Towards universal fit of PDFs, spin-PDFs and fragmentation functions and beyond

Nobuo Sato

Joint CTEQ Meeting and POETIC 7, 2016

Motivations for EIC: Issues with SIDIS talk by O. Gonzalez

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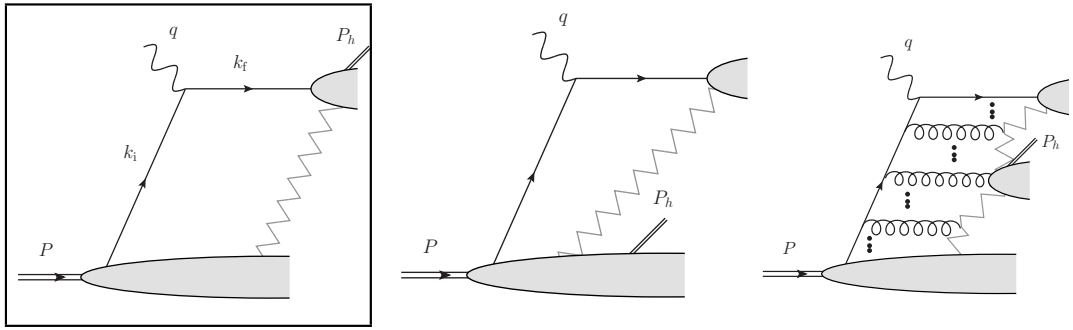

$$p_h \cdot k_i = \mathcal{O}(m^2)$$
$$p_h \cdot k_f = \mathcal{O}(Q^2)$$

- Define a *collinearity* parameter

$$R = \frac{(p_h \cdot k_f)}{(p_h \cdot k_f)} = \mathcal{O}(m^2/Q^2)$$

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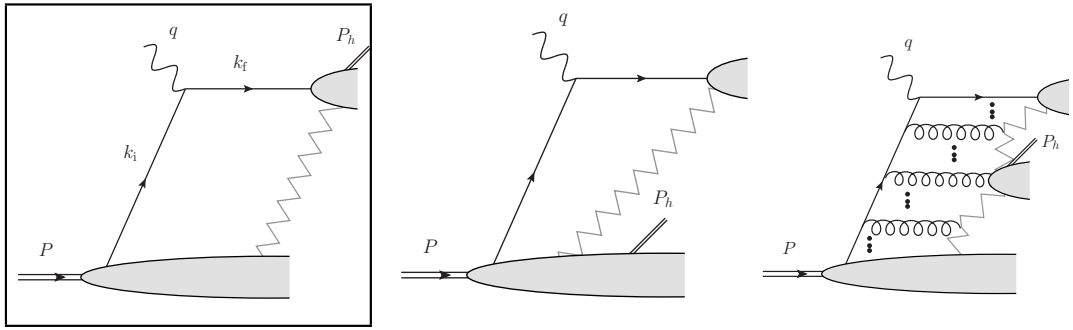

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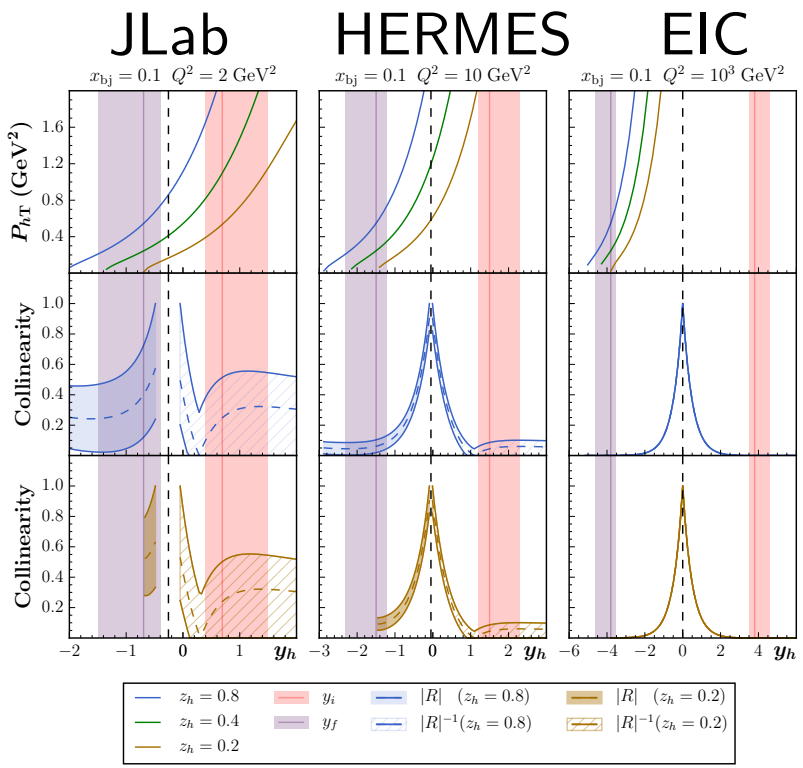
- Factorization seems to be optimal for EIC kinematics



Role of PDFs, SPDFs and FFs in TMDs

SIDIS cross section

$$\frac{d^5\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0 \Big[F_{UU} + \sin(\phi_h - \phi_s)\ F_{UT}{}^{\sin(\phi_h-\phi_s)} + \sin(\phi_h + \phi_s)\ \frac{2(1-y)}{1+(1-y)^2}\ F_{UT}{}^{\sin(\phi_h+\phi_s)} + \ldots \Big]$$

SIA cross section

$$\frac{d^5\sigma^{e^+e^-\to h_1 h_2 + X}}{dz_{h1} dz_{h2} d^2P_{h\perp} d\cos\theta} = \sigma_0 \left[\left(1+\cos^2\theta\right) Z_{uu}^{h_1 h_2} + \sin^2\theta \cos(2\phi_0) Z_{\text{collins}}^{h_1 h_2} \right]$$

Role of PDFs, SPDFs and FFs in TMDs

SIDIS structure functions (W+Y)

$$F_{UU} = H_{\rm SIDIS} \frac{1}{z_h^2} \int_0^\infty \frac{db\, b}{(2\pi)} J_0(q_{h\perp} b)\, \tilde{W}_{UU}(b_*) + Y_{UU}$$

$$F_{UT}^{\sin(\phi_h - \phi_s)} = -H_{\rm SIDIS} \frac{M_P}{z_h^2} \int_0^\infty \frac{db\, b^2}{(2\pi)} J_1\left(q_{h\perp} b\right)\, \tilde{W}_{UT}^{\sin(\phi_h - \phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h - \phi_s)}$$

$$F_{UT}^{\sin(\phi_h + \phi_s)} = H_{\rm SIDIS} \frac{M_h}{z_h^2} \int_0^\infty \frac{db\, b^2}{(2\pi)} J_1(q_{h\perp} b)\, \tilde{W}_{UT}^{\sin(\phi_h + \phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h + \phi_s)}$$

SIA structure functions (W+Y)

$$Z_{uu}^{h_1 h_2} = H_{\rm e^+e^-} \frac{1}{z_{h1}^2} \int_0^\infty \frac{db\, b}{(2\pi)} J_0(q_{h\perp} b)\, \tilde{W}_{UU}^{h_1 h_2}(b_*) + Y_{UU}^{h_1 h_2}$$

$$Z_{\rm collins}^{h_1 h_2} = H_{\rm e^+e^-} \frac{M_{h_1} M_{h_2}}{z_{h1}^2} \int_0^\infty \frac{db\, b^3}{(2\pi)} J_2(q_{h\perp} b)\, \tilde{W}_{collins}^{h_1 h_2}(b_*) + Y_{collins}^{h_1 h_2}$$

Role of PDFs, SPDFs and FFs in TMDs

SIDIS W

$$\tilde{W}_{UU}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm f_1}(Q,b)-S_{\rm NP}^{\rm D_1}(Q,b)}\ \widetilde{F}_{UU}(b_*)$$

$$\tilde{W}_{UT}^{\sin(\phi_h-\phi_s)}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm f_{1T}^{\perp}}(Q,b)-S_{\rm NP}^{\rm D_1}(Q,b)}\ \widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(b_*)$$

$$\tilde{W}_{UT}^{\sin(\phi_h+\phi_s)}(b_*) \equiv e^{-S_{pert}(Q,b_*)-S_{\rm NP}^{\rm h_1}(Q,b)-S_{\rm NP}^{\rm H_1^{\perp}}(Q,b)}\ \widetilde{F}_{UT}^{\sin(\phi_h+\phi_s)}(b_*)$$

SIA W

$$\tilde{W}_{uu}^{h_1h_2}(b_*) \equiv e^{-S_{\rm pert}(Q,b_*)-S_{\rm NP}^{D_1}(Q,b)-S_{\rm NP}^{\bar{D}_1}(Q,b)}\ \widetilde{Z}_{uu}^{h_1h_2}(b_*)$$

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Role of PDFs, SPDFs and FFs in TMDs

SIDIS W (small b contribution)

$$\widetilde{F}_{UU}(b_*) = \sum_q e_q^2 \left(C^{\mathrm{f}_1}_{q\leftarrow i} \otimes f_1^i(x_B,\mu_b)\right) \left(\hat{C}^{\mathrm{D}_1}_{j\leftarrow q} \otimes D_{h/j}(z_h,\mu_b)\right) ,$$

$$\widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(b_*) = \sum_q e_q^2 \left(C^{f_{1T}^\perp}_{q\leftarrow i} \otimes f_{1T}^{\perp(1)i}(x_B,\mu_b)\right) \left(\hat{C}^{\mathrm{D}_1}_{j\leftarrow q} \otimes D_{h/j}(z_h,\mu_b)\right) .$$

$$\widetilde{F}_{UT}^{\sin(\phi_h+\phi_s)}(b_*) = \sum_q e_q^2 \left(\delta C^{h_1}_{q\leftarrow i} \otimes h_1^i(x_B,\mu_b)\right) \left(\delta\hat{C}^{\mathrm{H}_1^\perp}_{j\leftarrow q} \otimes \hat{H}_1^{\perp(1)j}(z_h,\mu_b)\right)$$

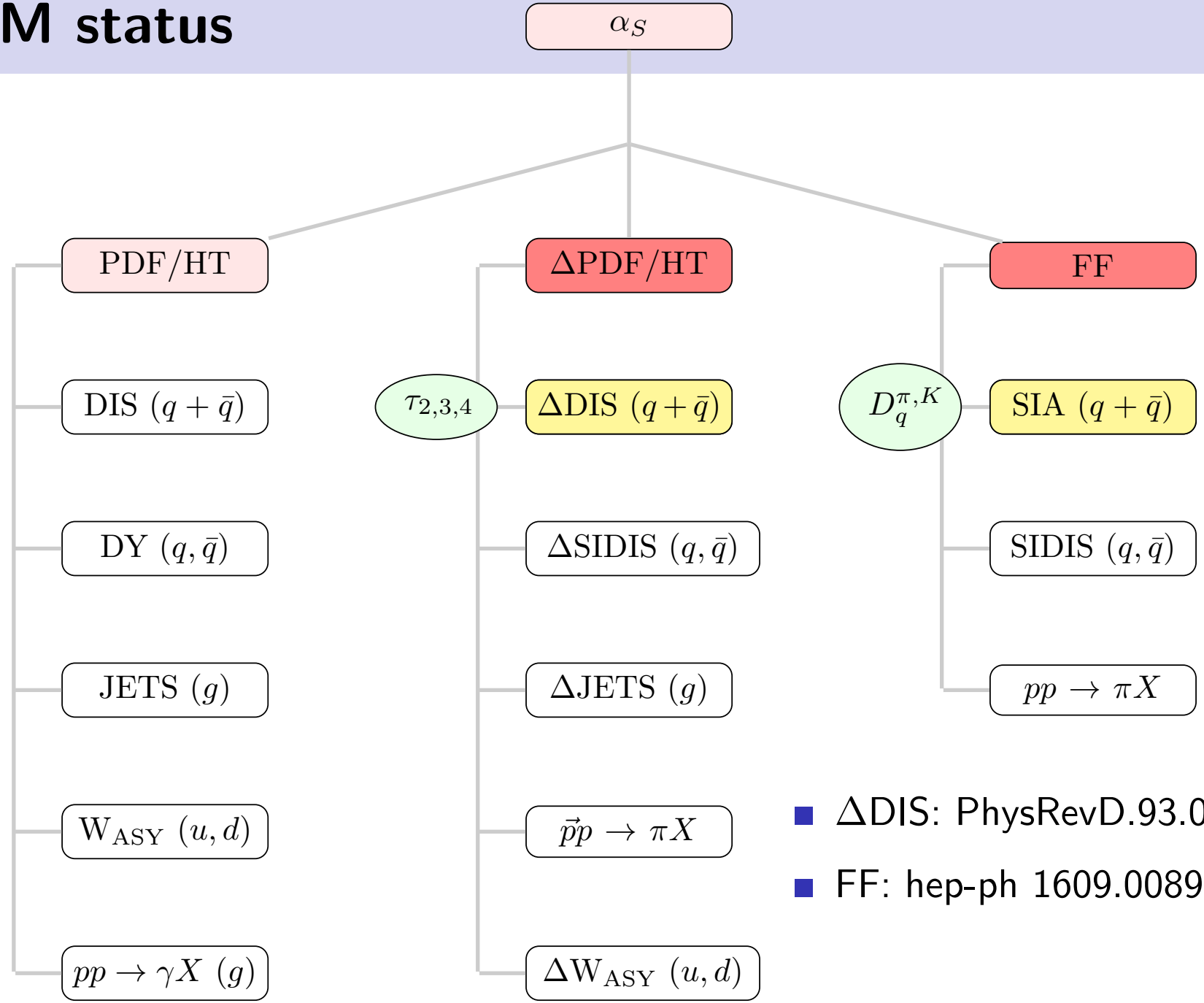
SIA at (small b contribution)

$$\tilde{Z}_{uu}^{h_1h_2}(b_*) = \sum_q e_q^2 \left(\hat{C}^{D_1}_{i\leftarrow q} \otimes D_{h/i}(z_{h1},\mu_b)\right) \left(\hat{C}^{\mathrm{D}_1}_{j\leftarrow \bar{q}} \otimes D_{h/j}(z_h,\mu_b)\right) .$$

$$\tilde{Z}_{collins}^{h_1h_2}(b_*) = \sum_q e_q^2 \left(\delta\hat{C}^{H_1^\perp}_{i\leftarrow q} \otimes H_1^{\perp(1)i}(z_{h1},\mu_b)\right) \left(\delta\hat{C}^{\mathrm{H}_1^\perp}_{j\leftarrow \bar{q}} \otimes H_1^{\perp(1)j}(z_h,\mu_b)\right)$$

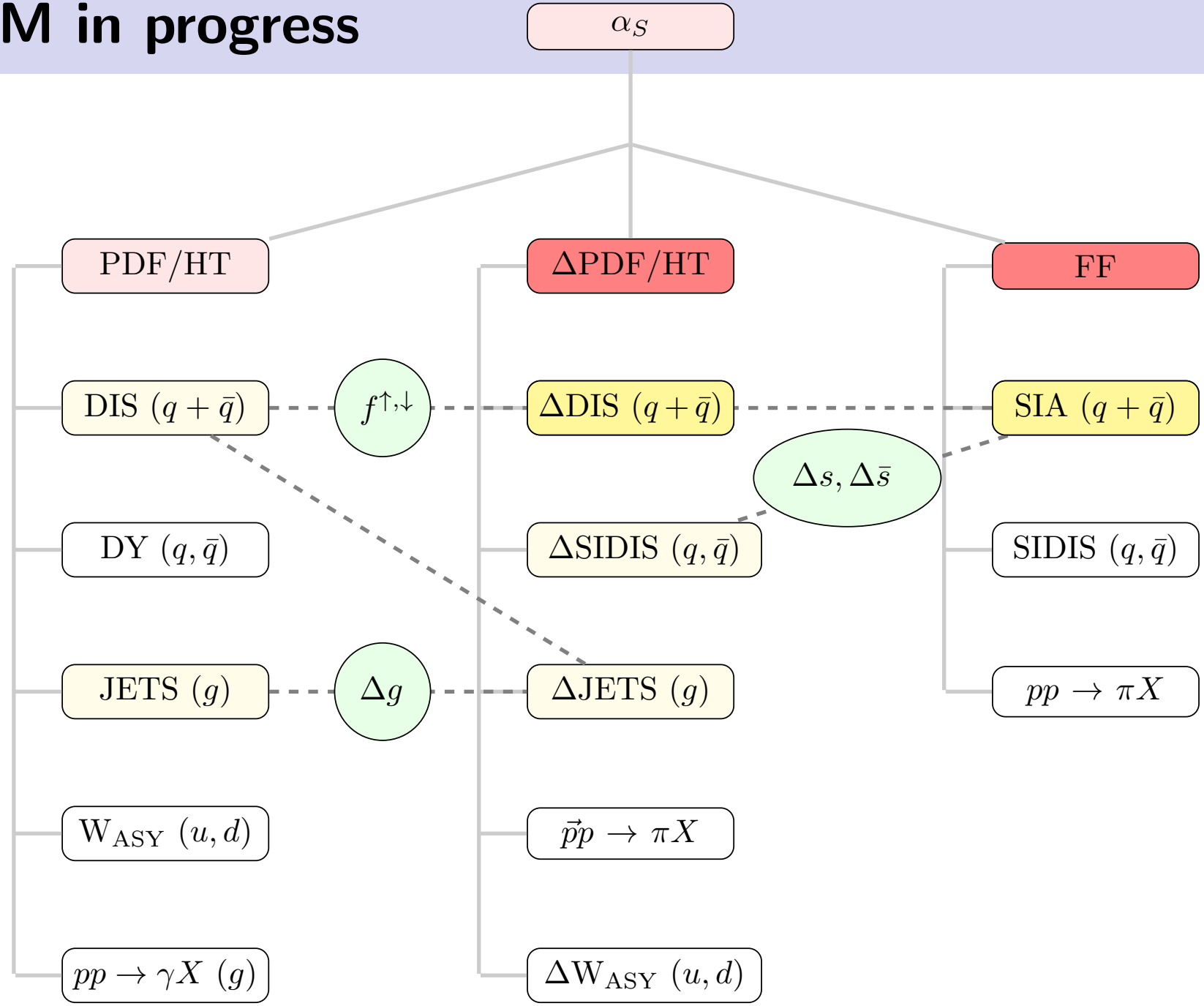
- Collinear PDFs, FFs play a role in TMDs.

JAM status



- ΔDIS: PhysRevD.93.074005
- FF: hep-ph 1609.00899

JAM in progress



Theory of fitting

The goal is to estimate:

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; [\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]]^2$$

- $\mathcal{O}$=$\Delta q, D_q^h, ...$
- $\sqrt{V[\mathcal{O}]} = 1\sigma$
- $\boldsymbol{a} \rightarrow$ PDF/FF parameters
- $\mathcal{P}(\boldsymbol{a}|data) \propto \mathcal{L}(data|\boldsymbol{a})\pi(\boldsymbol{a})$
- $\mathcal{L}(data|\boldsymbol{a}) \propto \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a})\right)$
- $\chi^2(\boldsymbol{a}) = \sum_i \left(\frac{D_i - T_i(\boldsymbol{a})}{\delta D_i}\right)^2$
- $\pi(\boldsymbol{a}) \rightarrow$*priors*. i.e $\prod_i \theta(a_i - a_i^0)$

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- **How to evaluate $E[\mathcal{O}], V[\mathcal{O}]$?**

Theory of fitting

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Monte Carlo methods

- $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \{\boldsymbol{a}_k\}$
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Maximum Likelihood

- Maximize $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \boldsymbol{a}_0$
- $\mathrm{E}[\mathcal{O}] \approx \mathcal{O}\left(\boldsymbol{a}_0\right)$
- $\mathrm{V}[\mathcal{O}] \approx$hessian, $\Delta\chi^2$ envelope,...

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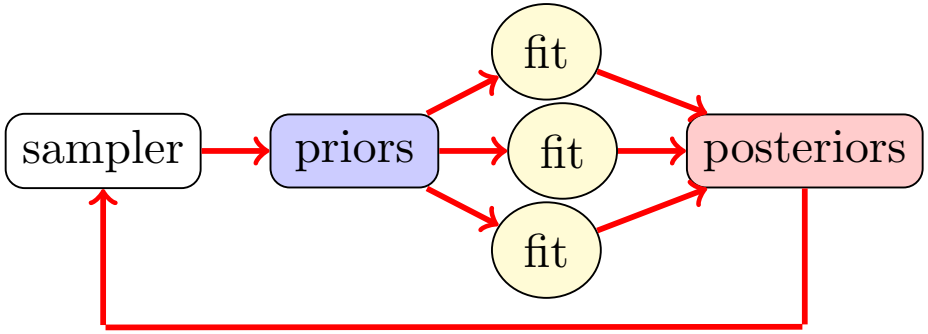
Iterative Monte Carlo analysis (IMC)



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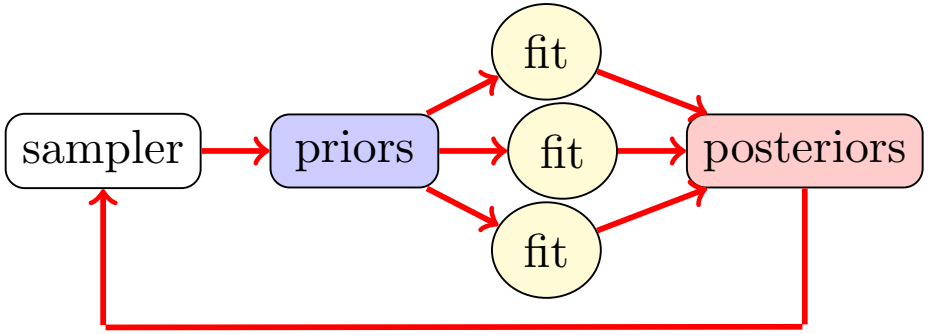

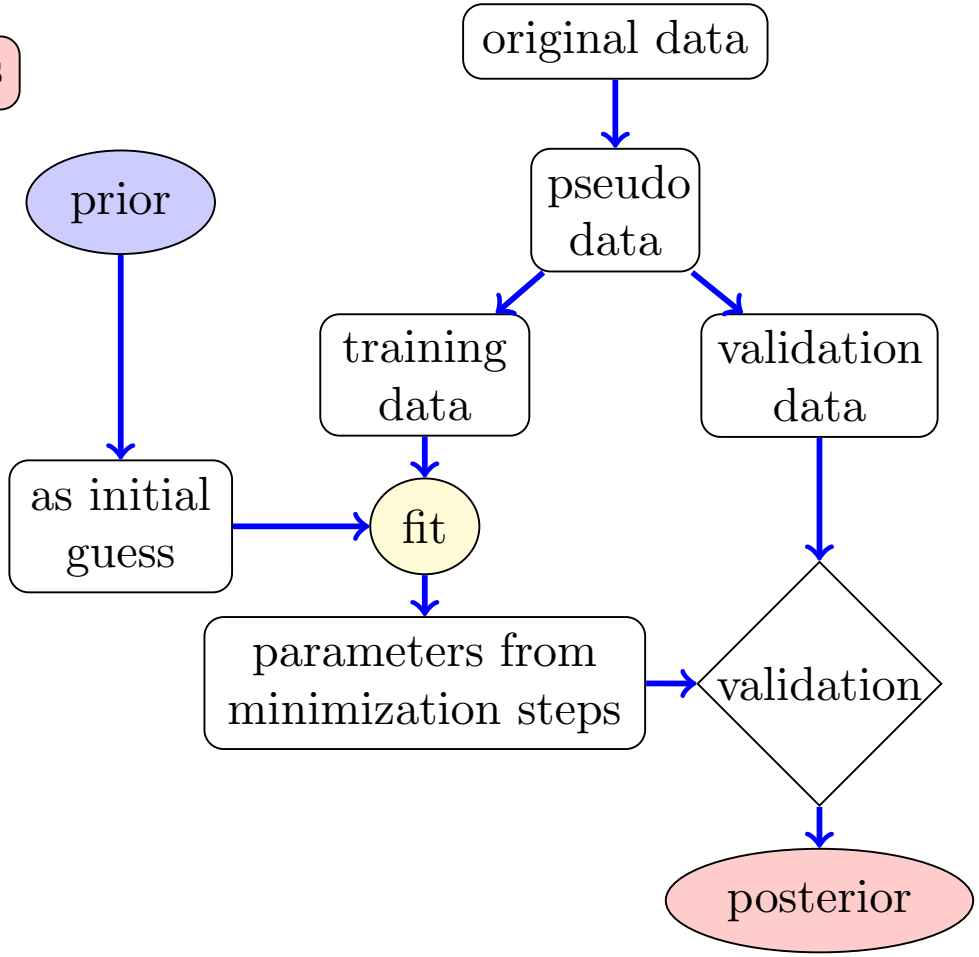

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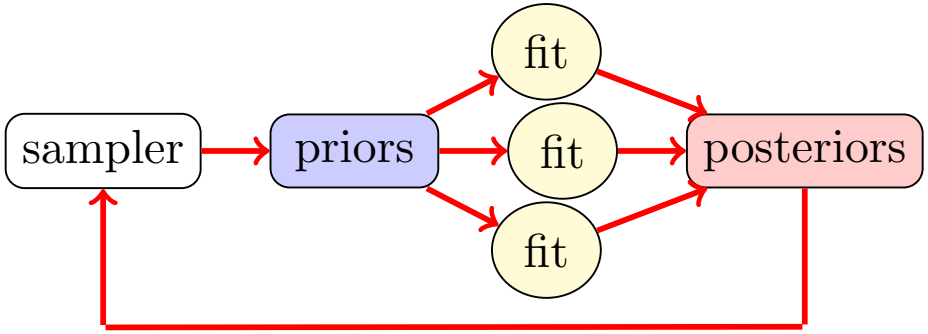
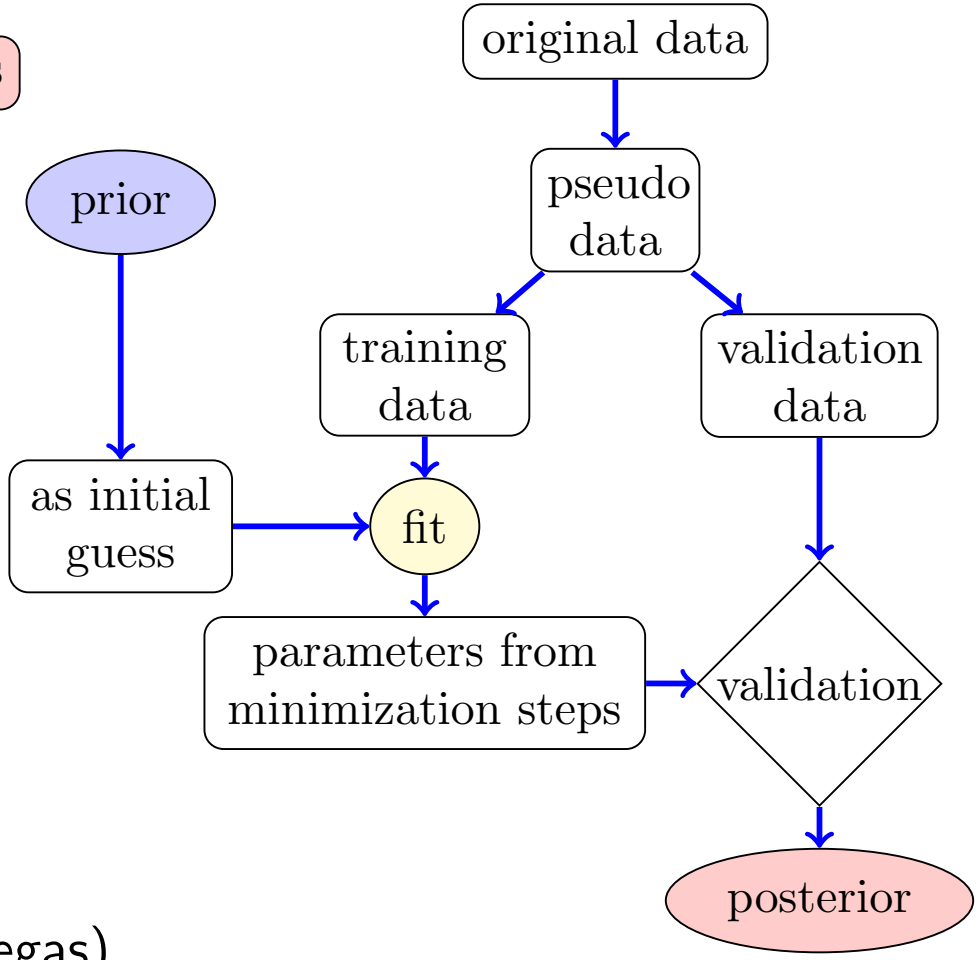
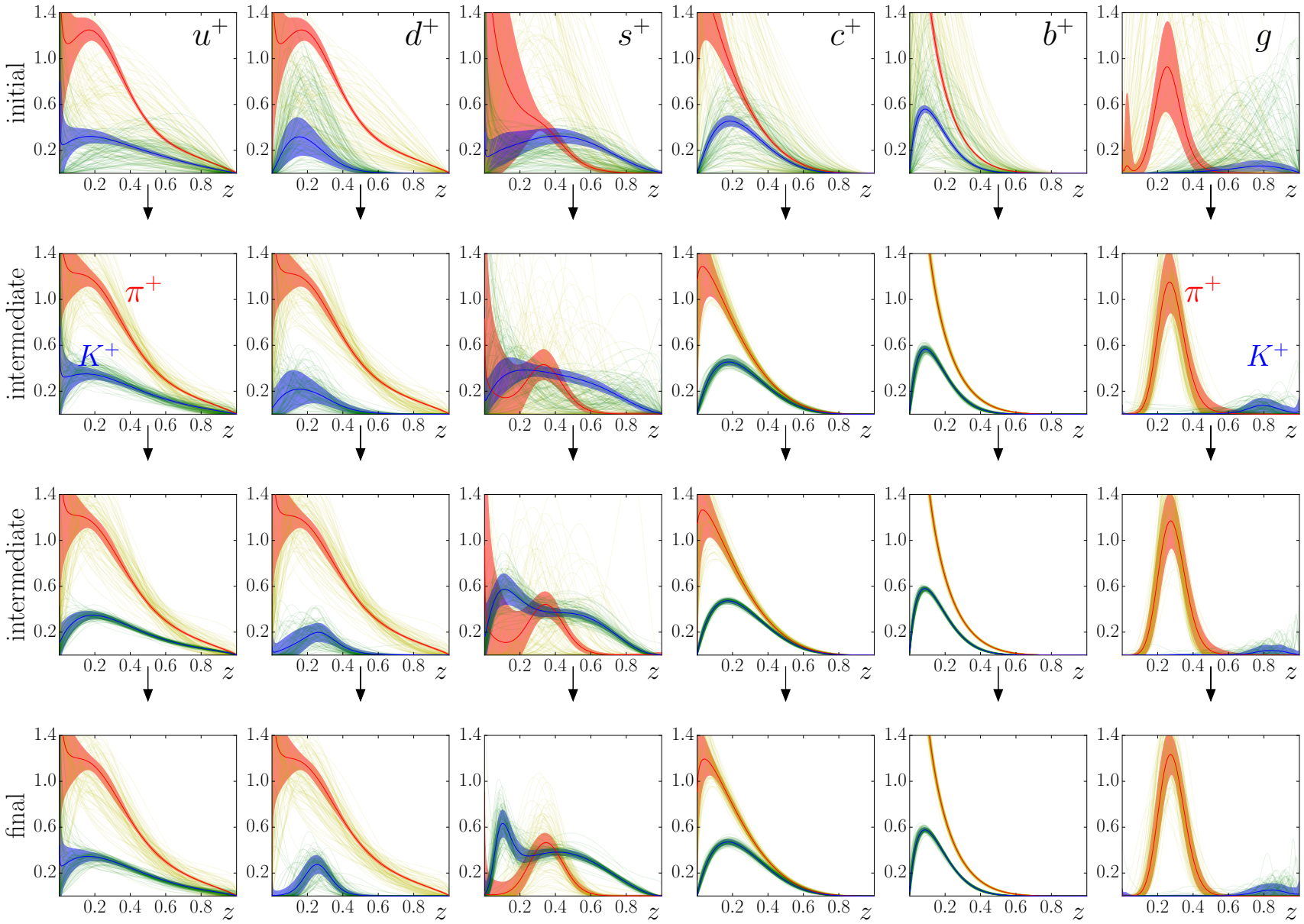




- Use traditional anzat
  $xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$
- Keep all the parameters free.
  No assumptions on the exponents
- Avoid over-fitting by Cross-Validation
- **Iterative procedure**
  $\rightarrow$ Adaptive MC integration (like in Vegas)
- Robust estimation of uncertainties

IMC in action (FF case)

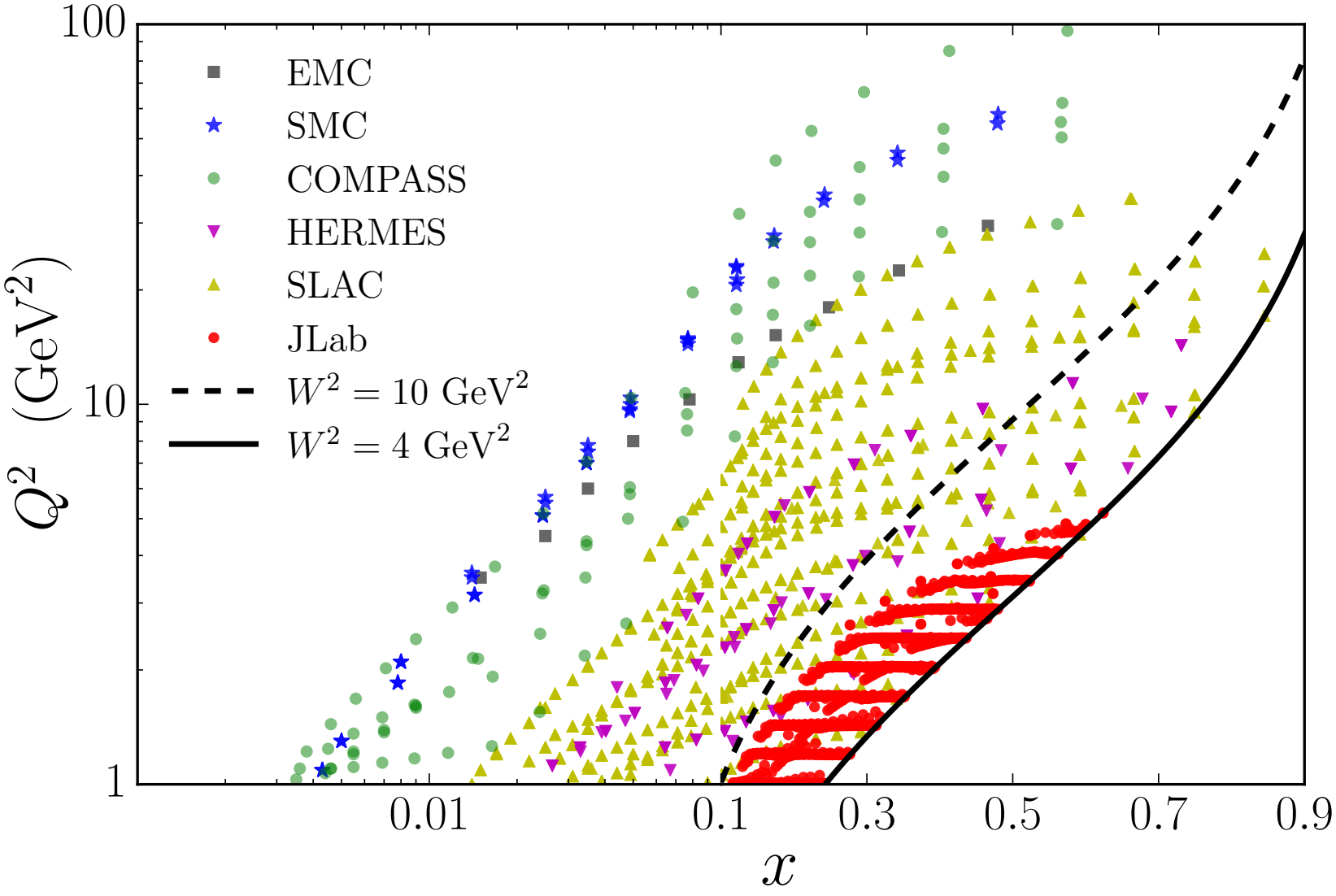


Spin PDFs from polarized DIS

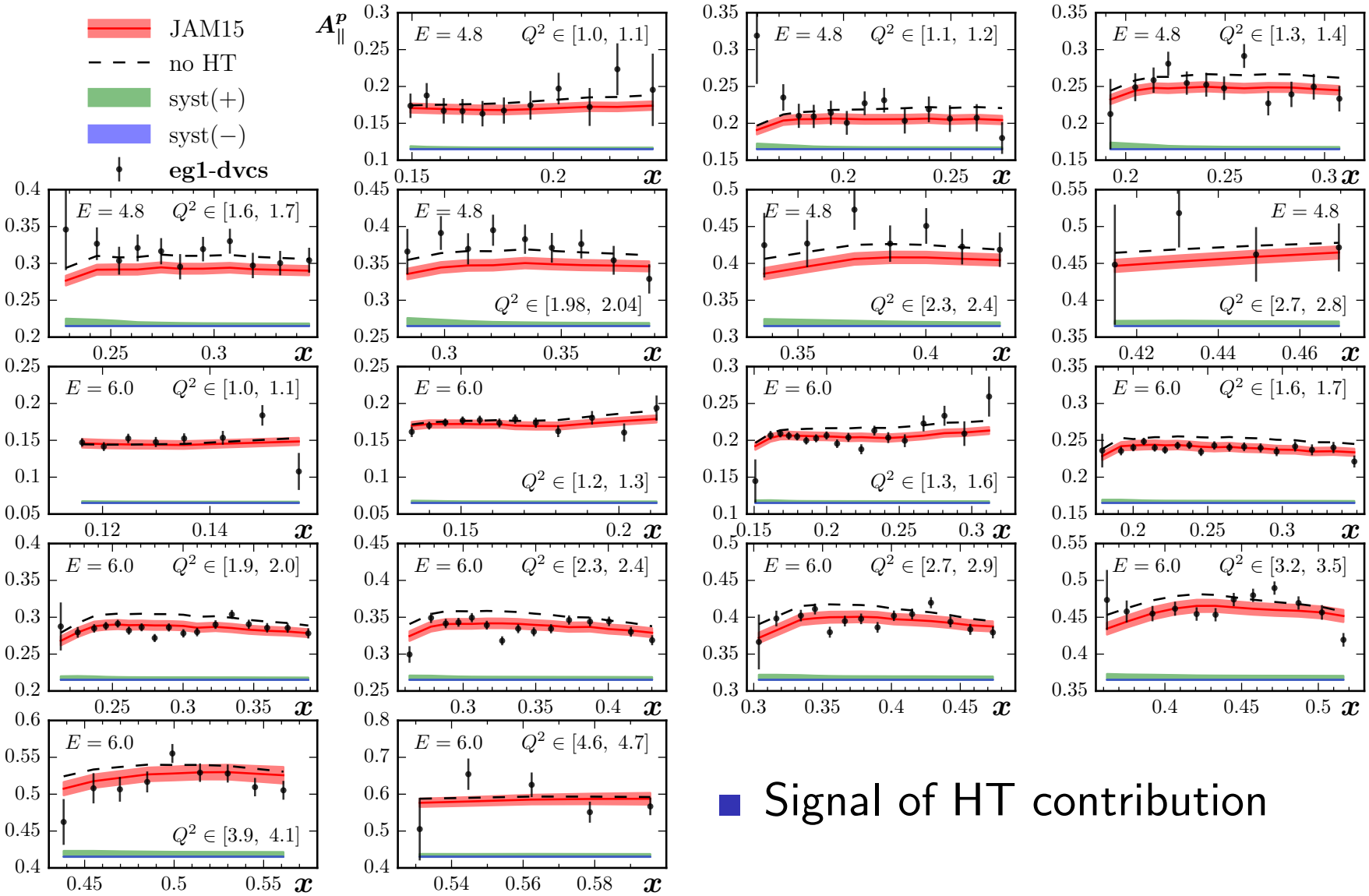
In Collaboration with:

- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S. E. Kuhn (Old Dominion U.)

Global polarized DIS data

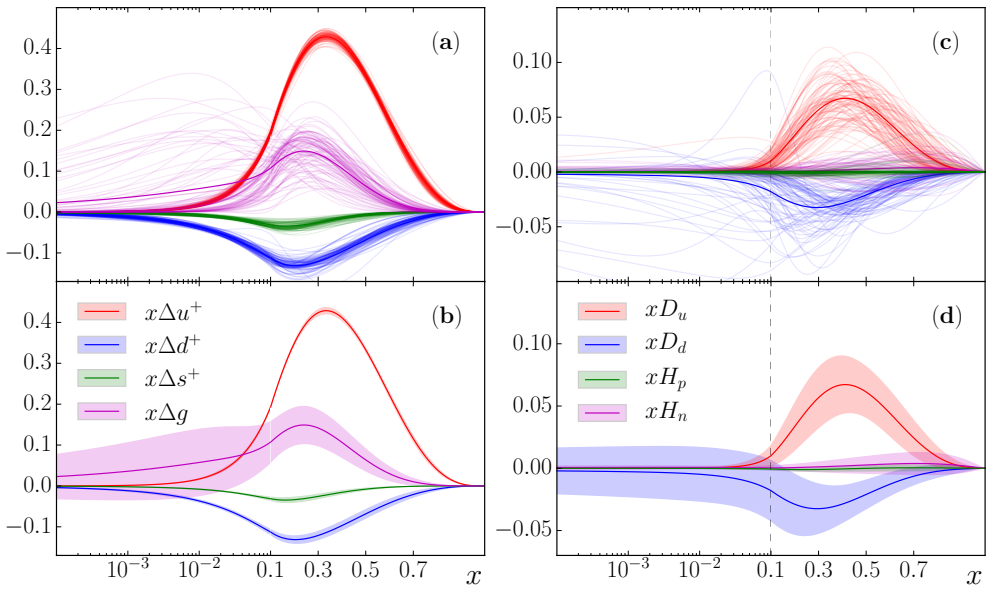


Data vs theory: proton JLab eg1-dvcs

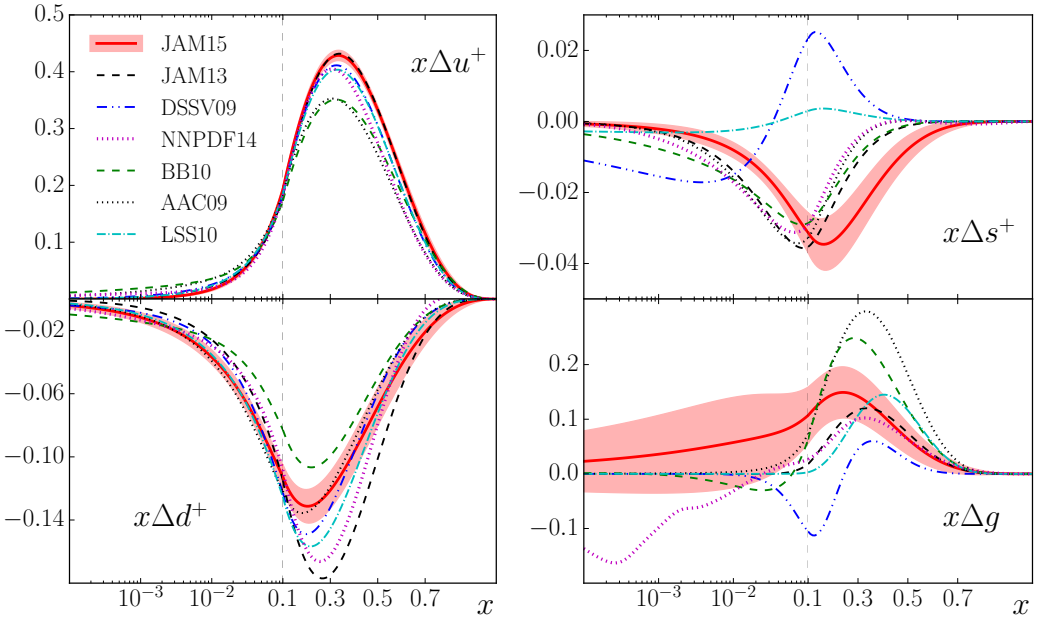


- Signal of HT contribution

Results



moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.011 ± 0.004	0.011 ± 0.004
d_2^n	-0.002 ± 0.002	-0.002 ± 0.002
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003



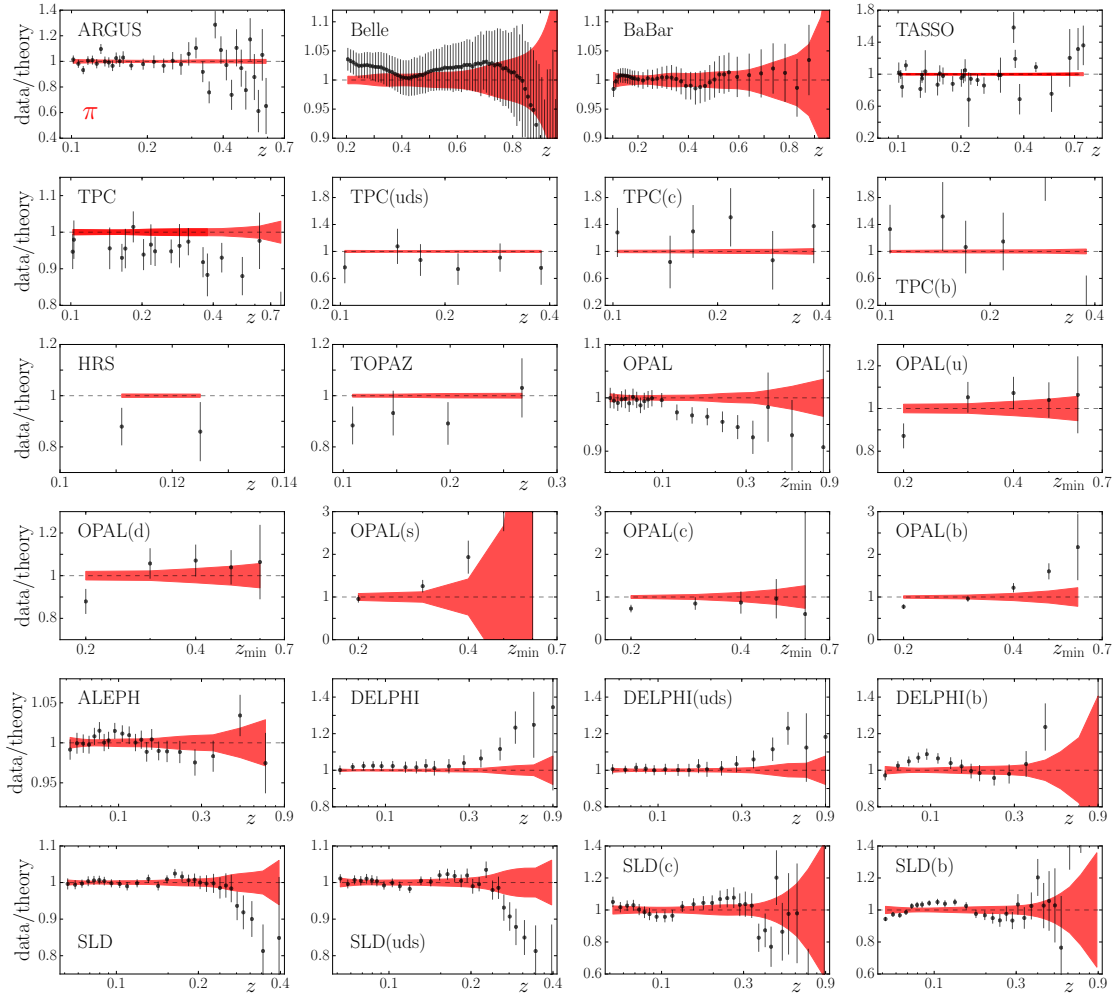
- $\chi^2/N_{npts} = 1.07$
- Sign of τ_3 distributions is the same as τ_2
- **Negative** Δs^+
- Δg compatible with the most recent DSSV fits
- moments of Δg not constrained.

Fragmentation Functions from SIA data

In Collaboration with:

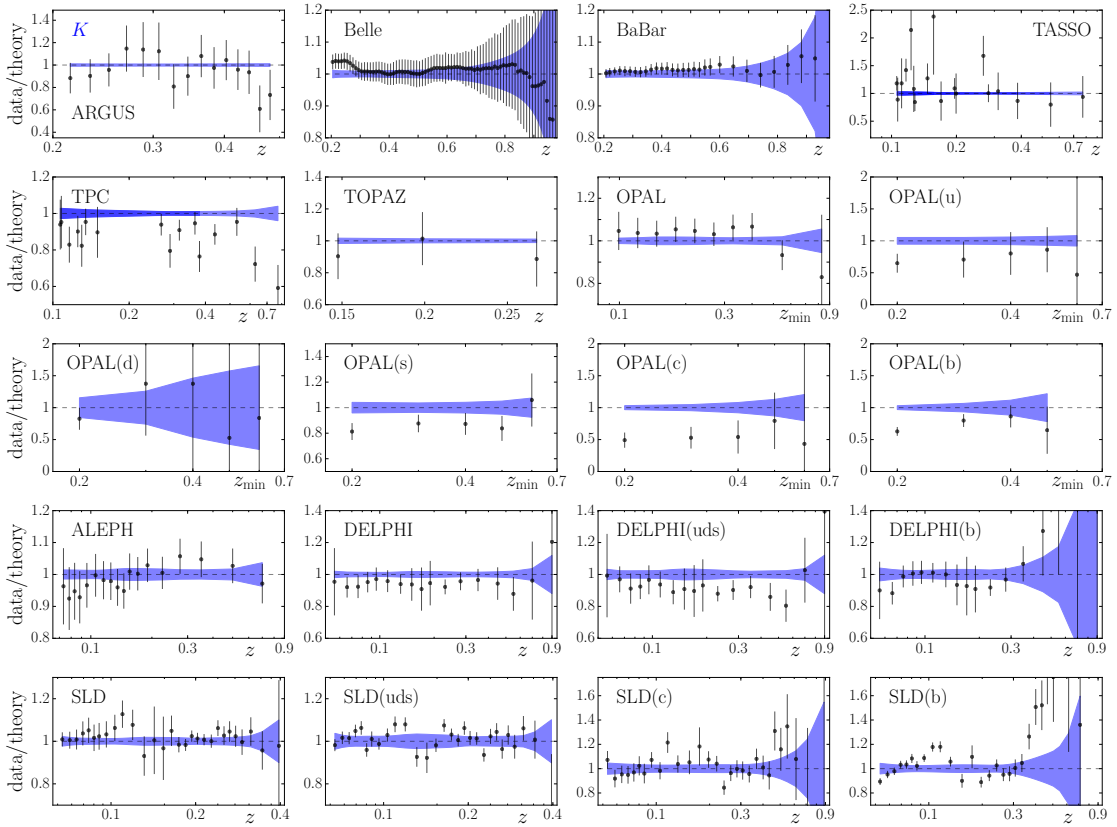
- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S.Kumano (KEK, J-PARC)
- M.Hirai (Nippon Institute of Technology)

π analysis $(\chi^2/N_{\rm npts} = 1.31)$



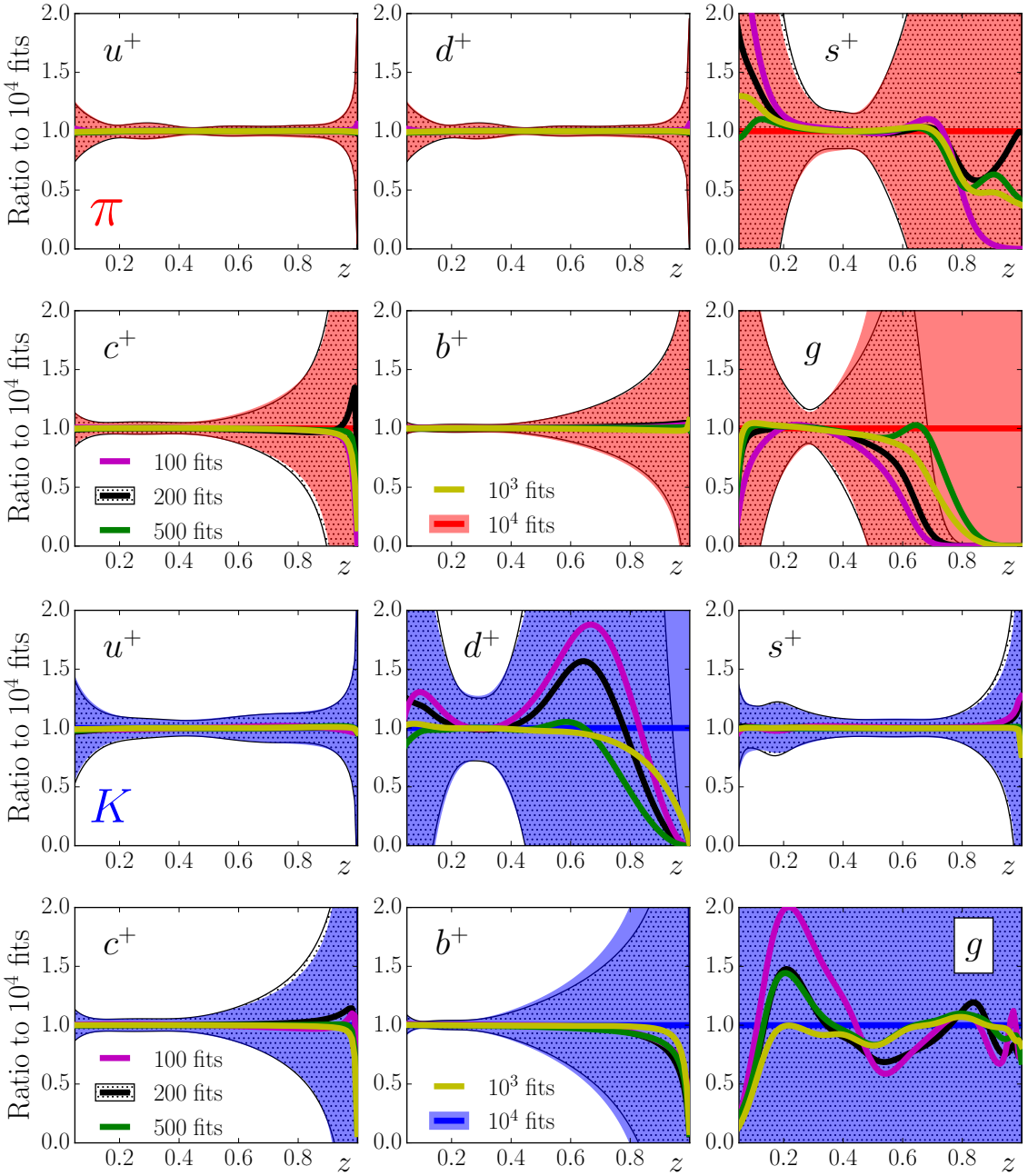
- $z_{\rm cut} > 0.1$ for low energies
- $z_{\rm cut} > 0.05$ for high energies
- We use BaBar prompt data set
- Belle data set needs 10% normalization
- Good agreement at low–z for inclusive data sets
- Data inconsistencies at large–z for $Q^2 = M_z^2$

K analysis $(\chi^2/N_{\mathrm{npts}} = 1.01)$



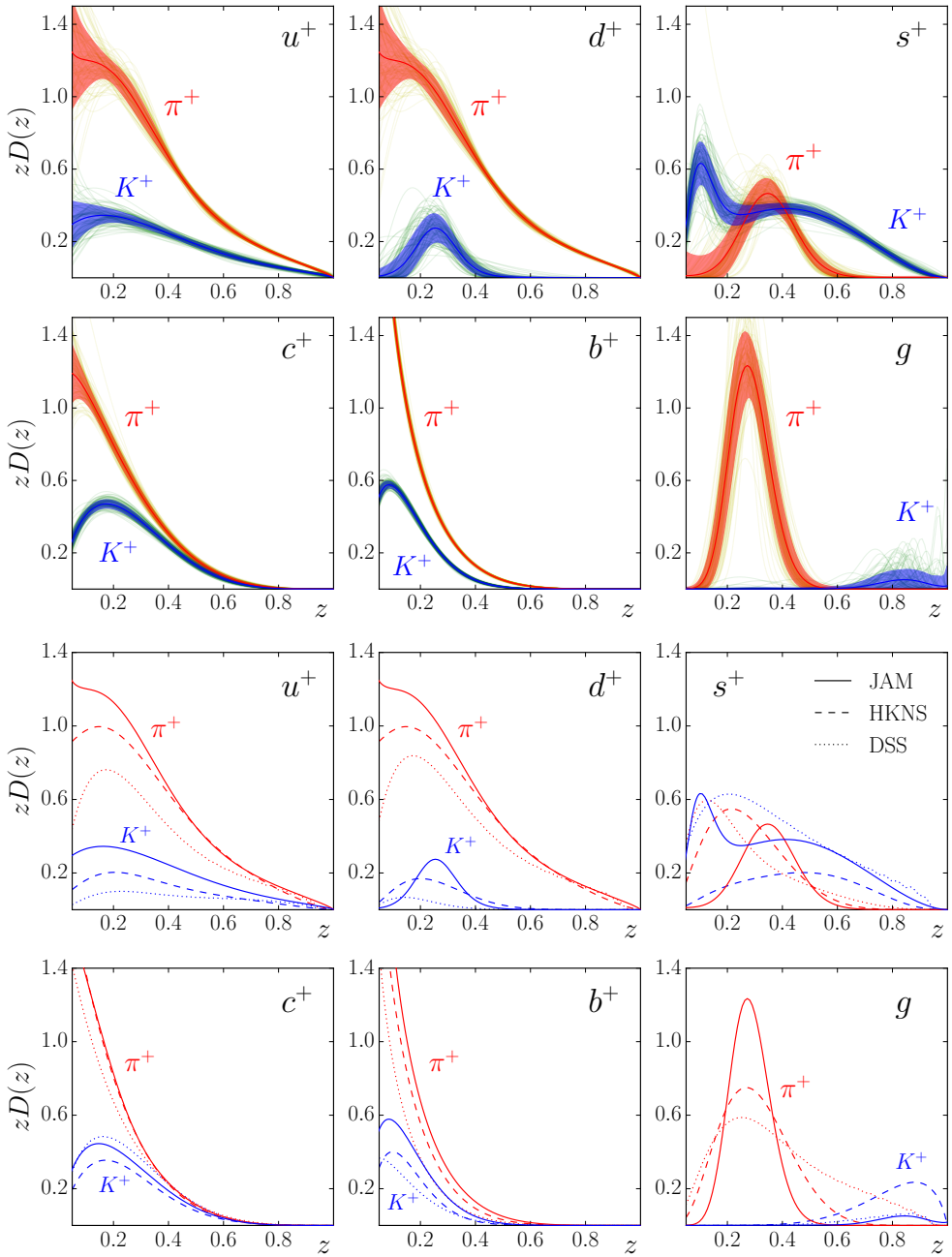
- $z_{\mathrm{cut}} > 0.2$ for low energies to avoid hadron mass corrections
- $z_{\mathrm{cut}} > 0.05$ for high energies
- Smaller χ^2 that π due to larger errors
- Consistent shapes across all z
- Inconsistencies mostly due to normalizations

Fragmentation functions



- Larger constraints on favored FFs than unfavored FFs
- More sensitivity on D_g^π than D_g^K
- D_g^K is unknown
- HQ tagged data provides similar constraints for π and K
- Convergence attained with ~ 200 posteriors

Fragmentation functions



- Similar behavior of unfavored $D^{\pi}_{s^+}$ and $D^{K}_{d^+}$
- In contrast the D^{π}_{g} and D^{K}_{g} behave differently
- The charm and bottom FFs become compatible at large–z
- Favored $D^{\pi}_{u^+}$ and $D^{K}_{s^+}$ have similar shape at large–z
- JAM $D^{K}_{s^+}$ is compatible with DSS. **Will it change the sign of Δs^+?**

Summary and outlook

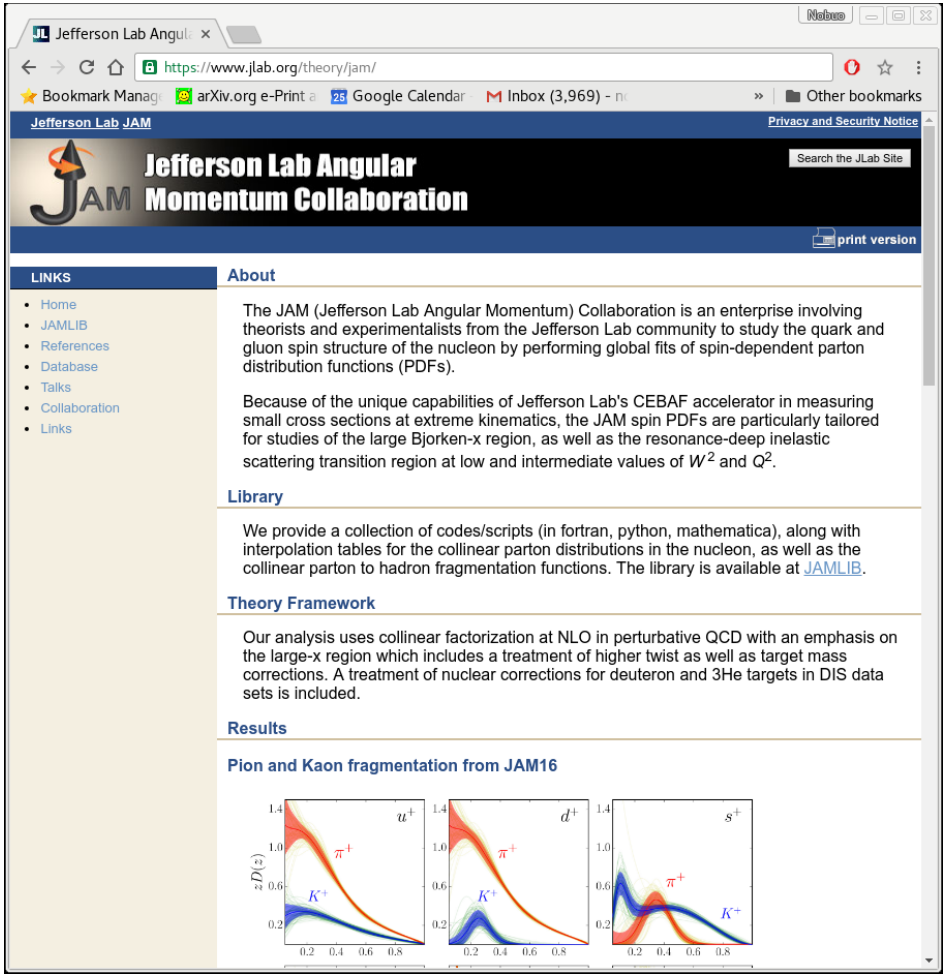
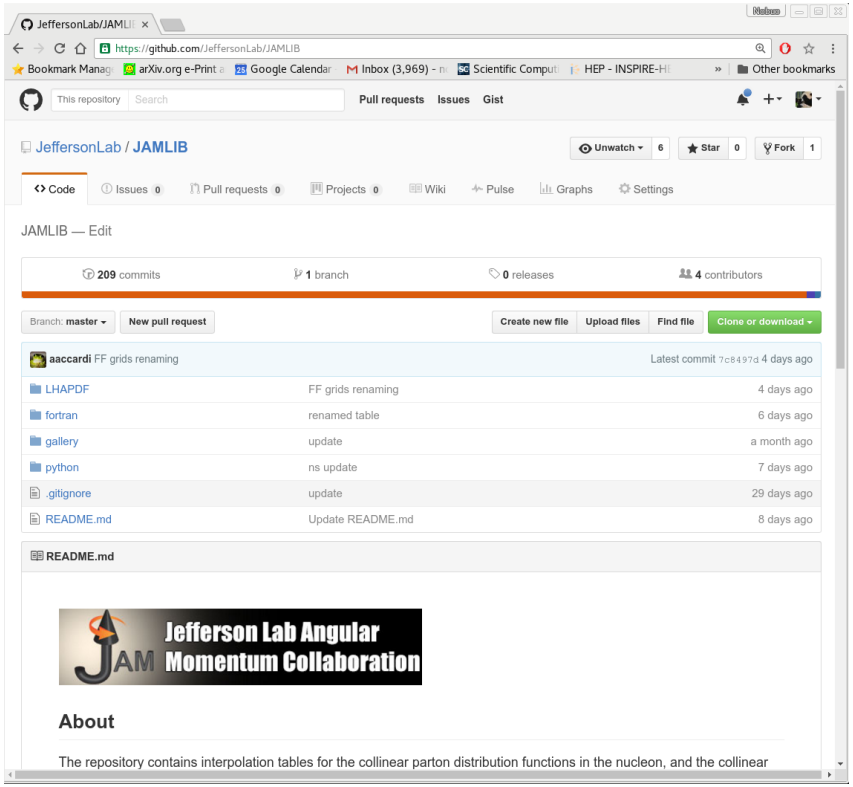
Polarized DIS data

- New JAM analysis to study impact of all JLab 6 GeV inclusive DIS data at low W and high x
- New extraction of LT & HT distributions
- Constraints on d_2

SIA data

- New study of SIA data including recent Belle and BaBar data
- New extraction of fragmentation functions
- The setup for combined pol DIS, SIA and pol SIDIS is ready!

JAMLIB

- JAM SPDFs and FFs are available at github
- Python, Fortran and LHAPDF interfaces provided

JAM for public