# RS2, AdS/CFT, and unparticle stuff

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Based on A.F., M. Giannotti, M. Graesser, arXiv: 0811xxx

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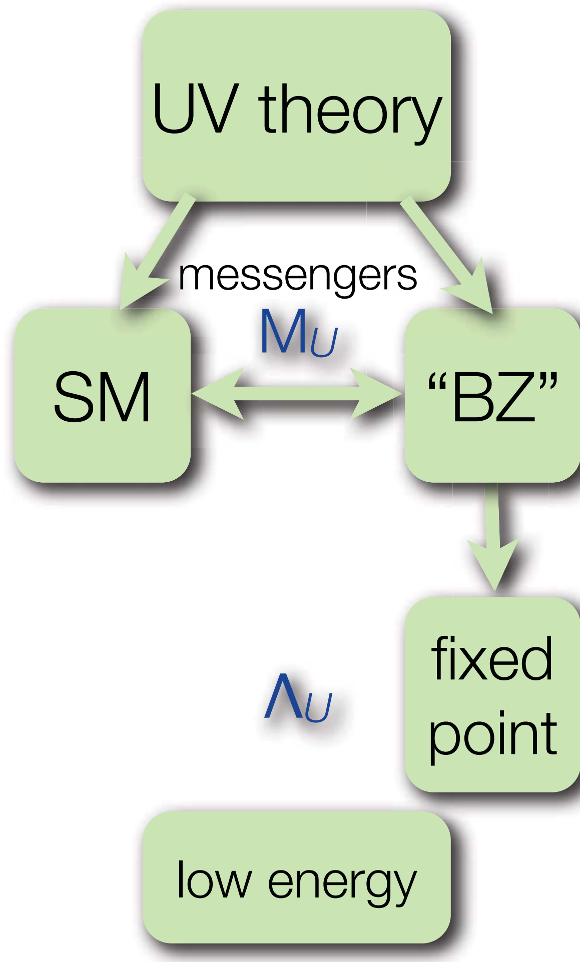
- Apologies for references: Georgi's unparticles have 200+ citations, RS 2 has 3000+

# Unparticles: alternative type of new physics

- Georgi I [hep-ph/0703260]
  - "Stuff with nontrivial scale invariance in the IR would be very unlike anything we have seen in our world.
  - ... could very well be a component of the physics above the TeV scale that will show up at the LHC.
  - ... would be a much more striking discovery than the more talked about [...] SUSY or extra dimensions, [which are] more new particles*.
  - ... would astonish us immediately."

*important footnote, to be discussed later.




$$\frac{c_0}{\Lambda^{d-1}} J^\mu_{SM} \mathcal{O}_\mu$$

# Unparticles, cont.

- Georgi II [arXiv: 0704.2457]
- Vector unparticle propagator
- Provides an additional channel for SM → SM scattering, i.e. $e^+e^- \to \mu^+\mu^-$
- Has a non-trivial phase,

  $$e^{-i\pi(d-2)}$$

  leading to interesting interference effects with SM
- Has a divergence at integer $d_U$, leading Georgi (and MANY subsequent authors) to consider

  $$1 < d_{\mathcal{U}} < 2$$

$$\int e^{iPx} \langle 0 | T(O^{\mu}_{\mathcal{U}}(x)\, O^{\nu}_{\mathcal{U}}(0)) | 0 \rangle \, d^4x$$
$$= i \frac{A_{d_{\mathcal{U}}}}{2\pi} \int_0^\infty (M^2)^{d_{\mathcal{U}}-2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{P^2 - M^2 + i\epsilon} \, dM^2$$
$$= i \frac{A_{d_{\mathcal{U}}}}{2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_{\mathcal{U}}\pi)} \left(-P^2 - i\epsilon\right)^{d_{\mathcal{U}}-2}$$

"I believe that this is a real effect. These integer values describe multiparticle cuts and the mathematics is telling us that we should not be trying to describe them with a single unparticle field."

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$$\frac{1}{(2\pi)^2}\frac{g_{\mu\nu}-2x_\mu x_\nu/x^2}{(x^2)^d}=\frac{(d-1)\Gamma(2-d)}{4^{d-1}\Gamma(d+1)}\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}(k^2)^{d-2}\left[g_{\mu\nu}-\frac{2(d-2)}{d-1}\frac{k_\mu k_\nu}{k^2}\right].$$

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This changes the rates of some processes, e.g. t → q + *U*

# Vector field in RS 2 background

- Instead of a Banks-Zaks type setup, let’s think about RS2 as an alternative realization of the unparticle scenario.

- As is well-known by the AdS/CFT correspondence, the RS 2 type theories should be connected to CFTs. As Georgi himself puts, they “can have unparticle-like behavior” [remember the important footnote on slide 3?]. We wish to clarify what exactly this “unparticle-like behavior” is.

- Simplest possible setup: a single massive vector field in the RS 2 background + SM fields on the brane. No strings, supersymmetry, complicated particle content, multiple branes, etc.

    - To paraphrase Witten [hep-th/9802150]: this is the case “where the most elegant statement is possible.”

# Unparticle-like behavior of vectors in RS 2

- At scales below the anti-deSitter (AdS) curvature, the effective theory is unparticle physics (CFT) *plus a set of contact interactions*.

- The contact interactions explicitly seen to dominate scattering amplitudes.

- The cancellations between the contact terms and the CFT at integer dimensions are trivially seen and reduce to the well-known properties of the Bessel functions.

- The unitarity bound on conf. dim. likewise easily follows, by considering the sign of the imaginary part of the longitudinal component of the propagator.

- At last, the correct CFT tensor structure is recovered once the longitudinal and transverse components of the propagator are combined.

# Brane-to-brane vector propagator in RS 2

- *Important*: impose Hartle-Hawking [Giddings,Katz,Randall, hep-th/0002091], also called radiative [Dubovsky,Rubakov, Tinyakov, hep-th/0006046] boundary conditions at $z \to \pm\infty$ (**outgoing waves from the brane**)

- *Important*: give the vector field bulk mass $m_5$. This break gauge symmetry, gives the longitudinal component, controls the conformal dimension via

$$d = 2 + \nu = 2 + \sqrt{1 + m_5^2/\kappa^2}$$

$$\begin{aligned}\Delta_{\rho\sigma}(p) &= \left(-\eta_{\rho\sigma} + \frac{p_\rho p_\sigma}{p^2}\right)\frac{1}{2}\left[p\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} - \kappa(\nu-1)\right]^{-1} \\ &- \frac{p_\rho p_\sigma}{p^2}\frac{1}{2m_5^2}\left[p\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} - \kappa(\nu+1)\right]\end{aligned}$$

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$$\Delta^{(T)}(p^2)$$

$$\begin{aligned}
\Delta_{\rho\sigma}(p) \quad &= \quad \left(-\eta_{\rho\sigma} + \frac{p_\rho p_\sigma}{p^2}\right)\frac{1}{2}\left[p\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} - \kappa(\nu-1)\right]^{-1} \\
&- \quad \frac{p_\rho p_\sigma}{p^2}\frac{1}{2m_5^2}\left[p\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} - \kappa(\nu+1)\right]
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&- \quad \frac{p_\rho p_\sigma}{p^2}\frac{1}{2m_5^2}\underbrace{\left[p\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} - \kappa(\nu+1)\right]}_{\Delta^{(L)}(p^2)}
\end{aligned}$$

# Flat space limit

- For large $p$, reduces to the brane-to-brane propagator in flat 5d space.

$$\Delta^{flat}_{\mu\nu}(p^2) = \int_{-\infty}^{\infty} \frac{dp_5}{2\pi} e^{ip_5 0} \frac{-\eta_{\mu\nu} + p_\mu p_\nu / m_5^2}{p^2 - m_5^2 - p_5^2 + i\epsilon}$$

$$-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m_5^2} = -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} + \frac{p_\mu p_\nu}{p^2} \frac{p^2 - m_5^2}{m_5^2}; \qquad \int_{-\infty}^{\infty} dp_5 (p^2 - m_5^2 - p_5^2 + i\epsilon)^{-1} = -i\pi\sqrt{p^2 - m_5^2 + i\epsilon}$$

$$\Delta^{flat}_{\mu\nu}(p^2) = \left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) \frac{1}{2} \frac{-i}{\sqrt{p^2 - m_5^2}} - \frac{p_\mu p_\nu}{p^2} \frac{i}{2m_5^2} \sqrt{p^2 - m_5^2}.$$

- Large $p$, fixed $\nu$ limit obvious, $pH_{\nu-1}(p)/H_\nu(p) \to ip$. To get nonzero $m_5$ also need a limit of large $m_5$ ($p$, $\nu >> \kappa$). Doable, $pH_{\nu-1}(p)/H_\nu(p) \to p \exp(i \arccos[m_5/p])$

- The cuts of the sqrt are physically important → continuum of KK modes escaping from the brane to $z$ infinity. Notice that both components are imaginary: all modes with $p^2 > m_5^2$ escape, no binding to the brane.

# Unitarity

$$\Delta_{\mu\nu}^{flat}(p^2) = \left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{2}\frac{-i}{\sqrt{p^2 - m_5^2}} - \frac{p_\mu p_\nu}{p^2}\frac{i}{2m_5^2}\sqrt{p^2 - m_5^2}.$$

- In the flat 5d limit, the **Im** parts of the components of the correlator must have certain signs: particles escape from the brane, not appear (Hartle-Hawking)! For the longitudinal component, this means $m_5^2 \geq 0$

$$\begin{aligned}\Delta_{\rho\sigma}(p) &= \left(-\eta_{\rho\sigma} + \frac{p_\rho p_\sigma}{p^2}\right)\frac{1}{2}\left[p\frac{H_{\nu-1}^{(1)}(x)}{H_\nu^{(1)}(x)} - \kappa(\nu - 1)\right]^{-1} \\ &- \frac{p_\rho p_\sigma}{p^2}\frac{1}{2m_5^2}\left[p\frac{H_{\nu-1}^{(1)}(x)}{H_\nu^{(1)}(x)} - \kappa(\nu + 1)\right]\end{aligned}$$

- Generalizes to curved space: to keep the right sign of the imaginary part of the longitudinal component, requires $m_5^2 \geq 0$. This means

$$d = 2 + \sqrt{1 + m_5^2/\kappa^2} \geq 3.$$

Saturates for conserved currents

# Expand in series

- Now consider the limit p << κ. Expand in series

$$
\begin{aligned}
\Delta^{(L)}(0,p) \;\simeq\; & \frac{\kappa}{2m_5^2}\Bigg[-(1+\nu)+\frac{(p/\kappa)^2}{2(\nu-1)}+\frac{(p/\kappa)^4}{8(\nu-1)^2(\nu-2)} \\
+\; & \frac{(p/\kappa)^6}{16(\nu-1)^3(\nu-2)(\nu-3)}+\frac{(5\nu-11)(p/\kappa)^8}{128(\nu-1)^4(\nu-2)^2(\nu-3)(\nu-4)}+\cdots \\
-\; & \frac{2\pi}{\Gamma(\nu)^2}(\cot\pi\nu-i)\left(\frac{p}{2\kappa}\right)^{2\nu}[1+\cdots]\Bigg].
\end{aligned}
$$

- Series of contact terms. Clearly dominates scattering for ν > 1.

- It takes longer to show that the last, nonanalytic term is relevant for some things. In fact, it represents the CFT (unparticle stuff)

# The CFT part

$$\begin{aligned}\Delta^{(L)}(0,p) \;\simeq\; & \frac{\kappa}{2m_5^2}\Bigg[-(1+\nu)+\frac{(p/\kappa)^2}{2(\nu-1)}+\frac{(p/\kappa)^4}{8(\nu-1)^2(\nu-2)} \\ & +\frac{(p/\kappa)^6}{16(\nu-1)^3(\nu-2)(\nu-3)}+\frac{(5\nu-11)(p/\kappa)^8}{128(\nu-1)^4(\nu-2)^2(\nu-3)(\nu-4)}+\cdots \\ & -\frac{2\pi}{\Gamma(\nu)^2}(\cot\pi\nu-i)\left(\frac{p}{2\kappa}\right)^{2\nu}[1+\cdots]\Bigg].\end{aligned}$$

- The non-analytic term has a cut and an imaginary part: continuum of states escaping from the brane into the bulk. (Notice that the phase is precisely that of Georgi, $-\pi\nu$!)

Dubovsky,Rubakov, Tinyakov, hep-th/0006046

- Cot $\pi\nu$ has poles at integer $\nu$. Observe the corresponding singularities on contact terms: the poles cancel, leaving physical log cut (cf. GIR). This is just the well-known property of the Bessel functions, which are finite and well behaved at integer or non-integer order. (p=0 is a branch point, hence the radius of convergence is zero. This is an asymptotic series.)

# Position space

- Schematically, $\Delta^{(L)}(p^2)$ has a part that is analytic on the real axis and a part that is nonanalytic.

- The Fourier transform to position space of the analytic part is exponentially cut off at large x (property of Fourier transforms). This gives a Yukawa-like interaction with the distance scale $\sim\kappa^{-1}$. Thus, the “contact terms” of GIR here are not really contact here, they are resolved. (The asymptotic series misses the nonpertubative exponential.)

- The Fourier transform of the nonanalytic part instead falls off as a power law, $\sim 1/(x^2)^d$. Light bulk modes dominate large distance interactions, creating a CFT.

# Position space propagator

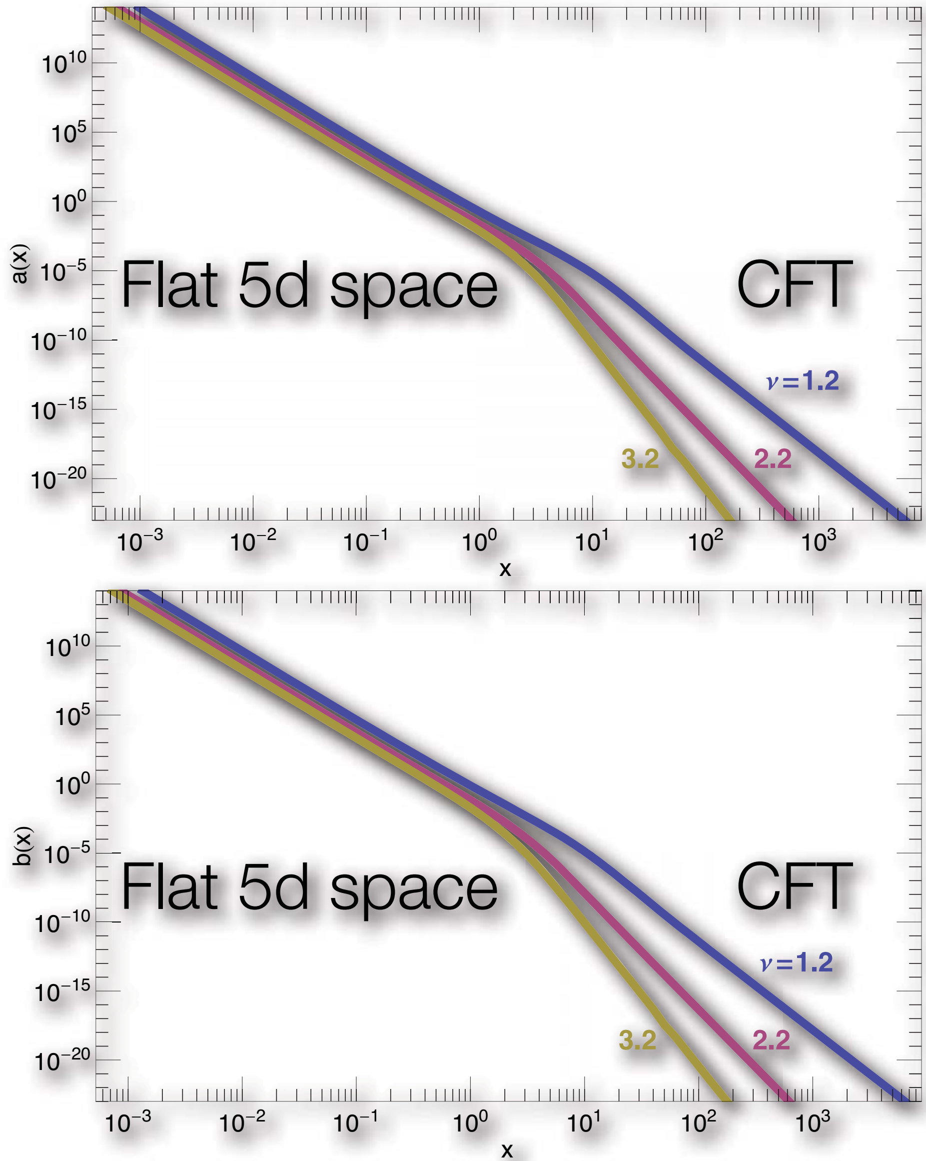
- Finally, Fourier transform the complete propagator to position space (Euclidean).

$$D_{ij}(x) = D_{ij}^{(T)}(x) + D_{ij}^{(L)}(x) = a(x)\ {}_{ij} + b(x)\frac{x_i x_j}{x^2}$$

- We see that **for the vector field case, the theory goes from the flat space behavior at short distances right into CFT at long distances**. There is not much happening in between.

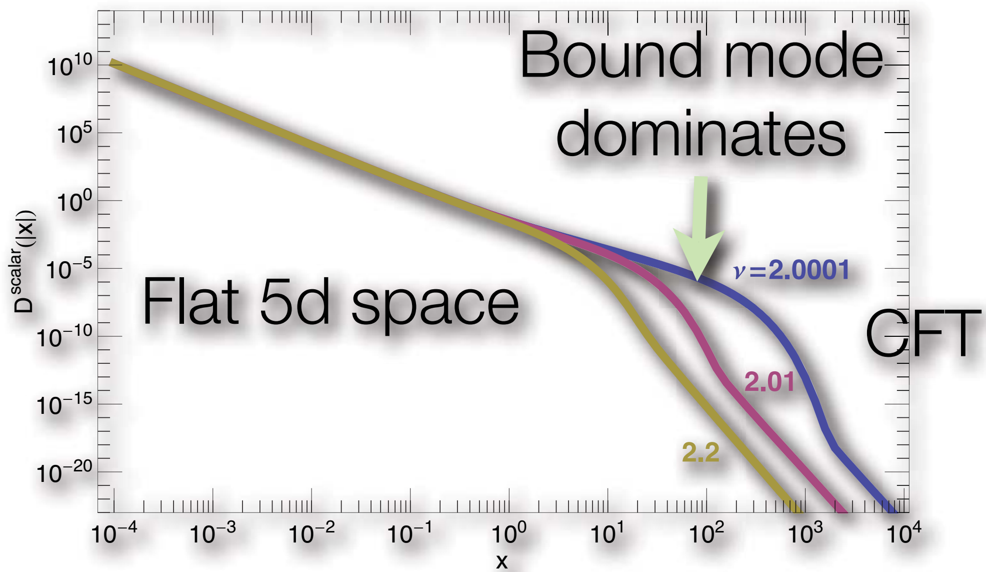
- In the CFT limit, we find the correct CFT tensor structure, $b(x)/a(x) = 2$.

Mück and Viswanathan, hep-th/0006046



# Compare to scalar in RS 2

- The transition from flat 5d to CFT does not have to be boring. Compare to the case of the scalar field, which has a mode bound to the brane that does not decouple as $m_5 \to 0$. There is a third regime, in which the theory looks 4-dimensional (cf. Rubakov, and others, *circa* 2000)



# Summary

- The RS 2 set up is a good realization of the unparticle scenario. All observations of Grinstein, Intriligator, Rothstein follow automatically and are extended beyond weak coupling. The arguments are embarrassingly simple!

- Practical advice to fellow phenomenologists: when in doubt, you may want to use the RS 2 realization of unparticles.

  - As an example, see A.F., M. Giannotti, *Astrophysical bounds on photons escaping into extra dimensions*, PRL 100 031602, (2008) -- unparticle paper without a single mention of “unparticles” ;-).

- Some of the properties of RS 2 described here don’t seem to be widely discussed in the literature.