Perturbative Uncertainties in Jet Bins

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May 10, 2011

Preliminary Remarks

The following discusses the *perturbative uncertainties* in the theory predictions of the exclusive jet cross sections at truth level, $\boldsymbol{\sigma_i^{\mathrm{true}}}$

- Formally they enter when the measured cross sections at truth level are compared to the SM predictions
- We have not yet discussed PDF+α_s uncertainties for the exclusive jet cross sections

The *experimental systematic uncertainty* due to the mismatch between truth-level jets and detector-level jets is a separate uncertainty. It enters

- either when the measured jet cross sections are corrected from detector level to truth level
- or equivalently the theory predictions are corrected from truth level to detector level. In this case the expected number of events in jet bin i is

$$\boldsymbol{N_i = \mathcal{L} \times \sigma_i^{\mathrm{true}} \times \varepsilon^{\mathrm{det}}}$$

where $\boldsymbol{\varepsilon^{\mathrm{det}}}$ contains the experimental efficiency, resolution, etc.

Perturbative Structure of Jet Cross Sections

$$\sigma_{\text{total}} = \underbrace{\int_0^{p_T^{\text{cut}}} \mathrm{d}p_T \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}}_{\sigma_0(p_T^{\text{cut}})} + \underbrace{\int_{p_T^{\text{cut}}}^{\infty} \mathrm{d}p_T \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}}_{\sigma_{\geq 1}(p_T^{\text{cut}})}$$

$$\begin{aligned}
\sigma_{\text{total}} &= 1 + \alpha_s + \alpha_s^2 + \cdots \\
\sigma_{\geq 1}(p_T^{\text{cut}}) &= \alpha_s (L^2 + L) + \alpha_s^2 (L^4 + L^3 + L^2 + L) + \cdots \\
\sigma_0(p_T^{\text{cut}}) &= \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}}) \\
&= \left[1 + \alpha_s + \alpha_s^2 + \cdots\right] - \left[\alpha_s (L^2 + L) + \alpha_s^2 (L^4 + \cdots) + \cdots\right]
\end{aligned}$$

where $L^2 = 2 \ln^2(p_T^{\text{cut}}/m_H)$ or $L^2 = \ln^2(\mathcal{T}^{\text{cut}}/m_H)$

- Perturbative series in σ_{total} and $\sigma_{\geq 1}(p_T^{\text{cut}})$ have different structures and are unrelated
- Apparent small uncertainties in $\sigma_0(p_T^{\text{cut}})$ arise from cancellation between two series with large corrections

Perturbative Uncertainties in Jet Bins

There is general agreement among theorists that one should hence treat the fixed-order perturbative series for σ_{total}, $\sigma_{\geq 1}$, $\sigma_{\geq 2}$ as independent with uncorrelated perturbative uncertainties, i.e.

- The *inclusive* jet cross sections are considered uncorrelated

$$\sigma_{\mathrm{total}},\ \sigma_{\geq 1},\ \sigma_{\geq 2} \quad \Rightarrow \quad C = \begin{pmatrix} \Delta^2_{\mathrm{total}} & 0 & 0 \\ 0 & \Delta^2_{\geq 1} & 0 \\ 0 & 0 & \Delta^2_{\geq 2} \end{pmatrix}$$

- The covariance matrix for the *exclusive* jet cross sections follows from

$$\sigma_0 = \sigma_{\mathrm{total}} - \sigma_{\geq 1}\,, \qquad \sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}\,, \qquad \sigma_{\geq 2}$$

$$\Rightarrow \quad C = \begin{pmatrix} \Delta^2_{\mathrm{total}} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} & 0 \\ -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} + \Delta^2_{\geq 2} & -\Delta^2_{\geq 2} \\ 0 & -\Delta^2_{\geq 2} & \Delta^2_{\geq 2} \end{pmatrix}$$

Perturbative Uncertainties For Jet Fractions

Equivalently, one can also transform to the exclusive jet fractions

$$f_0 = \frac{\sigma_0}{\sigma_{\rm total}} = 1 - \frac{\sigma_{\geq 1}}{\sigma_{\rm total}}\,, \quad f_1 = \frac{\sigma_1}{\sigma_{\rm total}} = \frac{\sigma_{\geq 1} - \sigma_{\geq 2}}{\sigma_{\rm total}}\,, \quad f_2 = \frac{\sigma_{\geq 2}}{\sigma_{\rm total}}$$

- The resulting covariance matrix for $\{\sigma_{\rm total}, f_0, f_1, f_2\}$ is

$$C_f = \frac{\Delta^2_{\rm total}}{\sigma^2_{\rm total}} \times \begin{pmatrix} \sigma^2_{\rm total} & \cdots & \cdots & \cdots \\ \sigma_{\rm total}(f_1+f_2) & \frac{\Delta^2_{\geq 1}}{\Delta^2_{\rm total}} + (f_1+f_2)^2 & \cdots & \cdots \\ -\sigma_{\rm total} f_1 & -\frac{\Delta^2_{\geq 1}}{\Delta^2_{\rm total}} - (f_1+f_2) f_1 & \frac{\Delta^2_{\geq 1}+\Delta^2_{\geq 2}}{\Delta^2_{\rm total}} + f_1^2 & \cdots \\ -\sigma_{\rm total} f_2 & -(f_1+f_2) f_2 & -\frac{\Delta^2_{\geq 2}}{\Delta^2_{\rm total}} + f_1 f_2 & \frac{\Delta^2_{\geq 2}}{\Delta^2_{\rm total}} + f_2^2 \end{pmatrix}$$

- Since $f_0 + f_1 + f_2 = 1$, C_f satisfies $\Delta(f_0 + f_1 + f_2) = 0$ exactly
- It would be simpler to just use $f_{\geq 1} = 1 - f_0 = f_1 + f_2$ and $f_{\geq 2} \equiv f_2$

Practical Implementation

In practice, the following inputs are used

- $\sigma_i = f_i \times \sigma_{\text{total}}$ with
  - f_i: MC (strictly speaking at truth level)
  - σ_{total}: Yellow Report (equivalent to HNNLO/FEHiP)
- $\delta\sigma_{\geq i} \equiv \dfrac{\Delta\sigma_{\geq i}}{\sigma_{\geq i}}$ from fixed order codes with
  - $\delta\sigma_{\geq 0} = \delta\sigma_{\text{total}}$: Yellow Report (equivalent to HNNLO/FEHiP)
  - $\delta\sigma_{\geq 1}$: HNNLO/FEHiP, or MCFM (are all identical)
  - $\delta\sigma_{\geq 2}$: HNNLO/FEHiP give LO (can later use MCFM for NLO)

Practical Implementation Continued

From above inputs we can get the full covariance matrix for $\{\sigma_0, \sigma_1, \sigma_{\geq 2}\}$:

- Using
$$\Delta\sigma_{\geq i} = \big[\delta\sigma_{\geq i}\big]_{\mathrm{FO}} \times \big[f_{\geq i}\big]_{\mathrm{MC}} \times \big[\sigma_{\mathrm{total}}\big]_{\mathrm{YR}}$$
  and plugging back into C on slide 3, one gets

$$C = \sigma_{\mathrm{total}}^2 \times \begin{pmatrix} \delta\sigma_{\mathrm{total}}^2 + (f_1+f_2)^2\,\delta\sigma_{\geq 1}^2 & -(f_1+f_2)^2\,\delta\sigma_{\geq 1}^2 & 0 \\ -(f_1+f_2)^2\,\delta\sigma_{\geq 1}^2 & (f_1+f_2)^2\,\delta\sigma_{\geq 1}^2 + f_2^2\,\delta\sigma_{\geq 2}^2 & -f_2^2\,\delta\sigma_{\geq 2}^2 \\ 0 & -f_2^2\,\delta\sigma_{\geq 2}^2 & f_2^2\,\delta\sigma_{\geq 2}^2 \end{pmatrix}$$

- Similarly, one can write C_f on slide 4 in terms of f_i and $\delta\sigma_{\geq 2}$

(Note: The relations $\sigma_0 = \sigma_{\mathrm{total}} - \sigma_{\geq 1}$ and $\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$ are in the end only used to correctly propagate the perturbative uncertainties.)

Numerical Example

From HNNLO (numbers from Jianming's slides for $p_T^{\mathrm{cut}} = 30\,\mathrm{GeV}$, $\eta^{\mathrm{cut}} = 3$):

	$(\mu_F/m_H,\ \mu_R/m_H)$								
	(0.5, 0.5)	(0.5, 1.0)	(0.5, 2.0)	(1.0, 0.5)	(1.0, 1.0)	(1.0, 2.0)	(2.0, 0.5)	(2.0, 1.0)	(2.0, 2.0)
	Cross sections in 0, 1 and 2-jet bin								
σ_0	30.1	28.5	26.6	30.1	28.6	26.8	30.3	28.6	27.0
σ_1	11.5	10.2	8.86	11.6	10.2	8.77	11.7	10.1	8.60
σ_2	3.95	2.64	1.84	3.57	2.39	1.66	3.24	2.17	1.51

	$\mu = 2m_H$	$\mu = m_H$	$\mu = m_H/2$	$\sigma \pm \Delta\sigma$	$\delta\sigma$
σ_{total}	37.11	41.19	45.55	41.33 ± 4.22	10.2%
$\sigma_{\geq 1}$	10.11	12.59	15.45	12.78 ± 2.67	20.9%
$\sigma_{\geq 2}$	1.51	2.39	3.95	2.73 ± 1.22	44.7%

- For simplicity, I've set $\mu_f = \mu_r \equiv \mu$ and symmetrized the central values and uncertainties
- In practice one should vary $\mu = \{m_H, m_H/4\}$ due to better convergence of perturbative series with central value at $\mu = m_H/2$.

Numerical Example Continued

- So we have

$$\delta\sigma_{\rm total} = 10.2\% \,, \qquad \delta\sigma_{\geq 1} = 20.9\% \,, \qquad \delta\sigma_{\geq 2} = 44.7\%$$

- Since I don't have the f_i from MC, *for illustration only* I will use those from HNNLO from the previous slide:

$$f_0 = 0.691 \,, \qquad f_1 = 0.243 \,, \qquad f_2 = 0.066$$

- Plugging above inputs into covariance matrix for $\{\sigma_0, \sigma_1, \sigma_{\geq 2}\}$ on slide 6 one gets

$$\delta\sigma_0 = 17.5\% \,, \qquad \delta\sigma_1 = 29.2\% \,, \qquad \delta\sigma_{\geq 2} = 44.7\% \,,$$

Correlation $$\widehat{C} = \begin{pmatrix} 1 & -0.49 & 0 \\ -0.49 & 1 & -0.42 \\ 0 & -0.42 & 1 \end{pmatrix}$$

Numerical Example Continued

Similarly, the covariance matrix for $\{\sigma_{\mathrm{total}}, f_0, f_1, f_2\}$ (slide 4) yields

$$\delta\sigma_{\mathrm{total}} = 10.2\%\,,$$

$$\delta f_0 = 10.4\%\,, \qquad \delta f_1 = 31.0\%\,, \qquad \delta f_2 = 45.8\%\,,$$

$$\widehat{C} = \begin{pmatrix} 1 & 0.44 & -0.33 & -0.22 \\ 0.44 & 1 & -0.92 & -0.10 \\ -0.33 & -0.92 & 1 & -0.31 \\ -0.22 & -0.10 & -0.31 & 1 \end{pmatrix}$$

- Using the results above to compute $\delta\sigma_i$ from $\sigma_i = f_i \times \sigma_{\mathrm{total}}$ exactly reproduces the results on the previous slide (as it should).

Remarks/Questions on Use of MC

Remember $N_i = \mathcal{L} \times \sigma_i^{\mathrm{true}} \times \varepsilon^{\mathrm{det}}$

When getting f_i from MC at truth level

- Computing $\sigma_i^{\mathrm{true}} = f_i \times \sigma_{\mathrm{total}}$ one does a theory calculation of σ_i^{true} by combining the NNLO for σ_{total} with a parton-shower resummation + hadronization for the "shape".
- For the uncertainties, one effectively translates the perturbative uncertainties from a fixed-order calculation of σ_i^{true} to this MC calculation

$\Rightarrow$ Should be ok in the short term, can be improved in the long term by using a resummed calculation (e.g. to reweight the MC truth)

When getting f_i^{det} from MC at detector level, f_i^{det} contains a piece of $\varepsilon^{\mathrm{det}}$

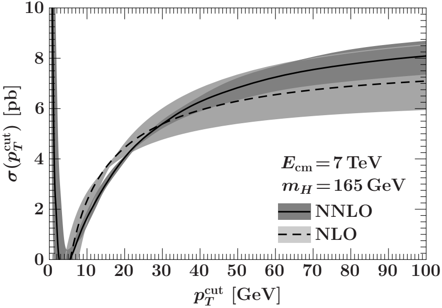
- Now $\sigma_i^{\mathrm{det}} = f_i^{\mathrm{det}} \times \sigma_{\mathrm{total}}$ but the pert. uncertainties apply to σ_i^{true}
- Using f_i^{det} in the theory uncertainties applies them to σ_i^{det} instead

$\Rightarrow$ This might not matter numerically, but seems conceptually incorrect? (It's like applying theory uncertainties by reweighting MC at detector level instead of truth level)

Some Plots

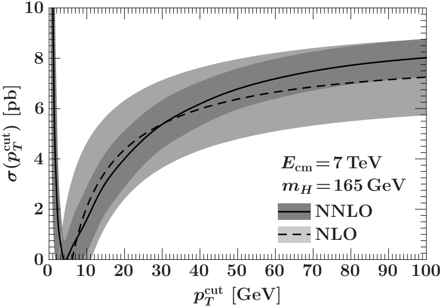
Fixed-Order Scale Uncertainties

Using naive scale variation for σ_0



$\Longrightarrow$

Using above procedure for σ_0

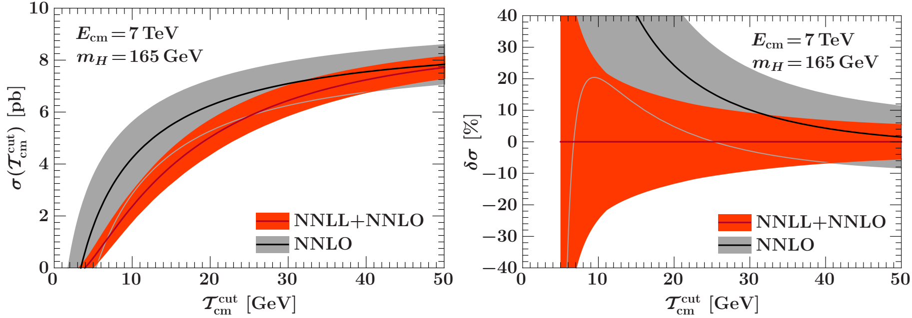


New procedure

- Uncertainties reproduce naive scale variation at large cut values
- Larger uncertainties at small cut values

  $\longrightarrow$ Now explicitly take into account large logarithmic corrections

Comparison to Resummation



cut	order	$\delta\sigma_{\mathrm{total}}$	$\delta\sigma_{\geq 1}$	$\delta\sigma_0$
$\mathcal{T}_{cm}^{cut} = 20$ GeV	NNLO	8.5%	28%	16%
$\mathcal{T}_{cm}^{cut} = 20$ GeV	NNLL+NNLO	5.2%	21%	13%

- NNLO uncertainties now consistent with those from NNLL+NNLO resummation