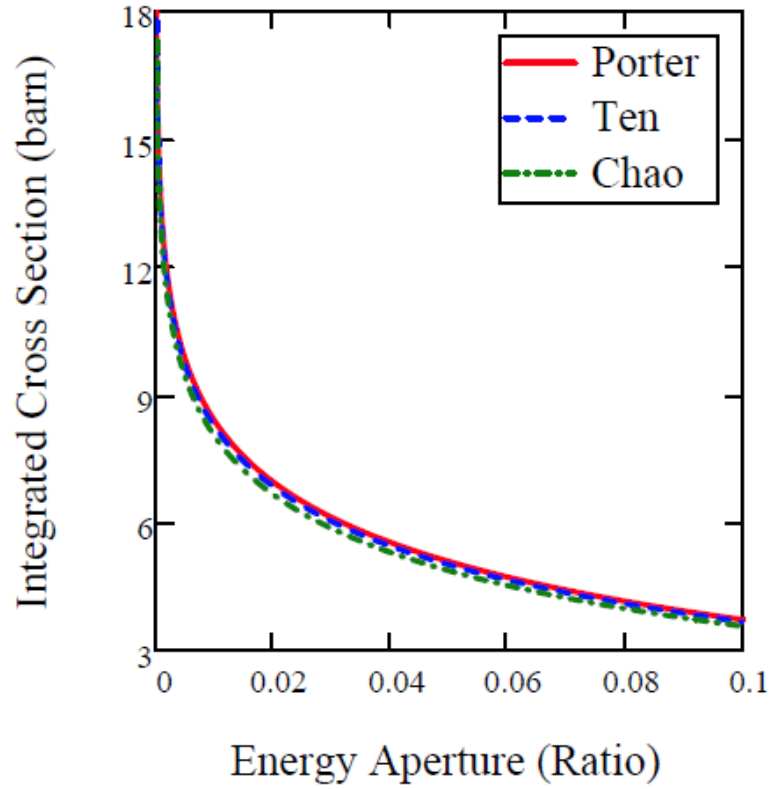
$$I_{loss} = I_{avg} \int_0^{s_{coll}} n_{gas}(s)\sigma_{brem}\left(\frac{\Delta E_{ape}}{E_0(s)}\right)ds$$

$$\left(\sigma_{brem}(u_{ape})\right)_{Porter} = \int_{u_{ape}}^{1}\left(\frac{d\sigma_{brem}}{dy}\right)_{Porter} dy$$

$$= \frac{16\alpha r_e^2}{3}\left\{\left(-\ln(u_{ape}) - \frac{5}{8} + u_{ape} - \frac{3}{8}u_{ape}^2\right)\left[Z^2\left(L_{rad} - f(\alpha^2 Z^2)\right) + ZL'_{rad}\right] - \frac{(Z+1)\left(\ln(u_{ape}) + 1 - u_{ape}\right)}{12}\right\}; \qquad u_{ape} = \frac{\Delta E_{ape}}{E_0}$$

$$\left(\frac{d\sigma_{brem}}{dy}\right)_{Porter} = \frac{4\alpha r_e^2}{y}\left\{\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)\left[Z^2\left(L_{rad} - f(\alpha^2 Z^2)\right) + ZL'_{rad}\right] + (1-y)(Z^2 + Z)/9\right\}$$

$$y = \frac{E_{photon}}{E_{electron}} \qquad L_{rad} = \ln\left(\frac{184.15}{Z^{1/3}}\right) \qquad L'_{rad} = \ln\left(\frac{1194}{Z^{1/3}}\right) \qquad f(x) = x\sum_{n=1}^{\infty}\frac{1}{n(n^2+x)}$$

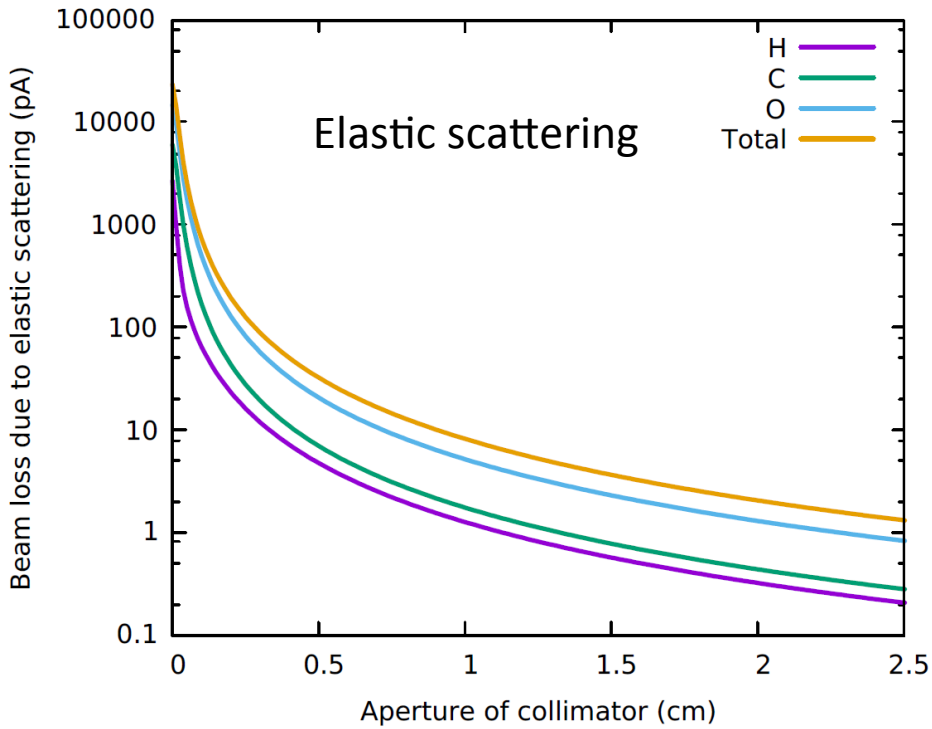


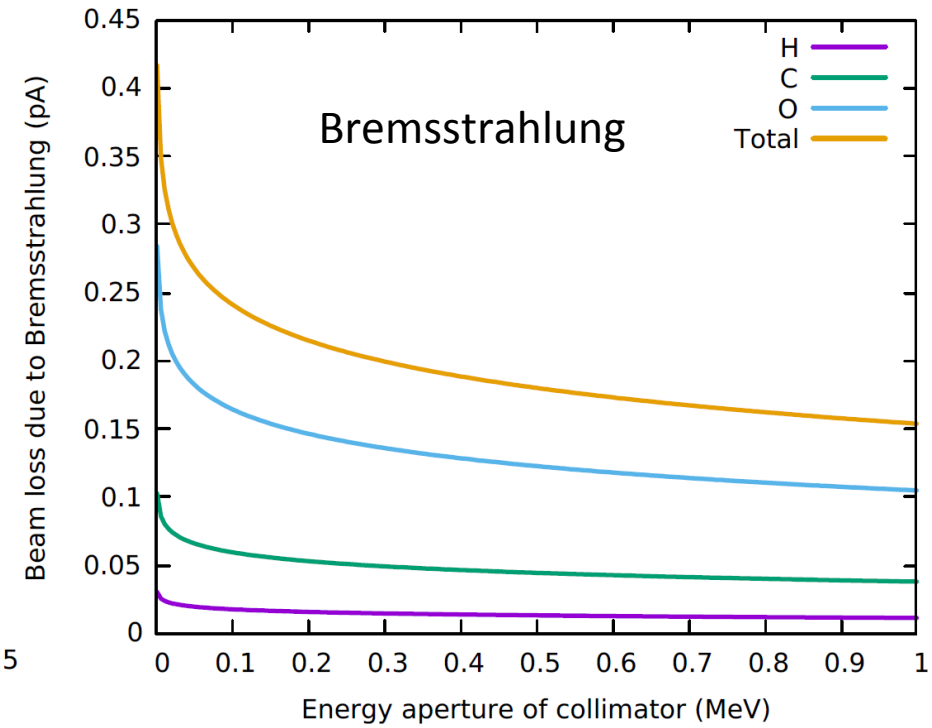
Reference:

1.'Luminosity lifetime at an asymmetric e+e- collider' NIM A302 (1991) 209-216, by F.C.Porter

Beam losses at initial operations

*The estimate assumes that the collimators locate at the last pass of the linac





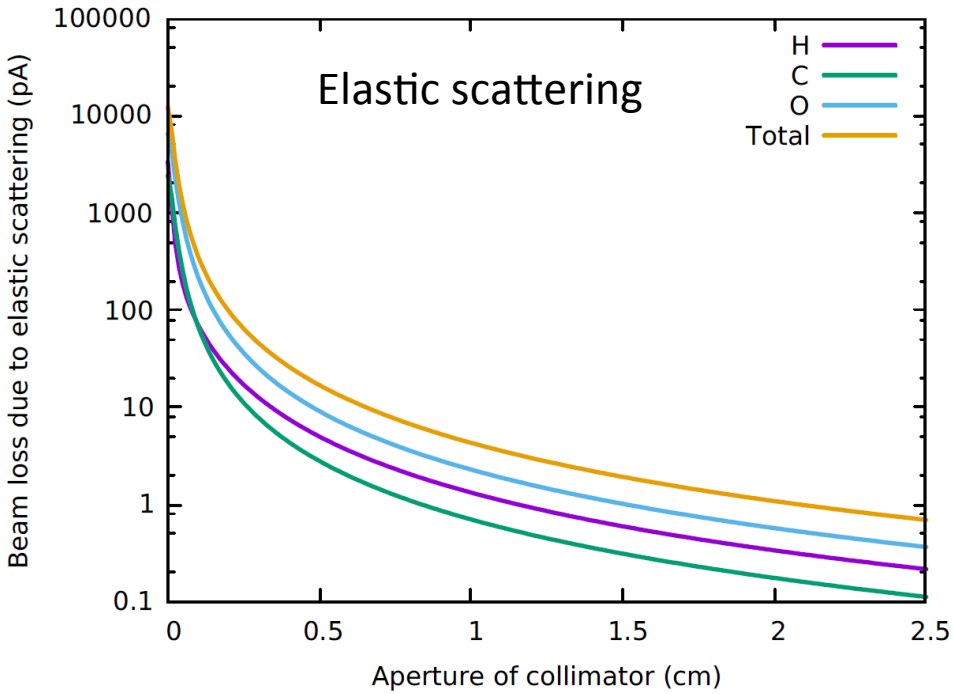
Partial pressure for warm sections, 1 nTorr	
H_2	50%
CO	30%
H_2O	20%

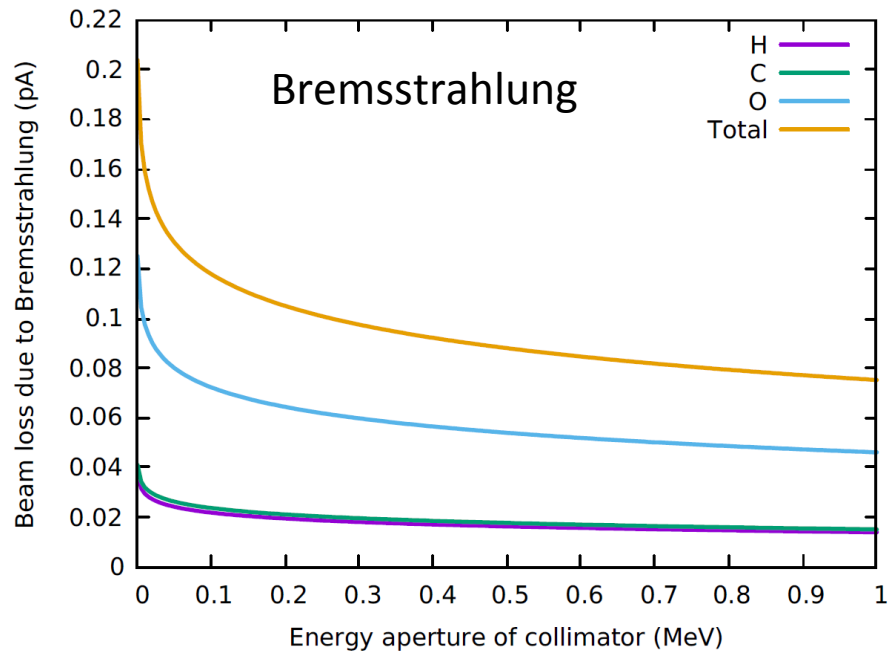
Partial pressure for cold sections, 0.001 nTorr	
H_2	100%

- For 2.5 cm aperture, the beam loss due to elastic scattering is 1.31 pA;
- For 1 MeV energy aperture, the beam loss due to Bremsstrahlung is 0.15pA

Beam losses at stable operations

*The estimate assumes that the collimators locate at the last pass of the linac





Partial pressure for warm sections, 1 nTorr	
H_2	78%
CO	12%
H_2O	10%

Partial pressure for cold sections, 0.001 nTorr	
H_2	100%

- For 2.5 cm aperture, the beam loss due to elastic scattering is 0.69 pA;
- For 1 MeV energy aperture, the beam loss due to Bremsstrahlung is 0.075 pA.

Summary

- Assuming that the limiting transverse aperture locates at the last linac pass, the analytical estimate shows that in the initial operation stage, the beam loss due to elastic scattering ranges from 1 pA (2.5 cm aperture) to 10 pA (1 cm aperture).
- Assuming that the limiting energy aperture locates at the last linac pass, the estimate shows that in the initial operation stage, the beam loss due to Bremsstrahlung ranges from 0.15 pA (0.1 MeV energy aperture) to 0.25 pA (1 MeV energy aperture).
- At the stable operation stage, the beam loss due to both processes reduces by a factor of 2.
- More accurate estimates can be achieved by numerical simulations (BMAD).

Backup Slides

Cross Section For Elastic Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{Z_i^2}{4} r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

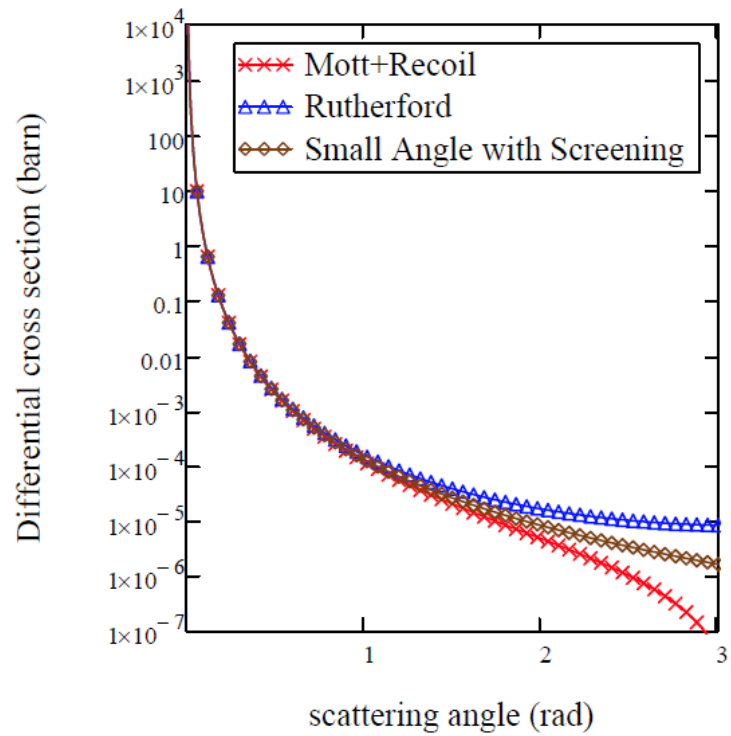
$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \left[1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right)\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott+\mathrm{Recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{1 + \sin^2(\theta/2)\frac{m_e \gamma \beta^2}{M_{nuclei}}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta << 1 + screening} = 4 Z_i^2 r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\left(\theta^2 + \theta_{\min}^2\right)^2}$$

$$\theta_{\min} = \frac{\hbar}{pa} \approx \frac{Z^{1/3}}{192} \frac{m_e c}{p} \approx \frac{2.6 \times 10^{-6}}{p(Gev)}$$

$$\theta_{\max} = \frac{\hbar}{pR} \approx \frac{Z^{1/3}}{192} \frac{m_e c}{p} \approx \frac{0.138}{p(Gev)}$$



Reference:

'Classical Electrodynamics' by Jackson, ch.13

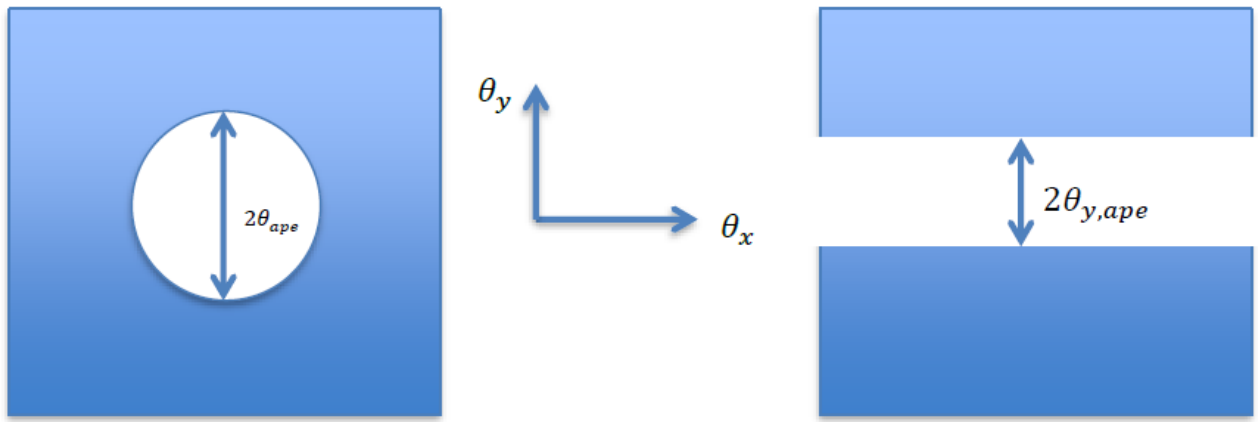
'Quantum Electrodynamics' by Landay&Lifshitz, p.632

'Handbook of acc. Physics and engineering' by Chao

Where the factor of 2 comes from?

$$\sigma(\theta_{scatter}) \sim \theta_{scater}^{-2} \sim \sin^2(\Delta\psi)$$

$$\sqrt{\frac{1}{2\pi}\int_0^{2\pi}\sin^2(\Delta\psi)d\Delta\psi} = \frac{1}{\sqrt{2}}$$



(a) (b)

Figure 1: angular aperture illustration of 1- D and 2- D. The horizontal direction is the horizontal deflection angle and the vertical direction is the vertical deflection angle. (a) 2-D angular collimator with aperture of θ_{ape}; (b) 1-D angular collimator with aperture of $\theta_{y,ape}$. For elastic scattering, eq. (7) suggests that, if $\theta_{ape} = \sqrt{2}\theta_{y,ape}$, the integrated cross-sections for (a) and (b) are the same. In other words, if a bunch of electrons incident into the collimator, the number of electrons passing through collimator (a) equals to the number of electrons passing through collimator (b) if their aperture satisfy $\theta_{ape} = \sqrt{2}\theta_{y,ape}$.

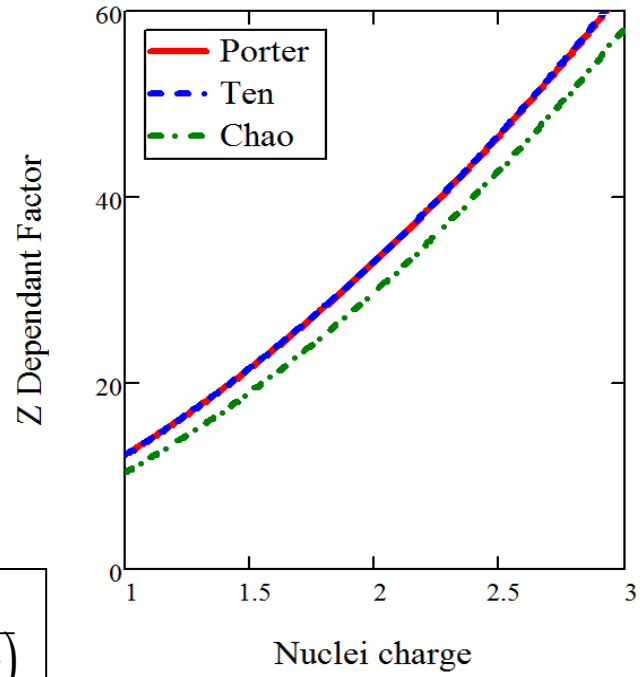
Cross Section For Bremsstrahlung

$$\left(\frac{d\sigma_{brem}}{dy}\right)_{Porter} = \frac{4\alpha r_e^2}{y}\left\{\left(\frac{4}{3}-\frac{4}{3}y+y^2\right)\left[Z^2\left(L_{rad}-f\left(\alpha^2 Z^2\right)\right)+ZL'_{rad}\right] + (1-y)\left(Z^2+Z\right)/9\right\}$$

$$y=\frac{E_{photon}}{E_{electron}} \qquad L_{rad}=\ln\left(\frac{184.15}{Z^{1/3}}\right) \qquad L'_{rad}=\ln\left(\frac{1194}{Z^{1/3}}\right) \qquad f(x)=x\sum_{n=1}^{\infty}\frac{1}{n\left(n^2+x\right)}$$

$$\left(\frac{d\sigma_{brem}}{dy}\right)_{Ten} = \frac{4\alpha r_e^2}{y}\left(\frac{4}{3}-\frac{4}{3}y+y^2\right)\left[Z^2\left(L_{rad}-f\left(\alpha^2 Z^2\right)\right)+ZL'_{rad}\right]$$

$$\left(\frac{d\sigma_{brem}}{dy}\right)_{Chao} = \frac{4\alpha r_e^2}{y}\left(\frac{4}{3}-\frac{4}{3}y+y^2\right)\left[Z^2+Z\right]L_{rad}$$

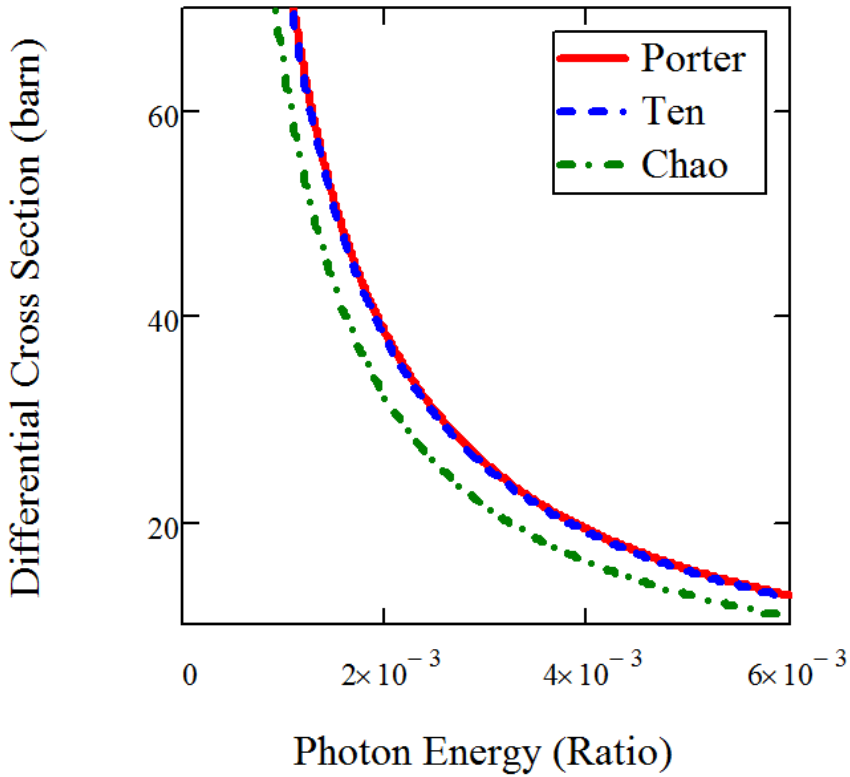


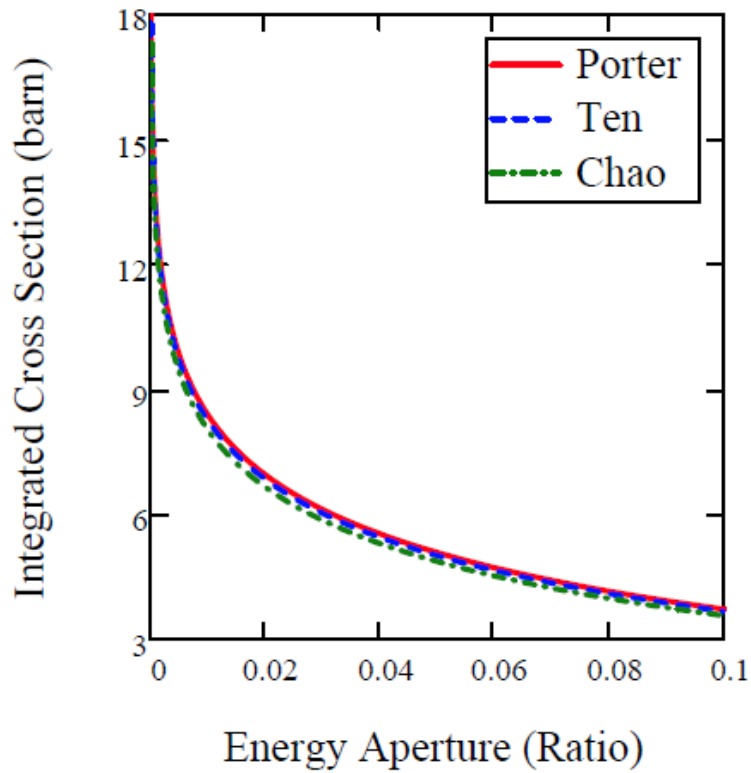
Reference:

1.'Handbook of Accelerator Physics and Engineering' by A.W.Chao p.213

2.'Beam-Gas and Thermal Photon Scattering in the NLC Main Linac as a Source of Beam Halo' by P. Tenenbaum, LCC-NOTE-0051(2001)

3.'Luminosity lifetime at an asymmetric e+e- collider' NIM A302 (1991) 209-216, by F.C.Porter





Only depends on $\Delta E / E$

$$\left(\sigma_{brem}(u_{\min})\right)_{Porter} = \int_{u_{\min}}^{1} \left(\frac{d\sigma_{brem}}{dy}\right)_{Porter} dy$$

$$= \frac{16\alpha r_e^2}{3}\left\{\left(-\ln(u_{\min}) - \frac{5}{8} + u_{\min} - \frac{3}{8}u_{\min}^2\right)\left[Z^2\left(L_{rad} - f\left(\alpha^2 Z^2\right)\right) + ZL'_{rad}\right] - \frac{(Z+1)\left(\ln(u_{\min}) + 1 - u_{\min}\right)}{12}\right\}$$