Axions For QCD and Dark Matter

Edward Witten

NNN15/UD2 Simons Center October 29, 2015

Light spin zero fields – which I will just call scalar fields – have been proposed for solving a variety of problems in particle physics and cosmology. Here are four:

(1) The dilaton of inflationary cosmology is not light by usual particle physics standards, but it was probably very light compared to the mass scale at which inflation occurred.

(2) The QCD axion is possibly needed to explain why CP is conserved by strong interactions.

(3) "Fuzzy dark matter" is an attempt to account for indications that conventional dark matter works perfectly above ~ 1 *kpc* but not below.

(4) Finally, some "quintessence" theories interpret dark energy in terms of a very light scalar that has not yet settled into its ground date.

Of these matters, I will really mostly talk about (2) and (3) today, with a few words on the others.

The problem that motivates the QCD axion is probably familiar. The strong interactions could potentially violate CP because of a topological interaction

$$\frac{\theta}{32\pi^2} \int d^4x \; \epsilon^{\mu\nu\alpha\beta} \mathrm{Tr}\, F_{\mu\nu} F_{\alpha\beta}.$$

A priori, one would expect to observe this effect with $\theta \sim 1$, but the failure so far to observe a neutron electric dipole moment tells us that instead $\theta \lesssim 10^{-10}$. The strong CP problem is the problem of explaining why this is.

One explanation would be that the up quark mass is 0, but lattice gauge theory seems to show that this is not so. There are also technical solutions in which CP is a spontaneously broken symmetry. The model has to be constructed to make the effective value of the CP-violating angle θ very small, even though the CP-violating CKM angle is not small. This is possible but the constructions required are a little technical.

The most attractive alternative is the QCD "axion." This is a very light scalar field a that has an approximate shift symmetry

$$a \to a + \mathrm{constant}.$$

The shift symmetry is supposed to be violated primarily by a coupling to the topological density of QCD:

$$\frac{a}{32\pi^2}\int d^4x\ \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\, F_{\mu\nu}F_{\alpha\beta}.$$

In other words, a is just the θ-angle, now regarded as a dynamical field. The effective value of θ is just the value that minimizes the vacuum energy. If we ignore the explicit CP violation in weak interactions (which can be shown to have very small effects), this value is $\theta = 0$.

One question about the axion is how natural the idea is. Is it natural to have a symmetry that is broken only by instantons? One partial answer to this question is that string theory models, which were definitely not invented with the strong CP problem in mind, turn out to always have axions. By an axion, I mean in this context, a scalar a whose shift symmetry $a \to a + \mathrm{constant}$ is violated only by instantons of one kind or another. In string theory there are lots of instantons: QCD instantons, instantons in other gauge groups, worldsheet instantons, gravitational instantons, and so on.

In this generalized sense of what we mean by an axion, string models always have axions – scalar fields that are massless and decouple in the zero momentum limit except for instanton effects of one kind or another. All string models have at least one or two axions and many string models have dozens or even hundreds of them. A string model will solve the strong CP problem if one of its axion-like modes gets mass primarily (to within one part in about 10^{10}) from QCD instantons.

The self-couplings of an axion can be described by an effective action

$$I = \int d^4x\sqrt{g}\left(\frac{F^2}{2}g^{\mu\nu}\partial_\mu a\partial_\mu a - \mu^4(1-\cos a)\right).$$

In a string theory model, F is usually read off at string tree level, but μ is "exponentially small" – it has to be computed from an instanton effect of one kind or another. An important point is that in anything that I will call an "axion," there is an exact symmetry $a \to a + 2\pi$, so the potential energy is a periodic function of a. I have just written the lowest harmonic. (Higher order terms $\cos ka$, $\sin ka$, with $k > 1$, are normally negligible.) For future reference, note that the canonically normalized scalar field is not a but

$$\phi = Fa.$$

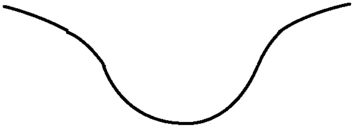
In many string theory models, the parameter F is roughly bounded above by the reduced Planck mass $M_{\rm Pl} \sim 2 \times 10^{18}$ GeV, and below by the traditional unification scale $M_G \sim 10^{16}$ GeV:

$$10^{18}\,\mathrm{GeV} \gtrsim F \gtrsim 10^{16}\ \mathrm{GeV}.$$

The upper bound is a problem for simple attempts to interpret one of the axions as the "inflaton" of inflationary cosmology. The problem is that with a symmetry $a \cong a + 2\pi$ and an upper bound on F, the canonically normalized scalar field $\phi = Fa$ cannot "roll" by more than $\sim F$, which if $F \lesssim M_{\mathrm{Pl}}$ is not quite enough for slow roll inflation. Because of this, string models of inflation follow a more roundabout path involving, for example, using axion monodromy (McAllister, Silverstein, and Westphal arXiv:0808.0706).

It appears that the upper bound on F is rather robust (see Banks, Dine, Fox, Gorbatov arXiv/0303252) except in situations in which μ is so large as to make a given axion irrelevant. This upper bound has been interpreted in terms of the idea that "gravity is the weakest force" (Arkani-Hamed, Motl, Nicolis, Vafa, hep-th/0601001).

The upper limit is also a problem for “axionic quintessence”:

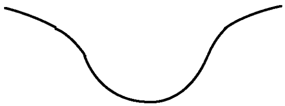


Basically an axion contributes to dark matter if it is already (in today’s universe) oscillating near the bottom of the potential, and it contributes to dark energy if its potential is so weak that it has not started to oscillate yet.

The upper bound $F \lesssim M_{\rm Pl}$ means that if a given axion has μ so small that that axion is not yet oscillating, then the potential $\mu^4(1 - \cos a)$ of this axion is too small to contribute more than a small part of the dark energy (Svrcek, hep-th/0607086). (I am oversimplifying this a little. Part of the story is that when F approaches the upper bound, μ becomes too large.)

For the QCD axion, a bigger problem is the lower bound $F \gtrsim M_G \sim 10^{16}$ GeV. Actually this bound is less robust than the upper bound. It can be avoided in some models with relatively large extra dimensions (Svrcek and EW, hep-th/0605206). However, if we want a model that retains the standard GUT relation between the strong, weak, and electromagnetic couplings, leading to $\sin^2\theta_W \sim .23$, then it seems very hard to avoid the lower bound.

The lower bound on F is a problem for cosmology because of a classic argument (Preskill, Wise, and Wilczek; Abbott and Sikivie; Dine and Fischler, all from 1983) which shows that a QCD axion with $F > 10^{12}$ GeV contributes too much dark matter. We will re-examine this argument later, but the basic idea is that in the early universe the axion a would start at a random value, because the physics that sets the initial value of the axion angle doesn't "know" about the details of the quark mass matrix which determine what value of a would minimize the energy. So we start at a random point on the curve



potentially contributing too much dark energy/matter.

The axion is frozen at its initial value as long as the Hubble constant H exceeds the axion mass m. At $H \sim m$, the axion field begins to oscillate arond its minimum energy state. The oscillations behave as a form of dark matter, and are thinned out as $1/R^3$ in the usual way. But if the axion potential is too strong (i.e. the axion mass is too big) we potentially end up with too much dark matter.

To avoid this for a QCD axion, we need $F \lesssim 10^{12}$ GeV. The smaller value of F increases the axion mass, and causes the axion field to start oscillating and redshifting away sooner. There is also a solid lower bound $F \gtrsim 10^9$ GeV from cooling of red giants. So standard cosmological assumptions place the QCD axion in the range $10^9\ \mathrm{GeV} \lesssim F \lesssim 10^{12}\ \mathrm{GeV}$. The ADMX experiment (which relies on the $a\gamma\gamma$ coupling and looks at axion-to-photon conversion in the presence of a magnetic field) is in the process of exploring this region.

Can cosmology be reconciled with a QCD axion at the value $\sim M_G$ that is most suggested by string theory? There have been a few suggestions. "Late" decay of moduli or other massive particles, injecting energy into the universe and diluting the axions, can reduce the axion share of dark matter. This mechanism can lift the upper bound on F, almost all the way to the GUT scale. Another suggestion is that, for anthropic reasons, the assumption of a random initial value for a is not realistic. Maybe a takes different values in different parts of the universe (in fact, in inflationary cosmology, one would expect this) and we can only live where the axionic dark matter is not too large. (For example, see Hertzberg, Tegmark, and Wilczek, arXiv:0807.1726.) Another suggestion is that the axion was heavier in the early universe so the usual cosmological analysis does not apply.

The ADMX hard experiment can find axionic dark matter in the usual cosmological window $10^{12}\,\mathrm{GeV} > F > 10^{9}\,\mathrm{GeV}$, but it is hard for this experiment to go much above 10^{12} GeV. For years, there were very few ideas about how to detect axions above this window or even in this window if they do not make up the dark matter. However, in the last couple of years, there have been some new ideas about how to do that.

The CASPEr experiment (Graham and Rajendran, arXiv:1306.6088) can detect QCD axions for F around the GUT scale and even above, assuming that they make up dark matter. It has two variants. CASPEr-Electric relies on the axion-gluon-gluon coupling and looks for a time-dependent electric dipole moment of the neutron. CASPEr-Wind looks at a direct axion-quark coupling

$$\frac{1}{F}\partial_\mu a\,\bar{q}\gamma^\mu\gamma_5 q$$

whose effect is that as the Earth moves relative to the axion field of the dark matter, an atomic nucleus precesses around the direction ∇a (as if this were a contribution to the magnetic field).

There is also a proposal (Arvanitaki and Geraci, arXiv:1403.1290) for a precision magnetometry experiment that can discover axions in and somewhat above the range where ADMX searches, but *without* having to assume that axions make up the dark matter. The idea is to look at the axionic force between a rotating mass and the spins in an NMR sample.

Finally, there is an astronomical method to study axions, by looking at spindown and superradiance of black holes. It is claimed that the non-observation of this effect already gives an upper bound on the QCD axion of $F < 10^{17}\,\mathrm{GeV}$ (Arvanitaki, Dimopoulos, et. al., arXiv:0905.4720; Arvanitaki and Dubovsky, arXiv:1004.3558).

In this discussion, I have assumed that the candidate QCD axion that one sees at tree level in any string theory model is the right one. This assumption forces us to either give up on Grand Unification (in the sense that we cannot maintain the usual GUT relation for $\sin^2\theta_W$) or else to take $F \gtrsim 10^{16}$ GeV. (In fact, if we look more closely, we find that F has to be pretty close to the lower bound or else there are non-QCD effects that violate the shift symmetry too strongly.) If we go down the second road, we need to modify standard cosmological assumptions in some way.

Is there an alternative? Instead of getting the QCD axion directly from the string, could we generate it at lower energies? Maybe, but it is actually a hard thing to do, because the demands on a QCD axion are very strong. If we don't take what the string seems to give us, we are back to not understanding whether/why it is natural to have a shift symmetry that is broken just by instanton effects.

So far, I have told you about several possible applications of string theory axions for which either the standard upper bound or the standard lower bound

$$10^{18}\,\mathrm{GeV} \gtrsim F \gtrsim 10^{16}\,\mathrm{GeV}$$

is problematical. However, there is one proposal for which this range is precisely what one would want. This is the idea of “fuzzy dark matter” (Hu, Barkana, and Gruzinov, astro-ph/0003365). (I have been a particle physics consulatant in a reassessment of fuzzy dark mattter: Hui, Ostriker, Tremaine and EW, to appear.)

The problem that fuzzy dark matter (FDM) aims to solve is that conventional Cold Dark Matter (CDM) works very well on length scales above about 1 kiloparsec, but it possibly has problems on smaller scales. For the most part, the status of these problems is not completely clear and certainly there might be alternative explanations. For example, dark matter haloes are not as "cuspy" as expected at the galactic center, and there are not as many mini-haloes (corresponding to dwarf galaxies) as suggested by some simulations. The FDM proposal is that the breakdown of CDM occurs because conventional CDM does not take into account the de Broglie wavelength of the dark matter particle.

To make this work, one wants a Compton wavelength of the FDM particle to be about .1 parsec, so that in a dwarf galaxy ($v/c \sim 10^{-4}$) the de Broglie wavelength is about 1 kiloparsec ($= 3 \times 10^3$ light years).

This means that the FDM particle has to have a mass $m \sim 10^{-22}\,\mathrm{eV}$. Of course, it has to be a boson because fermi statistics would prevent a fermion of that mass from having an interesting energy density. Is it reasonable to have a boson of that mass?

To decide if it is reasonable for an axion, we have to discuss the second parameter μ in the axion action:

$$I = \int d^4x\sqrt{g}\left(\frac{F^2}{2}g^{\mu\nu}\partial_\mu a \partial_\mu a - \mu^4(1-\cos a)\right).$$

Of course, this parameter determines the axion mass

$$m = \frac{\mu^2}{F}.$$

A rough estimate is

$$\mu^4 \sim F^2\Lambda^2 e^{-S}$$

where S is the action of the instanton that gives the axion its mass, and Λ is a parameter that measures possible suppression due to supersymmetry. Λ is highly uncertain:

$$10^4\,\mathrm{GeV} \lesssim \Lambda \lesssim 10^{18}\,\mathrm{GeV}.$$

Given the uncertainty in Λ, to get $m = 10^{-22}\, eV$, we want $S \cong 200 \pm 30$. Actually this is a perfectly reasonable range. In many simple models, one finds an axion with S close to $2\pi/\alpha_G$ (where α_G is the Standard Model coupling near the GUT scale). If we take this formula literally and use $\alpha_G \approx 1/25$, we get $S \approx 157$. That is only a rough estimate with plenty of uncertainty, but it does show that it is not surprising to get an axion mass of 10^{-22} eV.

An axion is not the only natural way I know to get a boson with an exponentially small mass (which might turn out to be near 10^{-22} eV), but it is the only way I know in which one can also get the necessary dark matter density. Here is an idea that does not work. We can assume the existence of another gauge group beyond the Standard Model that is asymptotically free but more weakly coupled than QCD. At exponentially small energies, such a gauge force can become strong. If this happens at $10^{-22}\,eV$, we could definitely get bosons of that mass. But they could not behave as galactic dark matter; at an energy density of $.6\ \mathrm{GeV}/\mathrm{cm}^3$ (the local dark matter density near the Earth), the description by hadronic bound states with mass $\sim 10^{-22}\ \mathrm{eV}$ would not be useful and one would have instead a relativistic plasma built from the "partons" of the new group.

If we do have an axion with $S \sim 200$ and $m \sim 10^{-22}\,\mathrm{eV}$ making up the dark matter, how much dark matter would it make? This has been estimated by various authors (e.g. Arvanitaki et al, 2009; Kim and Marsh, 2015). As discussed before, we start the axion with a random initial value in the early Universe and hence an energy density $\sim \mu^4$. The axion field is frozen at its initial value until the Hubble constant H is $\sim m$. This happens at a temperature T_0 such that

$$m \sim \frac{T_0^2}{M_{\mathrm{Pl}}}.$$

At this point, the dark matter density is of order μ^4 and the radiation density is of order T_0^4. Subsequently, axions behave as massive particles whose density is redshifted in the usual way: the ratio of dark matter to radiation grows as $1/T$.

The temperature of matter-radiation equality in our universe is roughly $T_1 \sim 1 \text{ eV}$. So we want

$$\frac{\mu^4}{T_0^4}\frac{T_0}{T_1} \sim 1,$$

which with $\mu^2 = mF$ leads to

$$F \sim \frac{1}{2} \times 10^{17}\,\text{GeV}.$$

Finally an application that requires an axion with F in the most natural range! The temperature T_0 at which the FDM axion begins to oscillate is

$$T_0 \sim 500 \text{ eV}.$$

If dark matter is indeed of the fuzzy variety, can we observe it otherwise than by observing the gravitational effects of dark matter? According to Arvanitaki et. al. (2009), a scalar field with mass $10^{-22}\,\mathrm{eV}$ might have an observable effect in superradiance from supermassive black holes. But in the long run, there is at least some hope of direct detection here on Earth, since it has been suggested (Kim and Marsh, 2015; P. Graham, private communication) that the CASPEr-Wind variant of the CASPEr experiment might ultimately be able to detect axionic dark matter at $\sim 10^{-22}\,\mathrm{eV}$.

What are the problems with FDM axions? Indirectly, there is a possible problem involving the QCD axion. We'd like, or at least I'd like, to hope that the strong CP problem is solved by a GUT-scale axion. But then some modification of standard cosmology is needed, and we can worry that it might ruin the estimate of the FDM dark matter abundance. However, this is hopefully not inevitable.

We should worry about the other string theory axions. Perhaps one of them has $S \sim 200$ and is the FDM axion, but what about the others? Axions with S greater than that of the FDM axion (so that they are even lighter than $10^{-22}\,\mathrm{eV}$) are not a problem. They just make small contributions to dark matter and/or dark energy. Axions with $S \lesssim 50$ and so mass $\geq 100\ \mathrm{TeV}$ may also not be a problem, as they may decay rapidly enough to cause no difficulty in cosmology. There is a dangerous window with $50 \leq S \leq 200$. We might have been tempted to say that maybe in the right model that describes our universe, there just are not any axions in that range of S. But this is in tension with wanting to put the QCD axion in that range.

There is also another potential problem, but we do not yet know if it is a problem. If tensor modes are observed in cosmology, i.e. if the tensor to scalar ratio r is measured to be nonzero, then this is a serious problem for a GUT-scale QCD axion, and it is also a problem, though less severe, for an FDM axion. The reason for the problem is that inflationary perturbations should affect the axion as well as the inflaton, but independent inflationary perturbations in two different scalar fields lead to isothermal (rather than adiabatic) cosmological perturbations. Of course we know that the observed perturbations are primarily adiabatic, so there is a bound on the axion fluctuations. This is one problem that one would love to have.