DK and D^*K scattering near threshold

C. B. Lang
Univ. Graz, Austria

in collaboration with

Sasa Prelovsek, Luka Leskovec, Daniel Mohler, Richard Woloshyn

Univ. Ljubljana, Slovenia Fermilab, USA TRIUMF, Canada

see also: Mohler et al., PRL 111, 222001 (2013), arXiv 1308.3157 and arXiv: 1403.8103

PDG Live

C S

$D_{s0}^*(2317)^\pm$

Γ_i	Mode	Fraction (Γ_i / Γ)
Γ_1	$D_{s0}^*(2317) \to D_s^+ \pi^0$	seen
Γ_2	$D_{s0}^*(2317) \to D_s^+ \gamma$	
Γ_3	$D_{s0}^*(2317) \to D_s^*(2112)^+ \gamma$	
Γ_4	$D_{s0}^*(2317) \to D_s^+ \gamma\gamma$	
Γ_5	$D_{s0}^*(2317) \to D_s^*(2112)^+ \pi^0$	
Γ_6	$D_{s0}^*(2317) \to D_s^+ \pi^+ \pi^-$	
Γ_7	$D_{s0}^*(2317) \to D_s^+ \pi^0 \pi^0$	not seen

$D_{s1}(2460)^\pm$

Γ_i	Mode	Fraction (Γ_i / Γ)
Γ_1	$D_{s1}(2460)^+ \to D_s^{*+} \pi^0$	$(.048 \pm .011) \times 10^1$
Γ_2	$D_{s1}(2460)^+ \to D_s^+ \gamma$	$(.018 \pm .004) \times 10^1$
Γ_3	$D_{s1}(2460)^+ \to D_s^+ \pi^+ \pi^-$	$(4.3 \pm 1.3) \times 10^{-2}$
Γ_4	$D_{s1}(2460)^+ \to D_s^{*+} \gamma$	<8 %
Γ_5	$D_{s1}(2460)^+ \to D_{s0}^*(2317)^+ \gamma$	$(3.7\ ^{+5.0}_{-2.4}) \times 10^{-2}$
Γ_6	$D_{s1}(2460)^+ \to D_s^+ \pi^0$	
Γ_7	$D_{s1}(2460)^+ \to D_s^+ \pi^0 \pi^0$	
Γ_8	$D_{s1}(2460)^+ \to D_s^+ \gamma\gamma$	

$D_{s2}^*(2573)$

Γ_i	Mode	Fraction (Γ_i / Γ)
Γ_1	$D_{s2}(2573)^+ \to D^0 K^+$	seen
Γ_2	$D_{s2}(2573)^+ \to D^*(2007)^0 K^+$	not seen

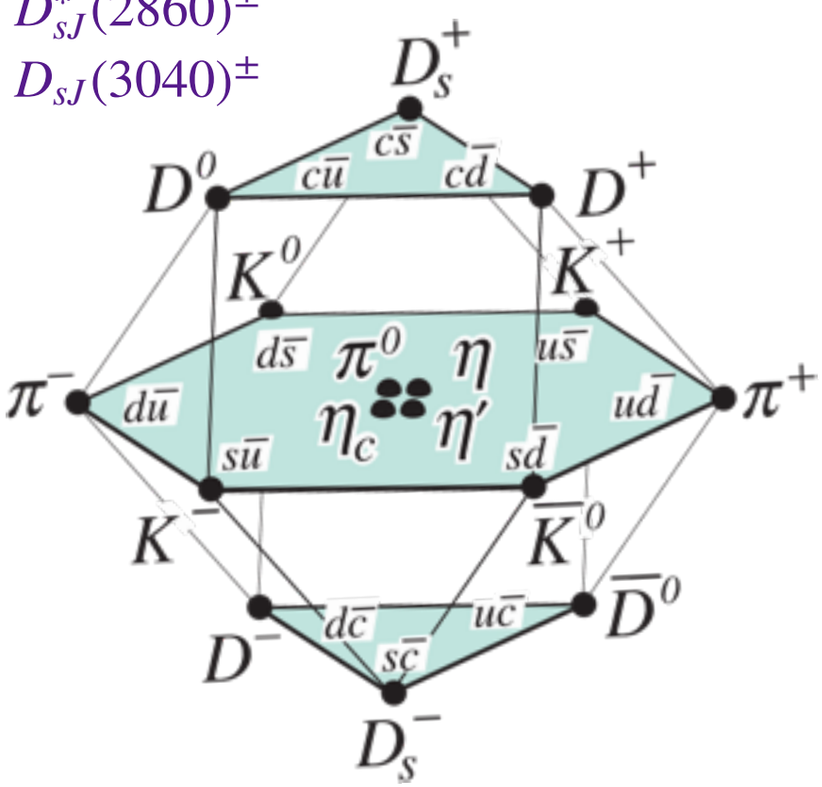
$D_{s1}(2536)^\pm$

Γ_i	Mode	Fraction (Γ_i / Γ)
Γ_1	$D_{s1}(2536)^+ \to D^*(2010)^+ K^0$	seen
Γ_2	$D_{s1}(2536)^+ \to (D^*(2010)^+ K^0)_{S-wave}$	
Γ_3	$D_{s1}(2536)^+ \to (D^*(2010)^+ K^0)_{D-wave}$	
Γ_4	$D_{s1}(2536)^+ \to D^+ \pi^- K^+$	
Γ_5	$D_{s1}(2536)^+ \to D^*(2007)^0 K^+$	seen
Γ_6	$D_{s1}(2536)^+ \to D^+ K^0$	not seen
Γ_7	$D_{s1}(2536)^+ \to D^0 K^+$	not seen
Γ_8	$D_{s1}(2536)^+ \to D_s^{*+} \gamma$	possibly seen
Γ_9	$D_{s1}(2536)^+ \to D_s^+ \pi^+ \pi^-$	seen

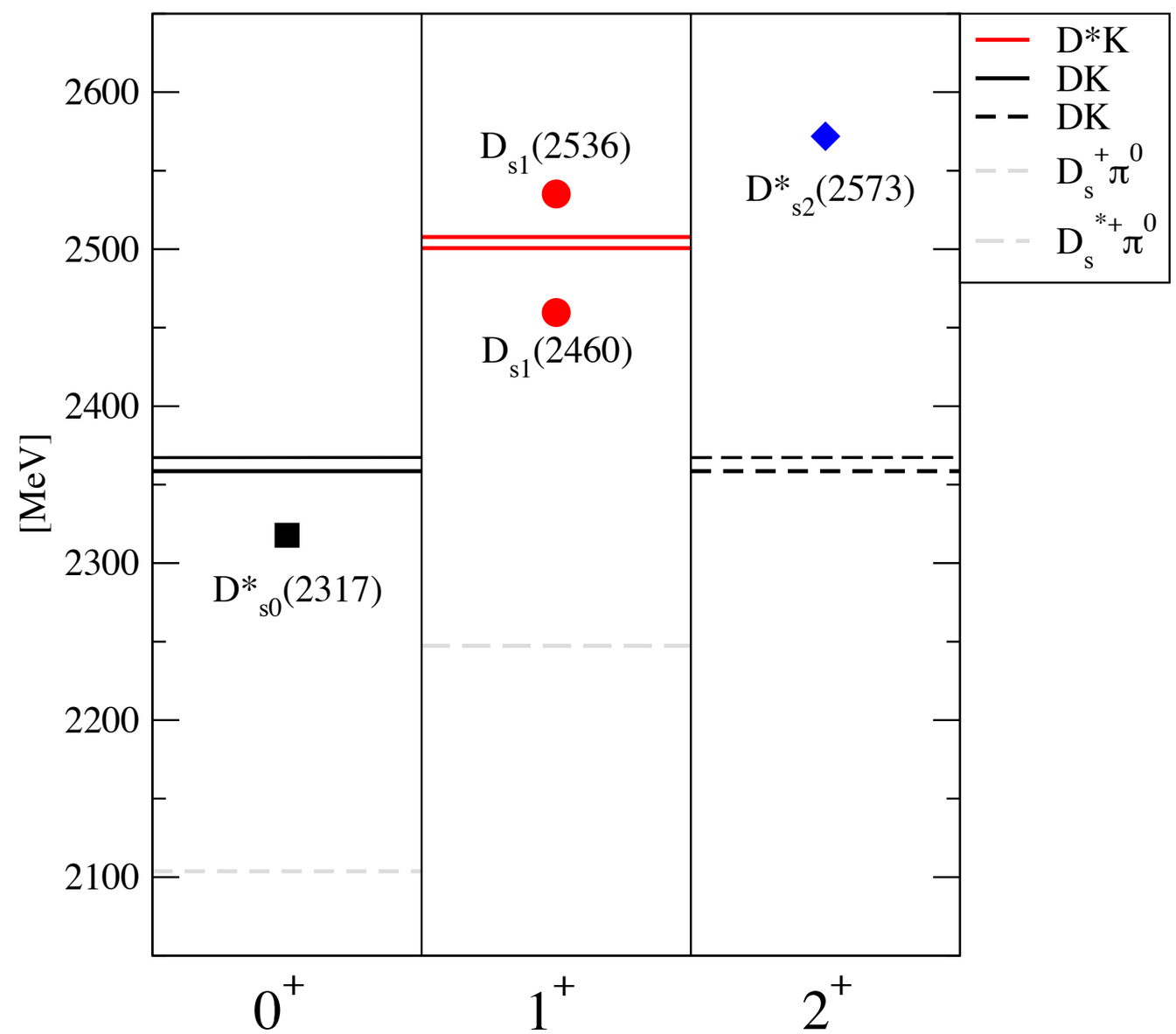
CHARMED, STRANGE MESONS

Particles

	HQL:
$D_s^{\pm}$	
$D_s^{*\pm}$	
$D_{s0}^*(2317)^{\pm}$	s-wave
$D_{s1}(2460)^{\pm}$	s-wave
$D_{s1}(2536)^{\pm}$	d-wave
$D_{s2}^*(2573)$	d-wave
$D_{s1}^*(2700)^{\pm}$	
$D_{sJ}^*(2860)^{\pm}$	
$D_{sJ}(3040)^{\pm}$	



Experiment



CHARMED, STRANGE MESONS

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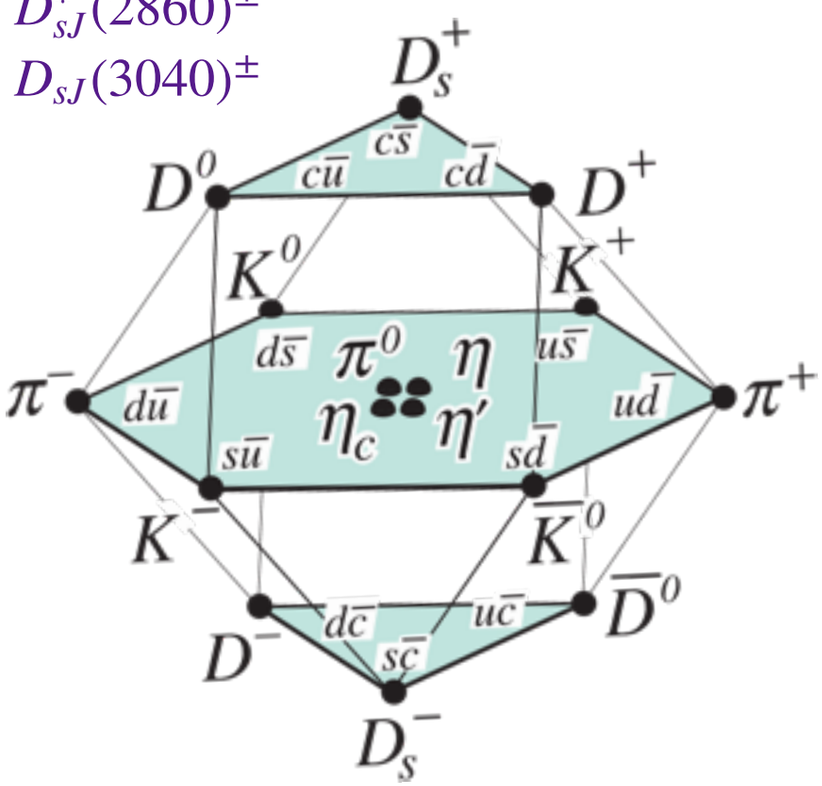
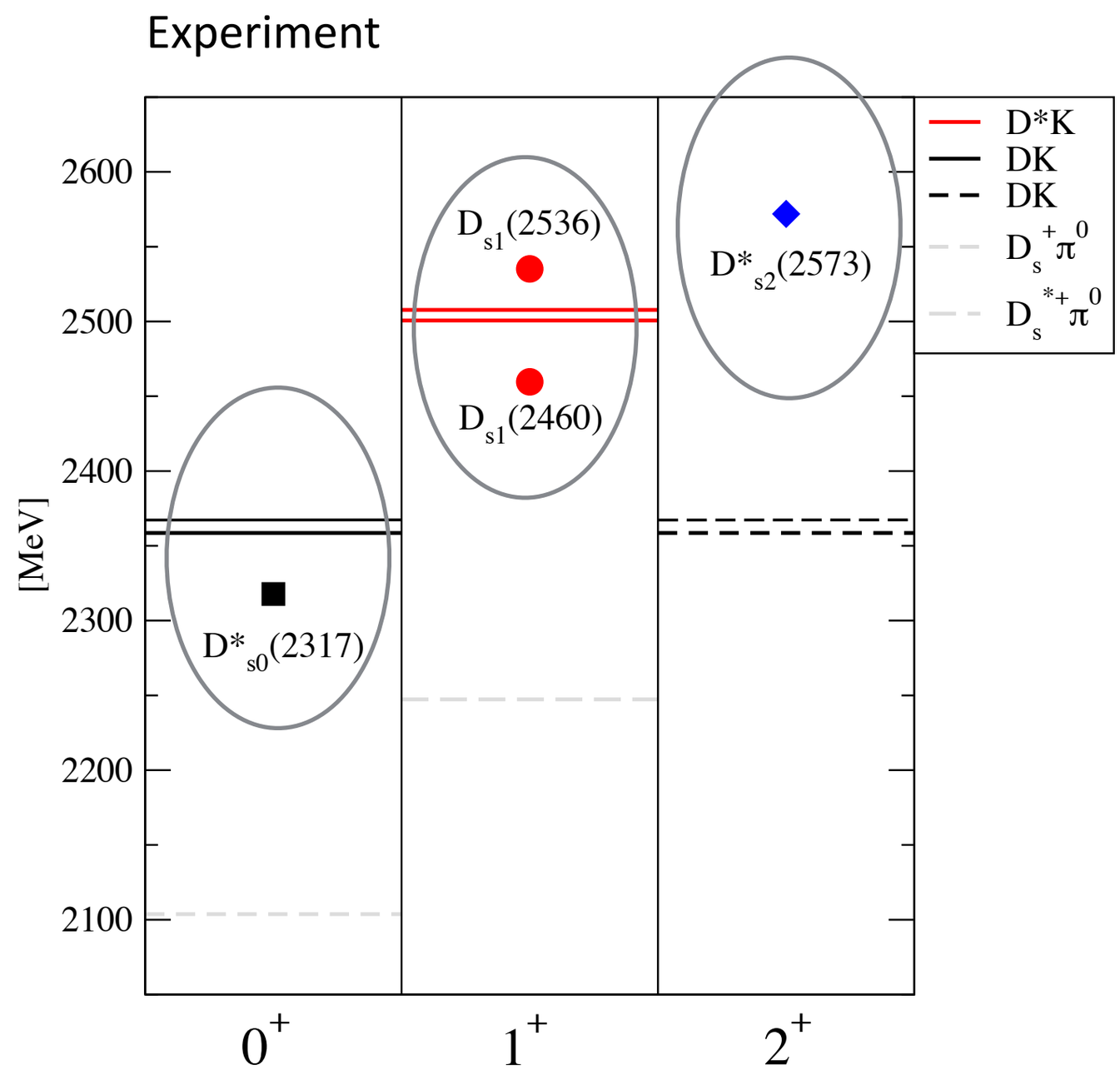
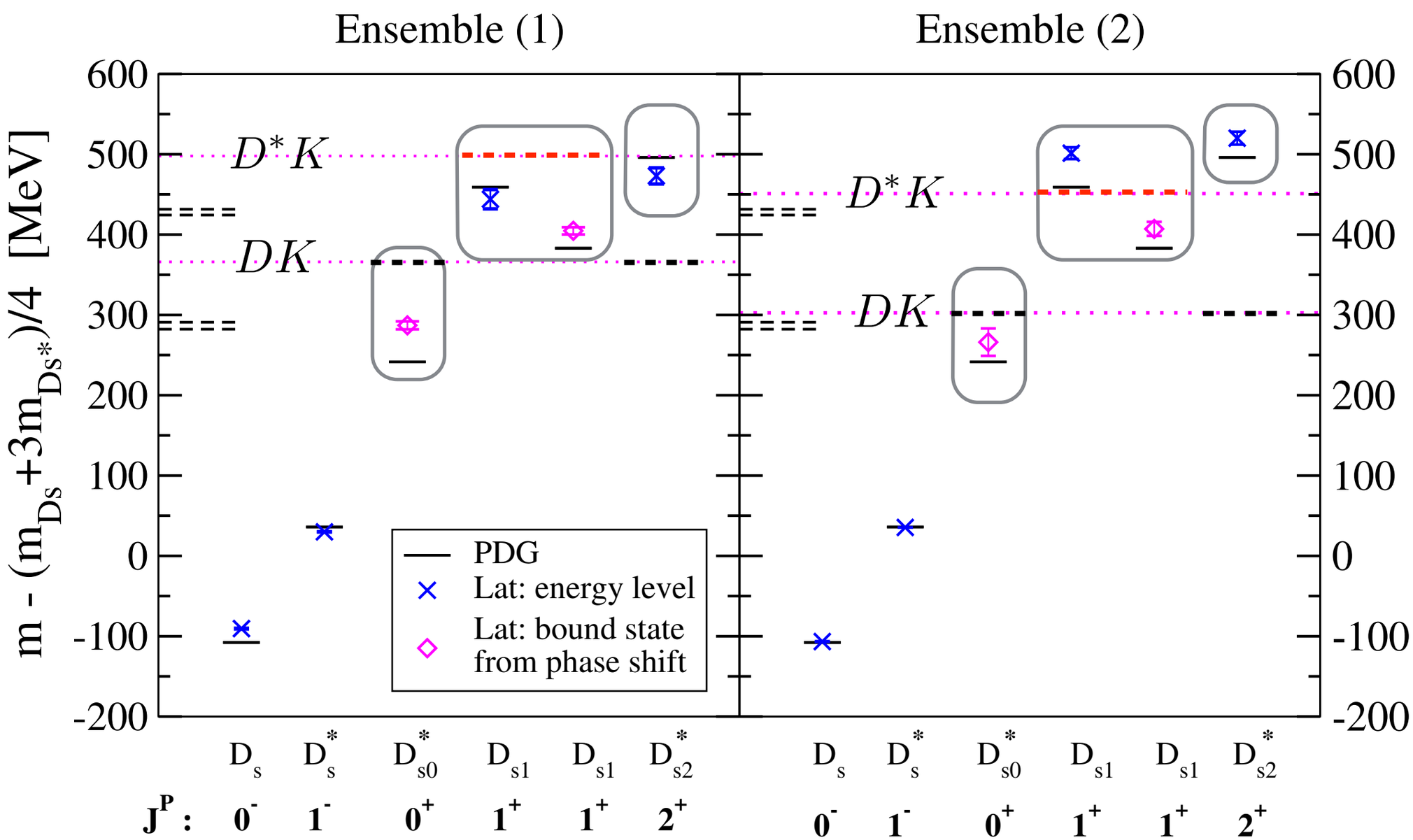




C.B. Lang (2014)

Results summary



Quark model:

$D_{s0}^*(2317)$ and $D_{s1}(2460)$ are *above* thresholds DK and D^*K

cf. Godfrey/Isgur PRD 32, 189 (1985)

But: threshold effects may be important

van Beveren/Rupp PRL 91(2003) 012003
Godfrey, PRD 72, 054029 (2005)

Lattice QCD:

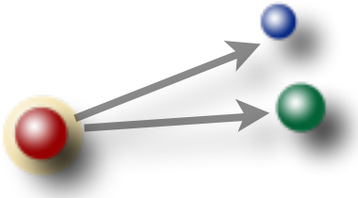
Single hadron (cs) studies give too high values

large pion mass: D_{s0}^* below threshold

small pion mass: D_{s0}^* above threshold

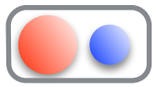
Namekawa et al., Phys. Rev. D 84, 074505 (2011)
Mohler/Woloshyn, Phys. Rev. D 84, 054505 (2011)
Bali et al., J. Phys. Conf. Ser. 426, 012017 (2013)
Bali et al., PoS LATTICE2011, 135 (2011),
Moir et al, JHEP 05, 021 (2013)
Kalinowski et al., A. Phys. Pol. B PS. 6, 991 (2013)
Wagner et al. 1310.5513.

Include meson meson interpolators!



Interpolators

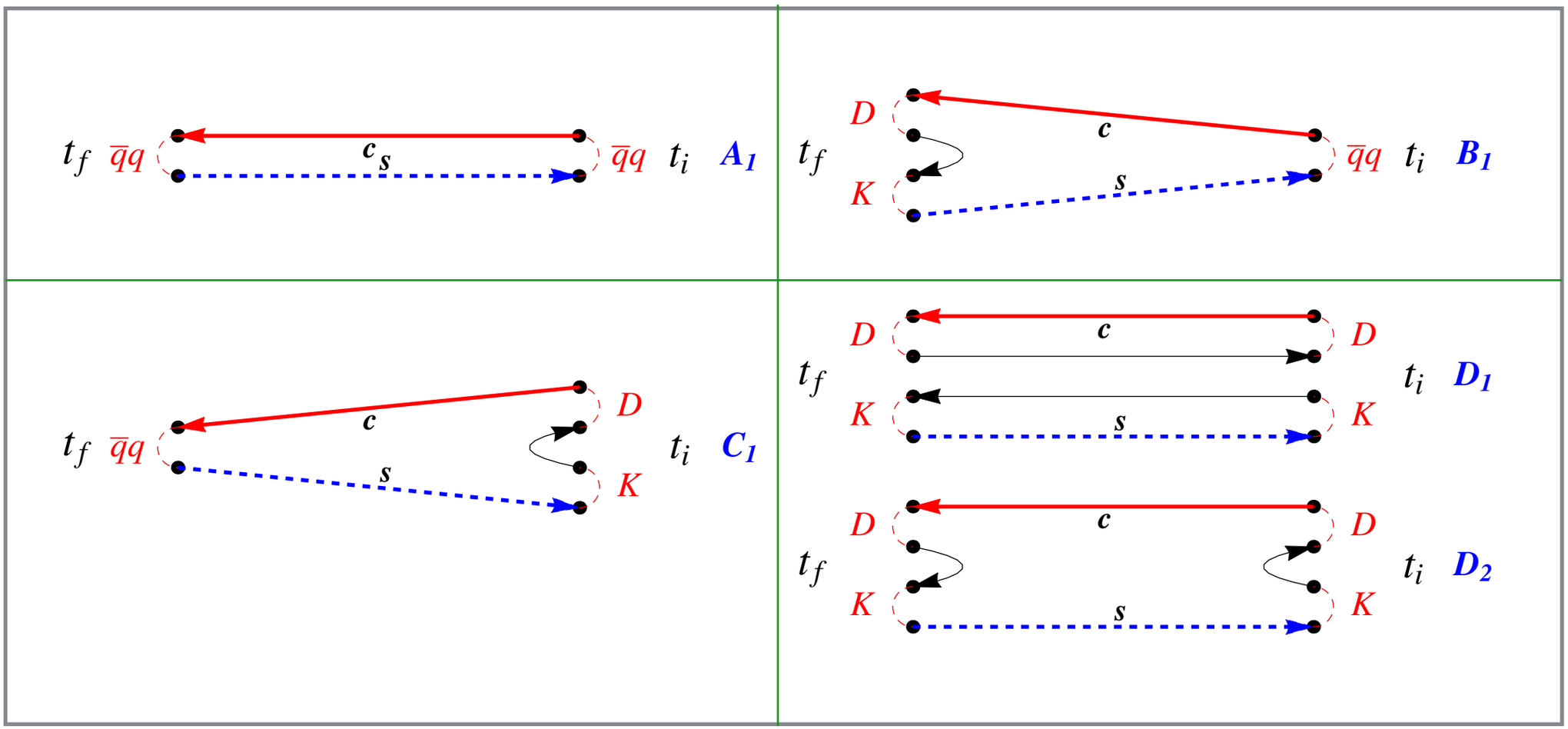
D_s



DK
D^*K

Lattice irrep	Quantum numbers J^{PC} in irrep	Interpolator label	Operator $\bar{s}\,c$	meson-meson
A_1^+	$0^+, 4^+, \ldots$	1	$\bar{q}q'$	$O_1^{DK} = [\bar{s}\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_5 c]\,(\vec{p}=0) + \{u \to d\}$,
		2	$\bar{q}\gamma_i \overrightarrow{\nabla}_i q'$	$O_2^{DK} = [\bar{s}\gamma_t\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_t\gamma_5 c]\,(\vec{p}=0) + \{u \to d\}$
		3	$\bar{q}\gamma_t\gamma_i \overrightarrow{\nabla}_i q'$	$O_3^{DK} = \sum_{\vec{p}=\pm e_{x,y,z}\ 2\pi/L} [\bar{s}\gamma_5 u]\,(\vec{p})\,[\bar{u}\gamma_5 c]\,(-\vec{p}) + \{u \to d\}$.
		4	$\bar{q}\overleftarrow{\nabla}_i \overrightarrow{\nabla}_i q'$	
T_1^+	$1^+, 3^+, 4^+, \ldots$	1	$\bar{q}\gamma_i\gamma_5 q'$	
		2	$\bar{q}\epsilon_{ijk}\gamma_j \overrightarrow{\nabla}_k q'$	
		3	$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j \overrightarrow{\nabla}_k q'$	$O_{1,k}^{D^*K} = [\bar{s}\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_k c]\,(\vec{p}=0) + \{u \to d\}$,
		4	$\bar{q}\gamma_t\gamma_i\gamma_5 q'$	$O_{2,k}^{D^*K} = [\bar{s}\gamma_t\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_t\gamma_k c]\,(\vec{p}=0) + \{u \to d\}$
		5	$\bar{q}\gamma_5 \overrightarrow{\nabla}_i q'$	$O_{3,k}^{D^*K} = \sum_{\vec{p}=\pm e_{x,y,z}\ 2\pi/L} [\bar{s}\gamma_5 u]\,(\vec{p})\,[\bar{u}\gamma_k c]\,(-\vec{p}) + \{u \to d\}$.
		6	$\bar{q}\gamma_t\gamma_5 \overrightarrow{\nabla}_i q'$	
		7	$\bar{q}\overleftarrow{\nabla}_i\gamma_j\gamma_5 \overrightarrow{\nabla}_i q'$	
		8	$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_j\gamma_5 \overrightarrow{\nabla}_i q'$	
T_2^+	$2^+, 3^+, 4^+, \ldots$	1	$\bar{q}\lvert\epsilon_{ijk}\rvert\gamma_j \overrightarrow{\nabla}_k q'$	
		2	$\bar{q}\lvert\epsilon_{ijk}\rvert\gamma_t\gamma_j \overrightarrow{\nabla}_k q'$	

Necessary contractions



2 Configuration ensembles

- Ensemble 1:
  - Hasenfratz et al., PRD 78, 014515 & 054511 (2008)
  - n_f=2 Wilson improved, 4 nHYP
  - 16^3x32, L_x=2 fm, 279 configs.
  - m_π=266 MeV, m_K=552 MeV

- Ensemble 2:
  - PACS-CS, Aoki et al, PRD 79, 034503 (2009)
  - n_f=2+1 Wilson improved, 3D HYP
  - 32^3x64, L_x=2.9 fm, 196 configs.
  - m_π=156 MeV, m_K=504 MeV

Propagators

- Distillation
  - HSC, Peardon et al., PRD80, 054506 (2009)
  - n_v=96
  - perambulators

$$\tau_{ij}^{\overline{\alpha}\overline{\beta}}(t',t) = v_i^*(t')\, G^{\overline{\alpha}\overline{\beta}}(t';\, t)\, v_j(t)$$

- Stochastic Distillation
  - Morningstar et al., PRD 83, 114505 (2011)
  - n_v=192, n_b=12,n_{ti}=8
  - stochastic sources

$$S_b^{\overline{\alpha}[r]}(\vec{x},c;t) = \sum_i v_i(\vec{x},c;t)\, \eta_{ib}^{\overline{\alpha}[r]}$$

  - (half) stochastic perambulators

$$T_{ib}^{[r]}(t,t') = v_i^*(t)\, G(t;t')\, S_b^{[r]}(t')$$

$$\langle M(t')\, M^\dagger(t)\rangle = -\mathrm{tr}\,[\phi(t')\tau(t',t)\phi(t)\tau(t,t')]$$

Quark mass parameters

u d	m_π=266(6)MeV	m_π=156(7)
s	valence m_Φ=1016(12) MeV m_K=552(7) MeV	partially quenched m_Φ=1018(14) MeV m_K=504(7) MeV $m_{\eta s}$=693(10)MeV $m_{\eta s}$=688(2) MeV from Dowdall et al., PRD 88, 074504(2013)
c	valence	valence

Fermilab method

Tune spin-average mass $\bar{m}$ (M_2 in the d.r.) for D, D_s and charmonium, respectively and determine $m - \bar{m}$

D.rel.: $$E(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3}$$

E.g. $\bar{m} = \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$

El Khadra et al., PRD 55, 3933 (1997),...,
C. Bernard et al., PRD 83, 034503 (2011)

Energy levels (a)

"Variational method"
Michael NPB259 (1985) 58
Lüscher/Wolff, NPB339 (1990) 222
Blossier et al., JHEP0904 (2009) 094

- Determine correlation matrix for many interpolators (lattice operators coupling to the given quantum channel)

$$C_{ij}(t) = \langle \mathcal{O}_i(t) | \mathcal{O}_j^\dagger(0) \rangle$$

- Solve the generalized eigenvalue problem, then

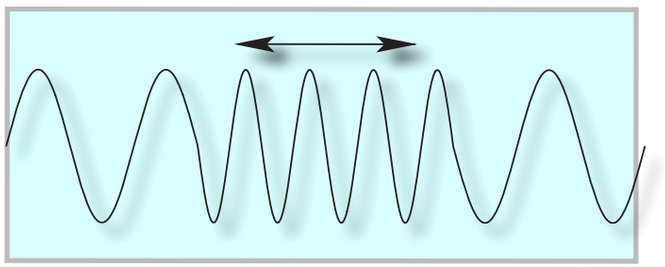
$$\lambda^{(n)} \sim \exp(-E_n t)$$

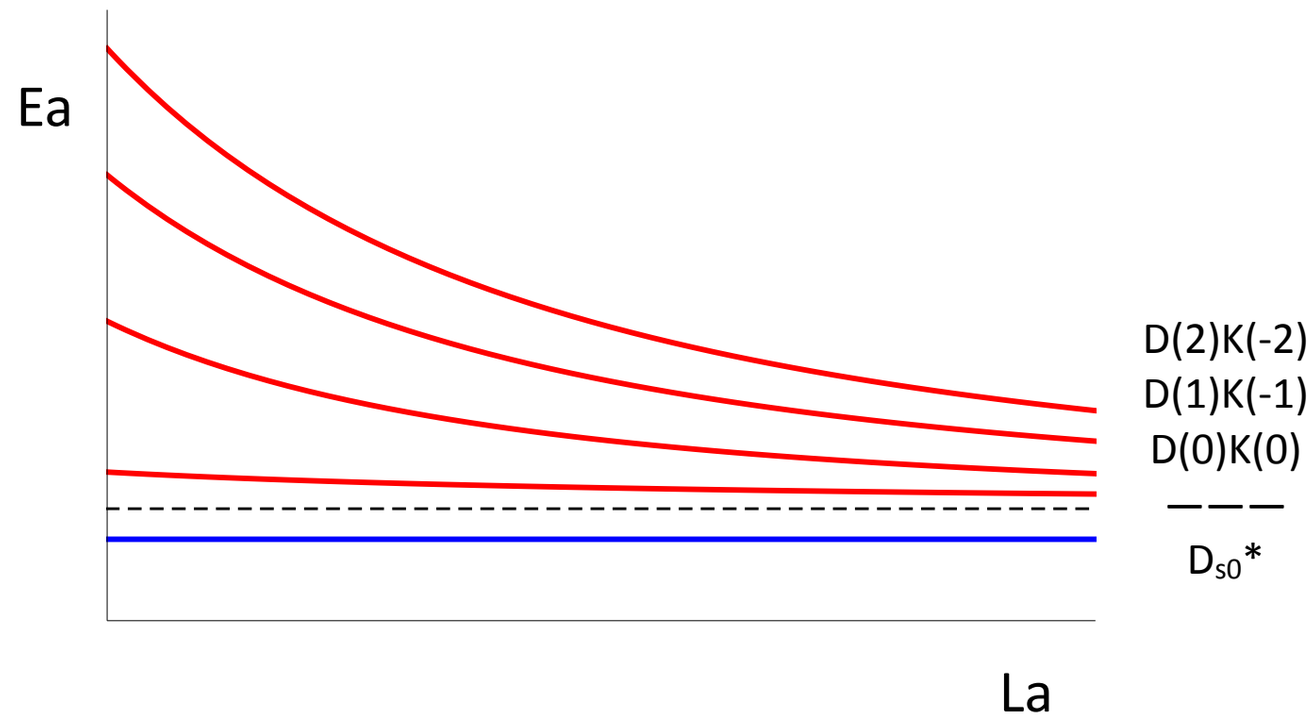
- The eigenstates approach the physical eigenstates when the operator basis is sufficiently complete

- Overlap factors $\langle n | \mathcal{O}_i \rangle$

Energy levels (b)

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237





Energy levels in finite volume ⟷ phase shift in infinite volume (in the elastic region)

UChPT model calculations:
Döring et al., Eur. Phys. J. A 47, 139&163 (2011)
Martinez Torres et al., PRD 85, 014027 (2012)
Albaladejo et al., PRD 88, 014510 (2013)

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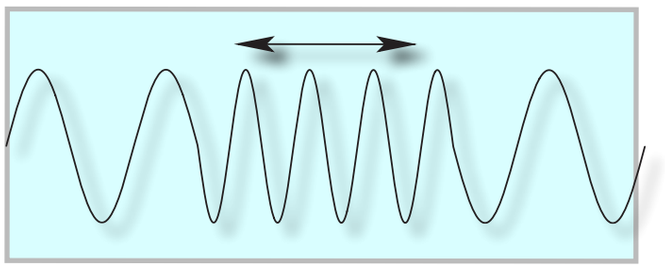

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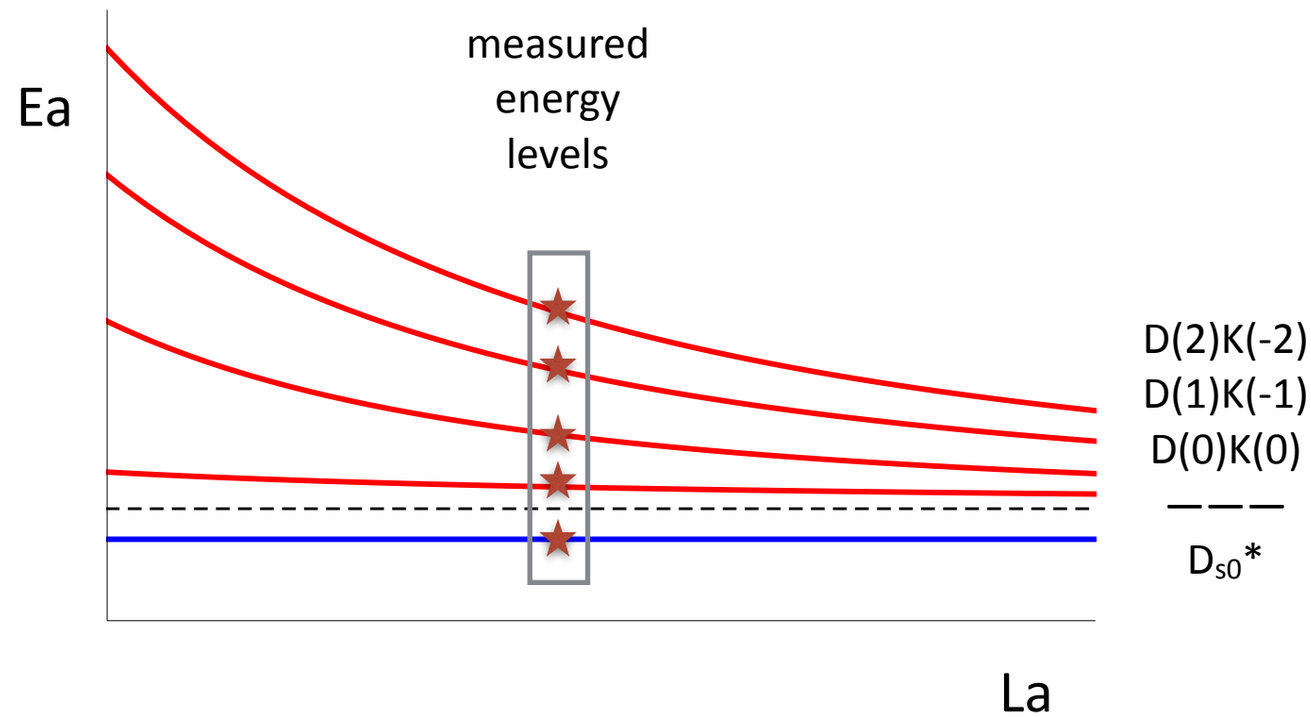

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Near threshold

$$T^{-1} = \begin{cases} K^{-1} - i\,p & \text{for } p^2 > 0 \\ K^{-1} + |p| & \text{for } p^2 < 0 \end{cases}$$

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

$$K^{-1} = \frac{2\mathcal{Z}_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$$\approx \frac{1}{a_0} + \frac{1}{2} r_0\, p^2$$

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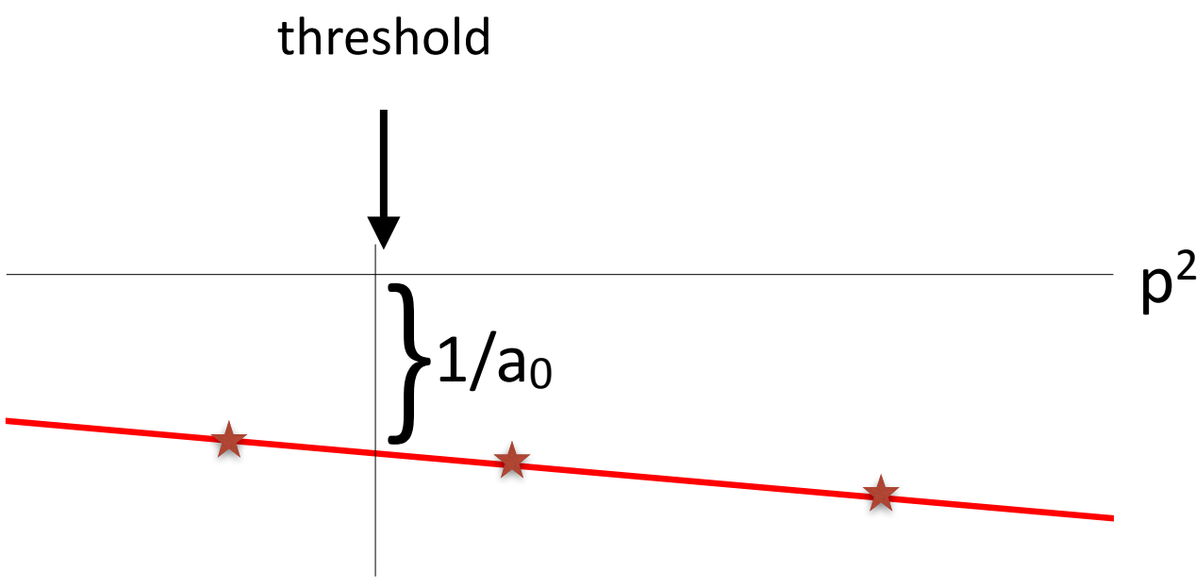

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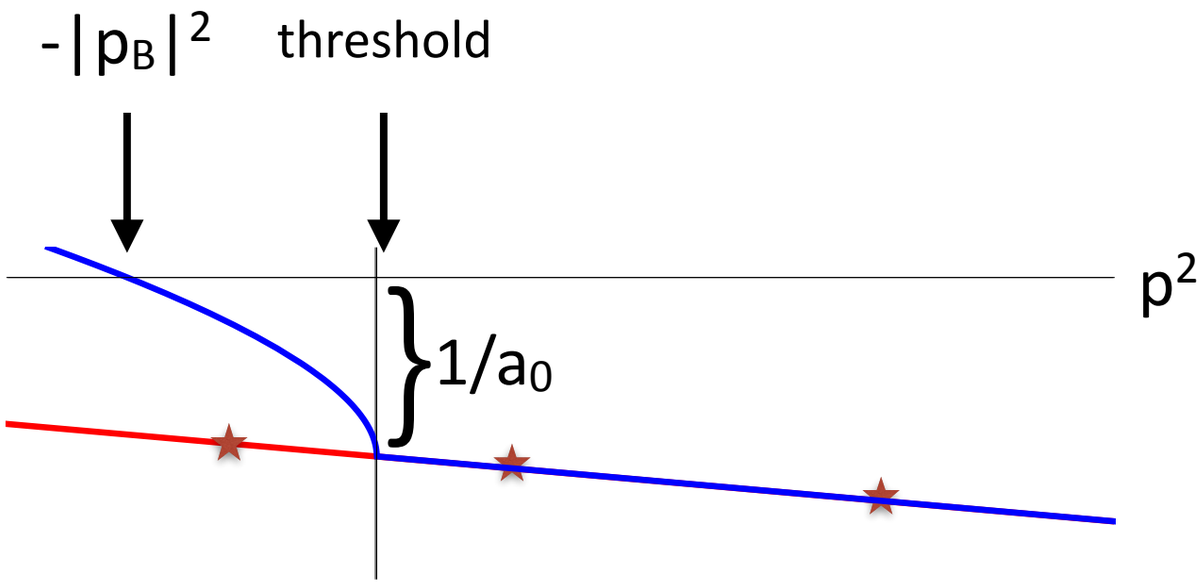

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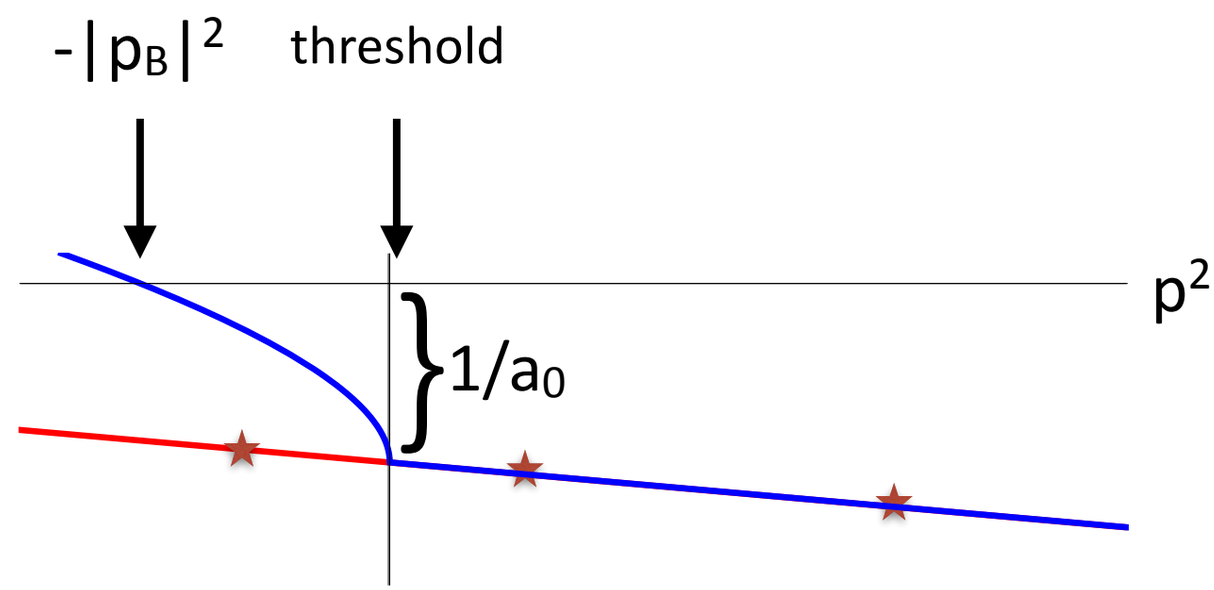
bound state



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$$K^{-1} = p\,\cot\delta(p) \quad \text{for } p^2 > 0$$

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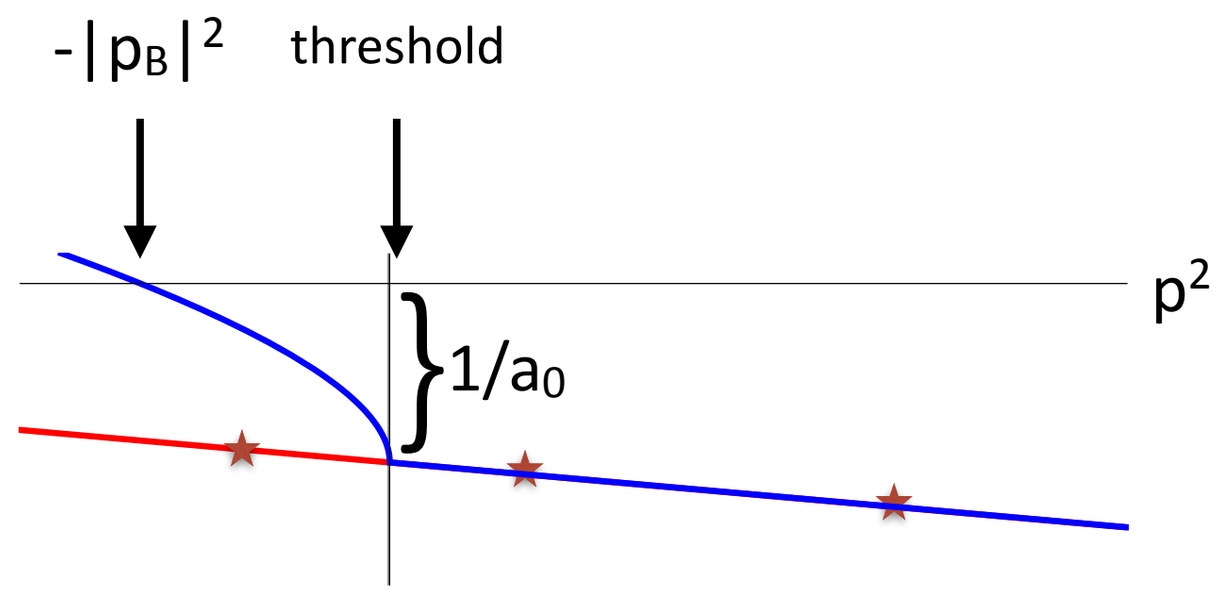

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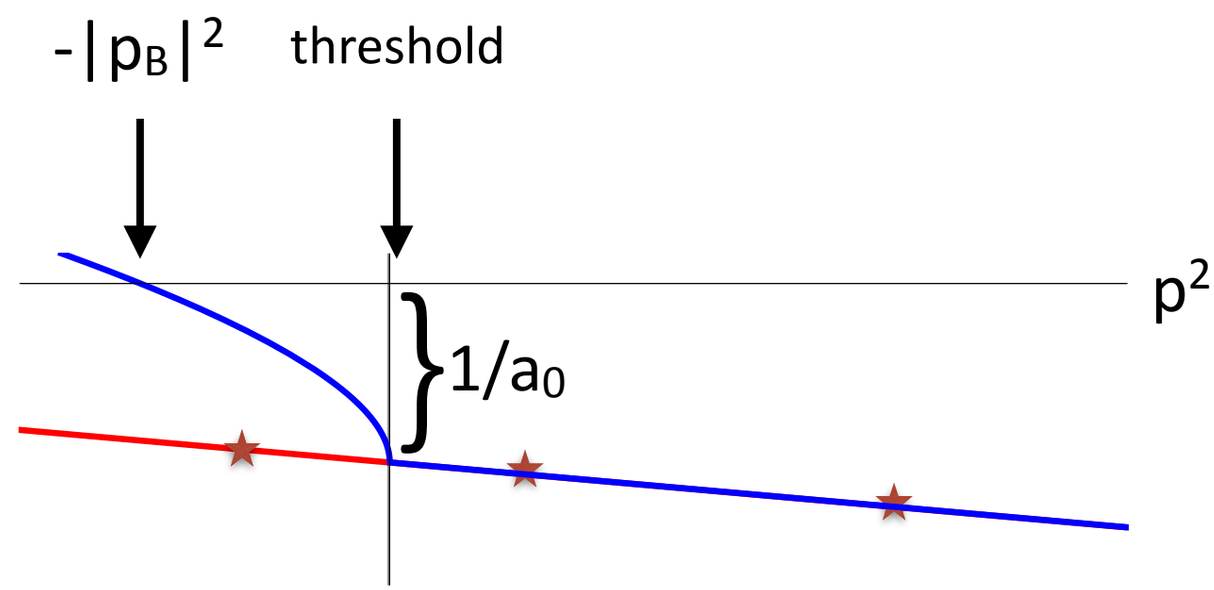
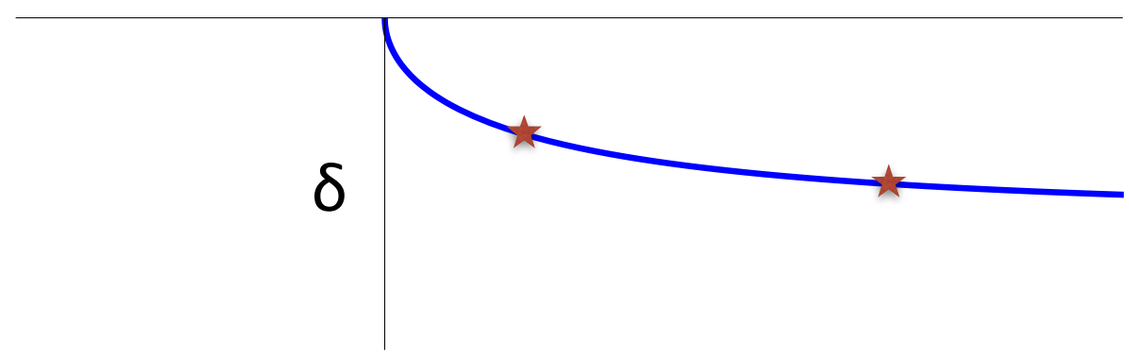
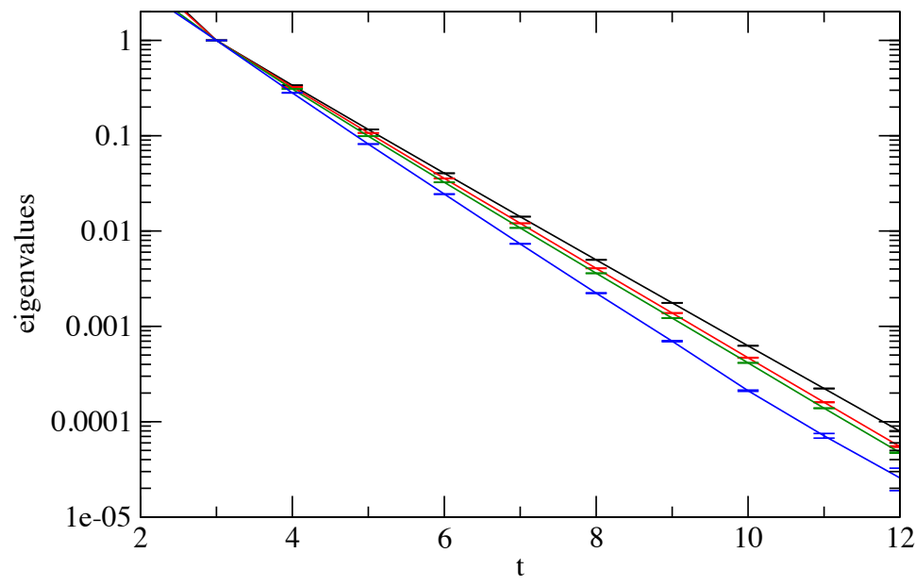
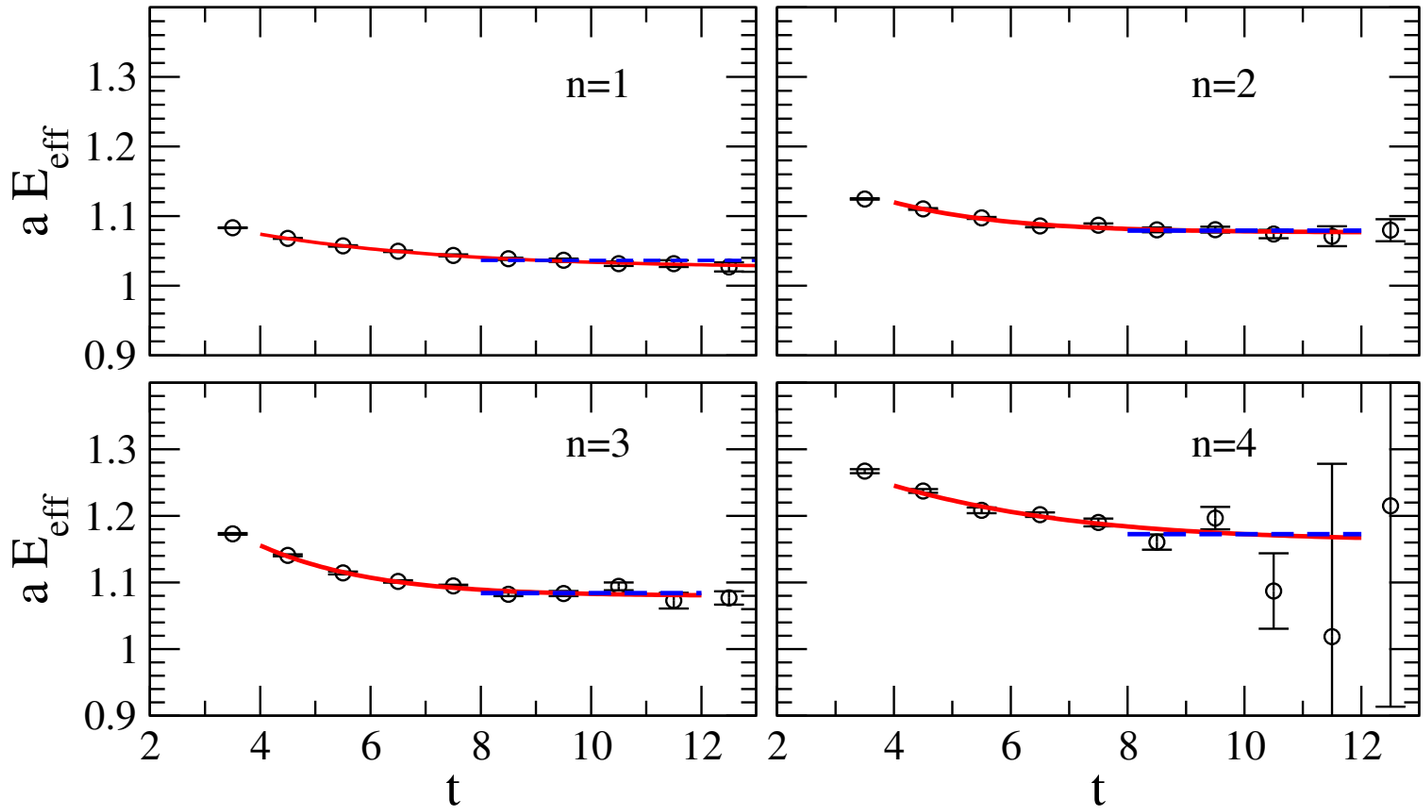




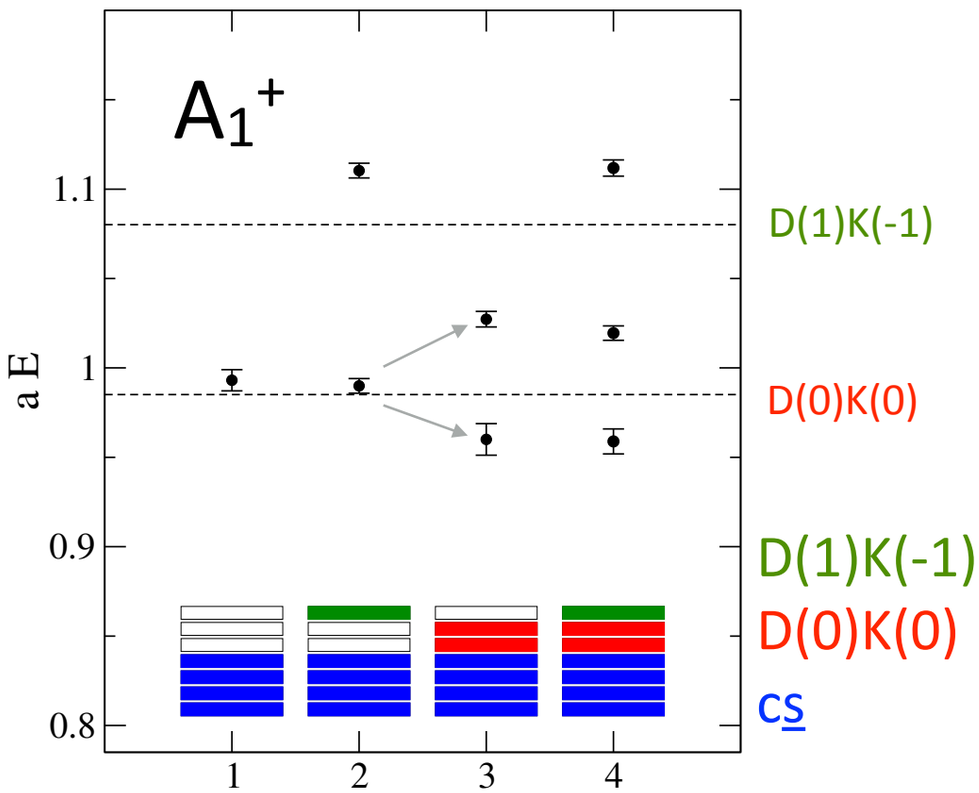
Eigenstates: example T_1^+, ensemble 2

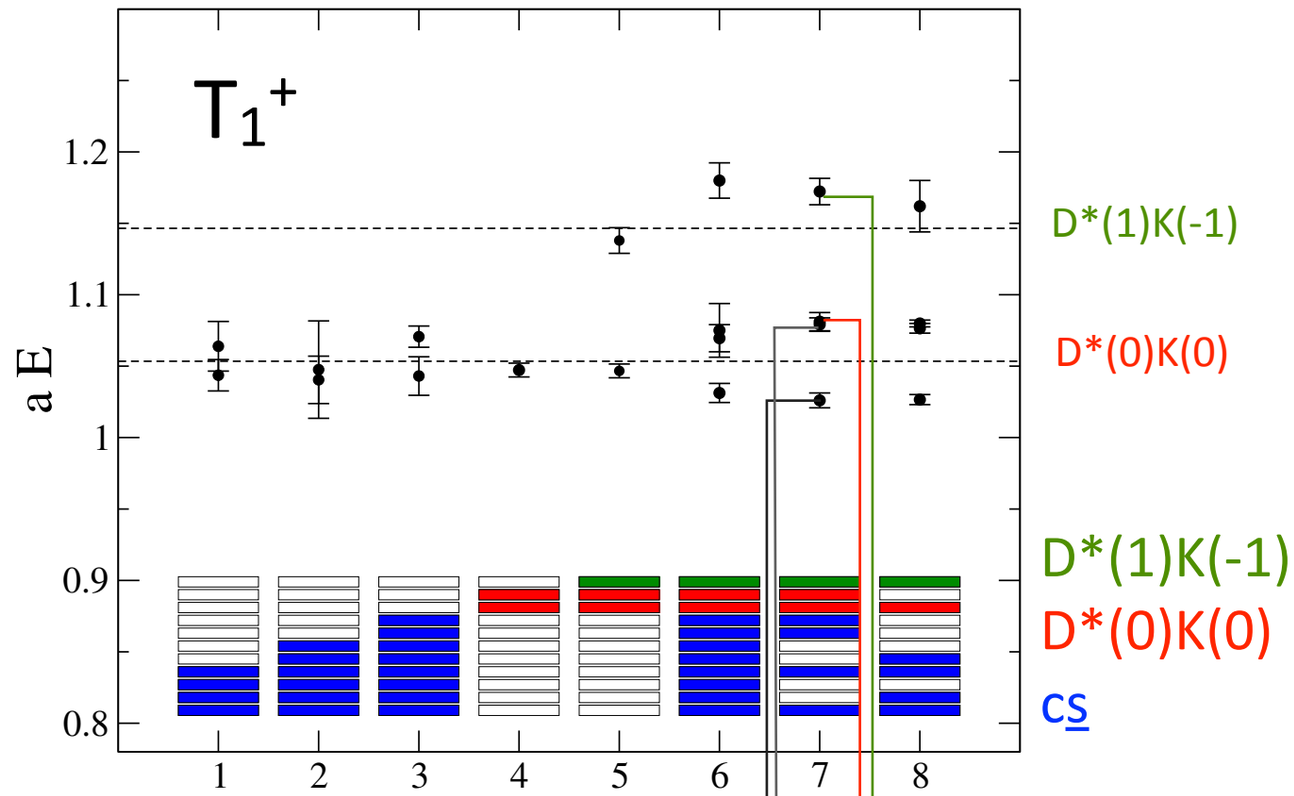


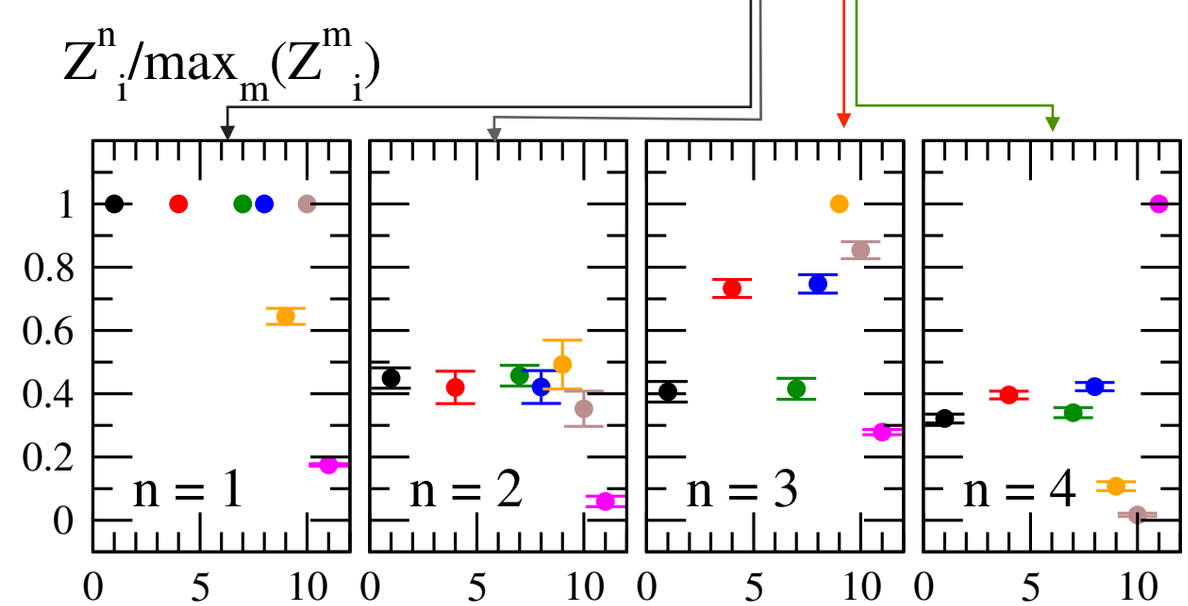


t_0=3
correlated
2-exp fit

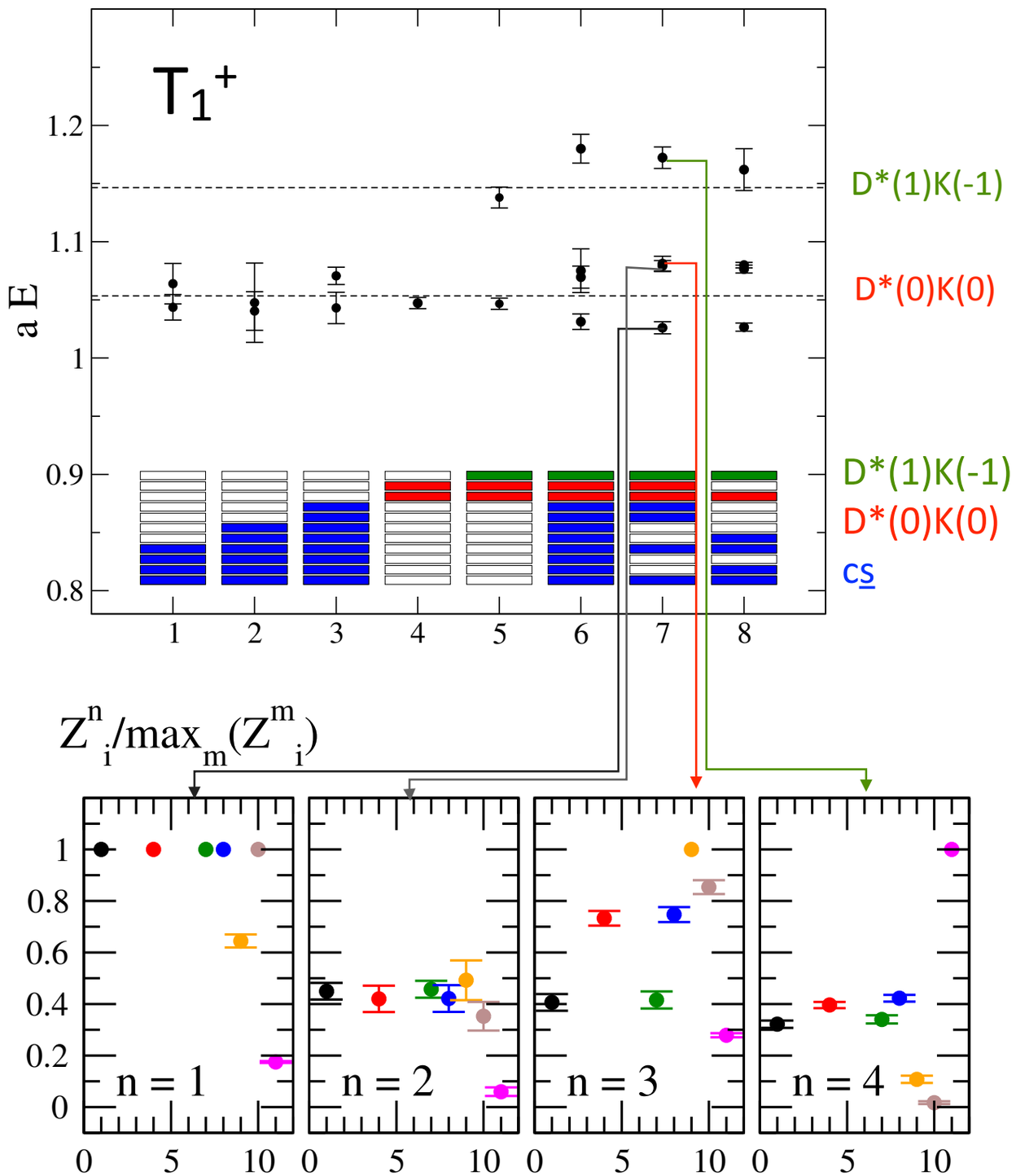
Composition of eigenstates (ensemble 2)







Identification of eigenstates (ensemble 2)



Energy levels

1.172(9)	$D^*(1)K(-1)$	shifted
1.081(6)	$D^*(0)K(0)$	shifted
1.079(5)	$D_{s1}(2536)$	couples in s-wave weakly to D*K
1.026(5)	$D_{s1}(2460)$	

Results T_1^+

$D_{s1}(2460)$

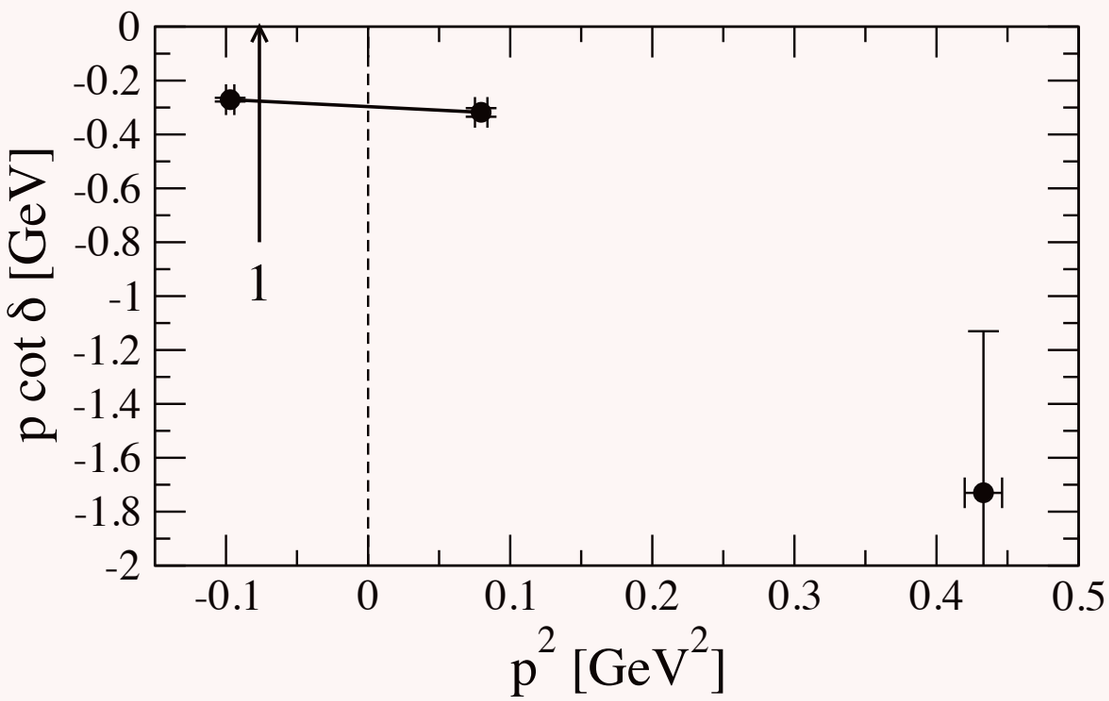
set	$a_0^{D^*K}$ [fm]	$r_0^{D^*K}$ [fm]	$m_K + m_{D^*} - m_B$ [MeV]	$m_B - \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$ [MeV]
Ensemble (1)				
	-0.665(25)	-0.106(37)	93.2(4.7)(1.0)	404.6(4.5)(4.2)
Ensemble (2)				
set 1	-1.15(19)	0.13(22)	43.2(13.8)(0.6)	408(13)(5.8)
set 2	-1.11(11)	0.10(10)	44.2(9.9)(0.6)	407.0(8.8)(5.8)
Experiment				
			44.7	383

$D_{s1}(2536)$

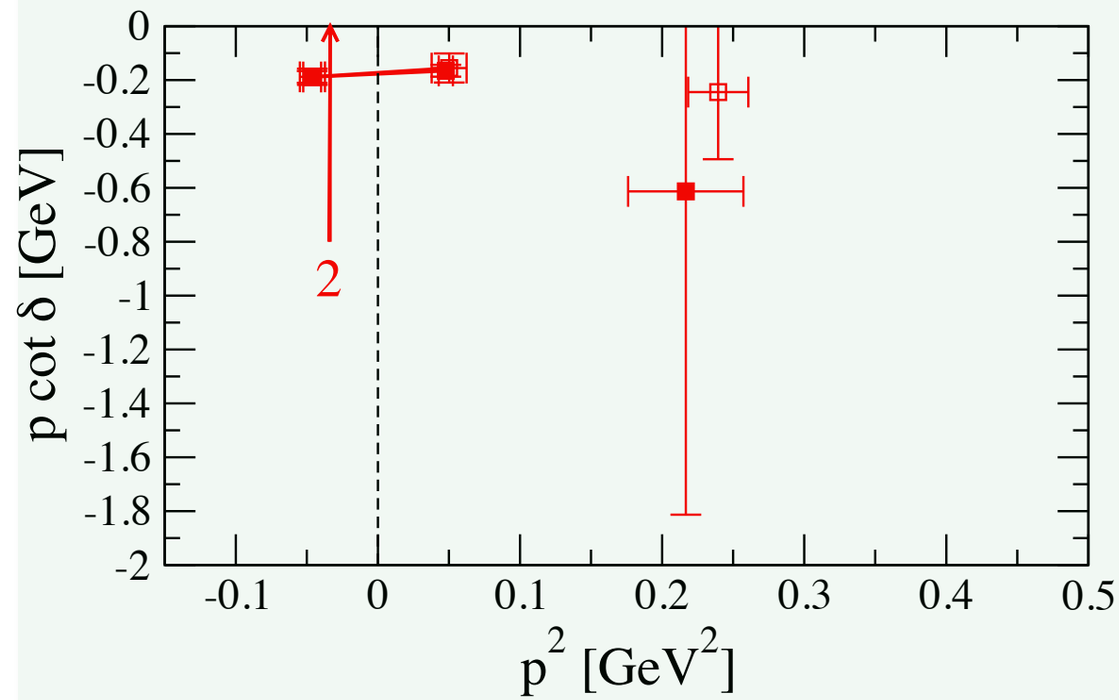
set	$m_{D_{s1}(2536)} - \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$ [MeV]	$m_{D_{s1}(2536)} - m_K - m_{D^*}$ [MeV]
Ensemble (1)		
	444(12)	-53(12)
Ensemble (2)		
set 1	507(10)	56(11)
set 2	501(8)	50(8)
Experiment		
	459	31

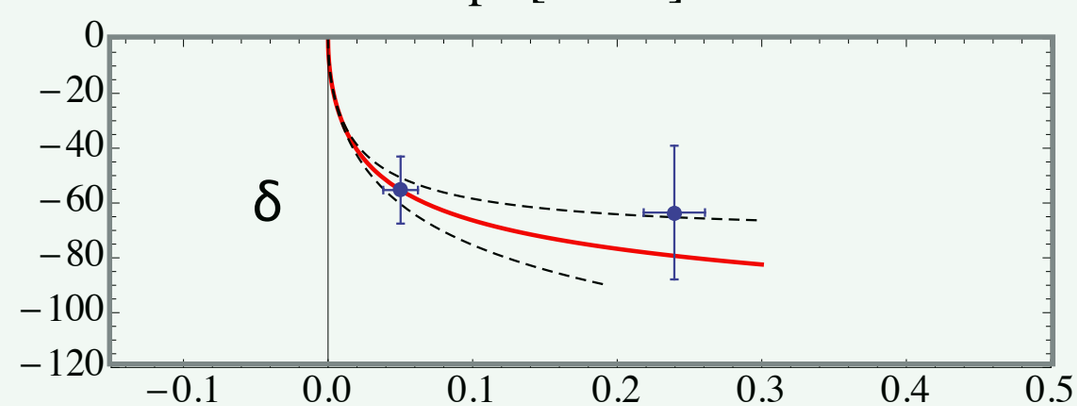
T_1^+ near threshold

Ensemble 1



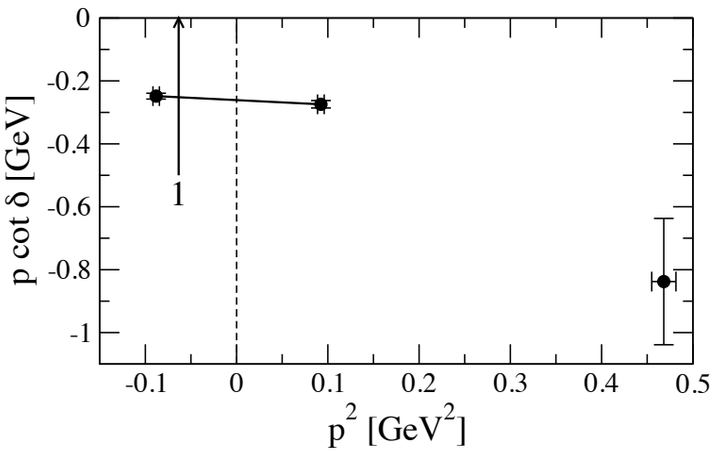
Ensemble 2

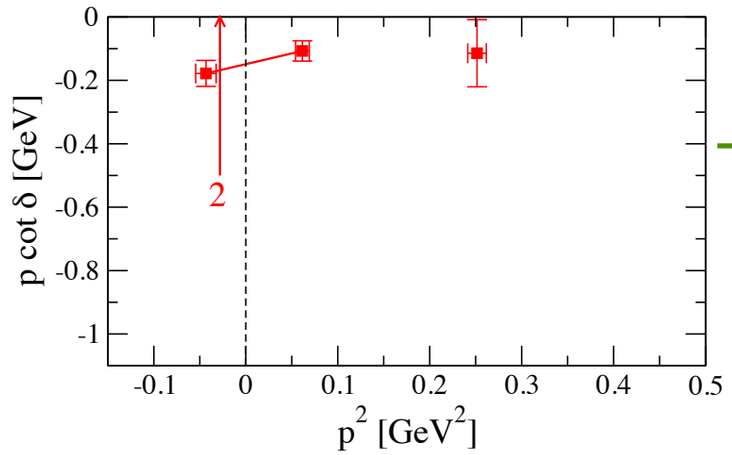




Results A_1+

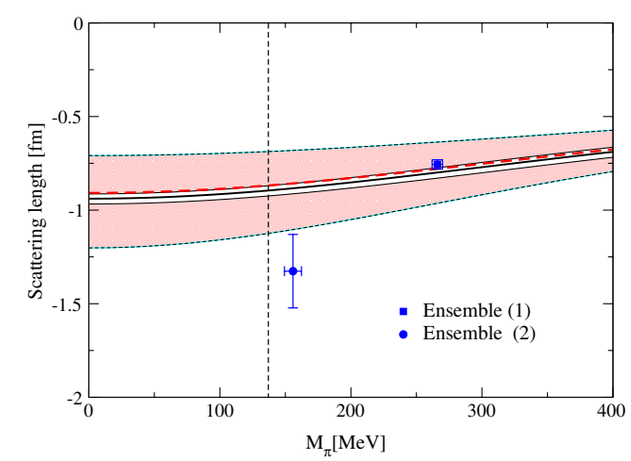
set	a_0^{DK} [fm]	r_0^{DK} [fm]	$D_{s0}^*(2317)$ $m_K + m_D - m_B$ [MeV]	$m_B - \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$ [MeV]
Ensemble (1)				
	-0.756(25)	-0.056(31)	78.9(5.4)(0.8)	287(5)(3)
Ensemble (2)				
	-1.33(20)	0.27(17)	36.6(16.6)(0.5)	266(17)(4)
Experiment				
			45.1	241.5





compatible with
Liu et al., PRD 87, 014508 (2013)

see also
Mohler et al., PRL 111, 222001 (2013)



Results T_2^+

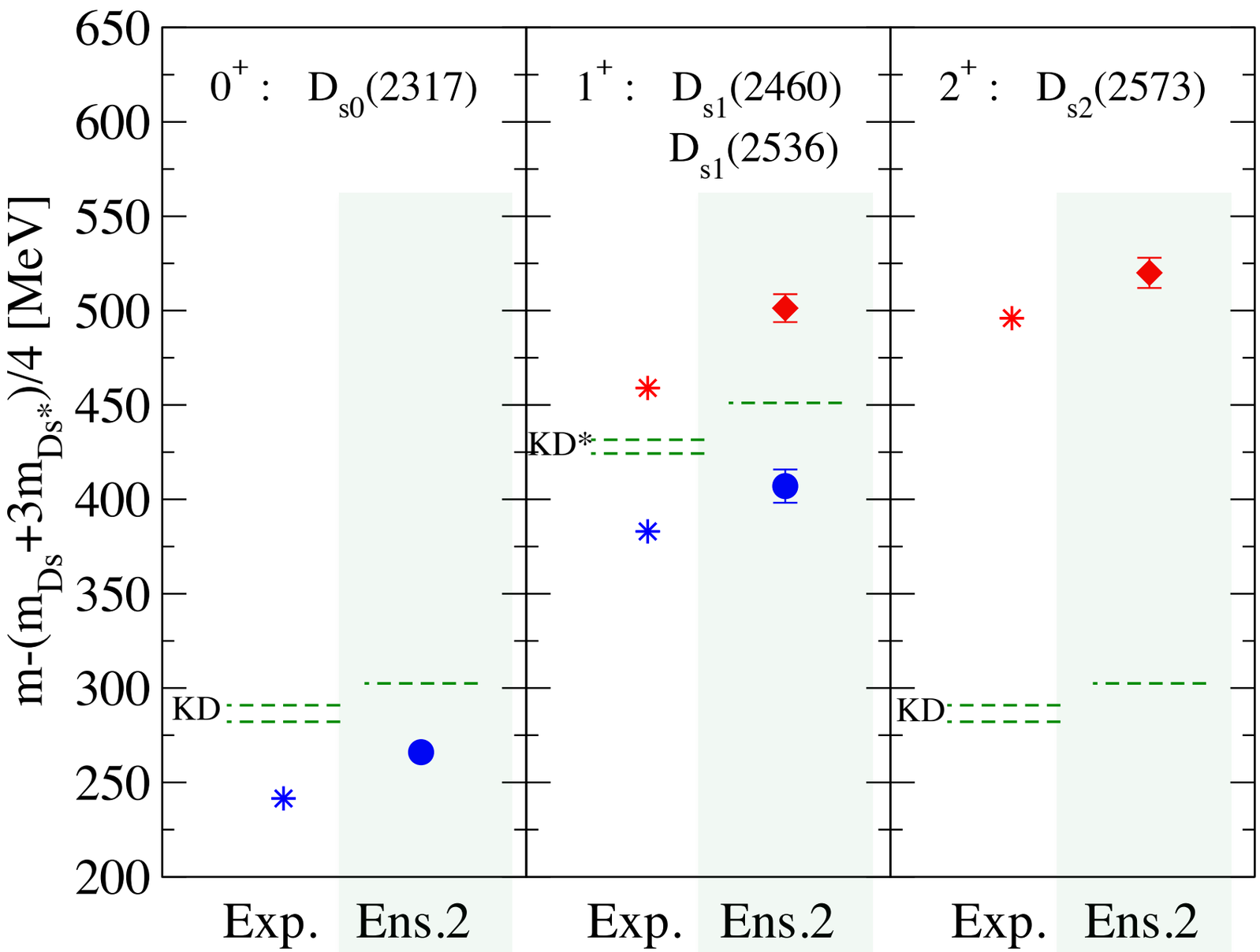
$D_{s2}^*(2573)$ ⟶ DK in d-wave

well described by $c\bar{s}$:

	$E-\bar{m}$
Ensemble 1	473(19)(5) MeV
Ensemble 2	520(8)(7) MeV
Experiment	496 MeV

C.B. Lang (2014)

Results summary



Thank you!